Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/166-6.2.2-e-x-^m-a+b-x^n-^p-cosh

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [111]. This is test number [166].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (111)	0.00 (0)
Mathematica	100.00 (111)	0.00 (0)
Maple	100.00 (111)	0.00 (0)
Fricas	100.00 (111)	0.00 (0)
Giac	63.96 (71)	36.04 (40)
Maxima	57.66 (64)	42.34 (47)
Sympy	23.42 (26)	76.58 (85)
Mupad	18.02 (20)	81.98 (91)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

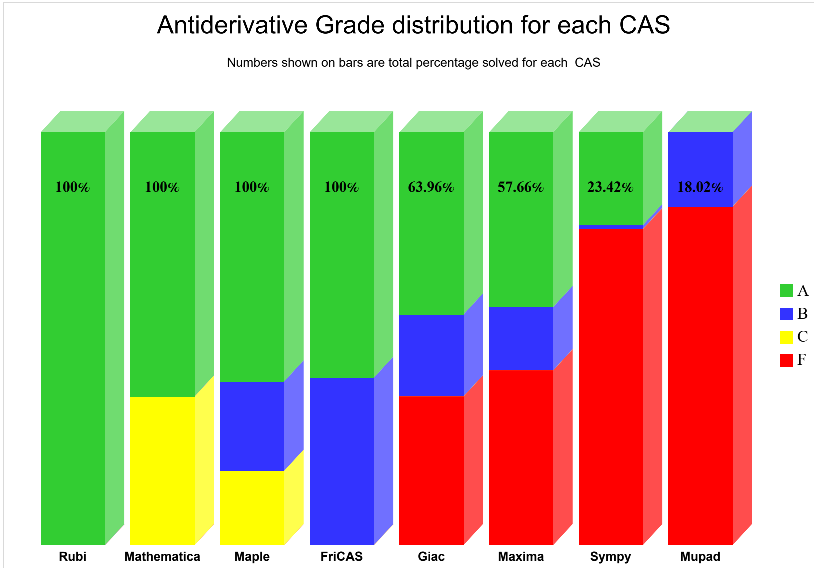
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

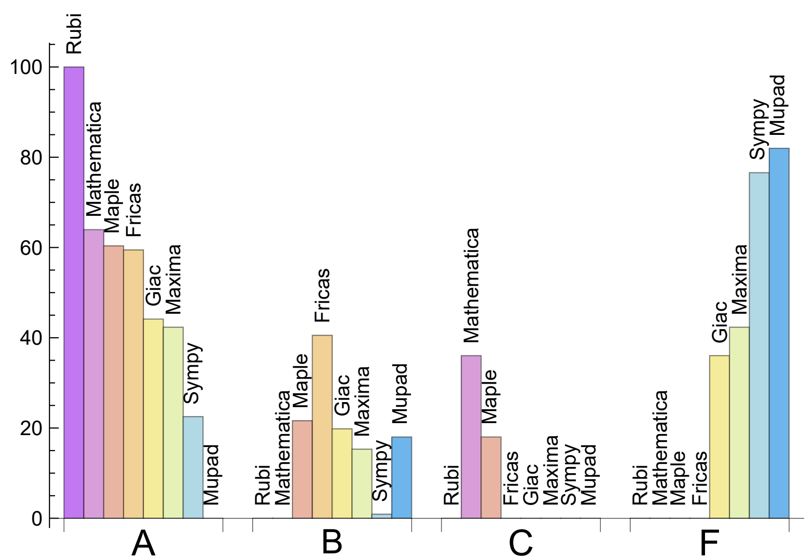
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	63.964	0.000	36.036	0.000
Maple	60.360	21.622	18.018	0.000
Fricas	59.459	40.541	0.000	0.000
Giac	44.144	19.820	0.000	36.036
Maxima	42.342	15.315	0.000	42.342
Sympy	22.523	0.901	0.000	76.577
Mupad	0.000	18.018	0.000	81.982

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Giac	40	92.50	0.00	7.50
Maxima	47	87.23	12.77	0.00
Sympy	85	69.41	30.59	0.00
Mupad	91	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Maple	0.25
Fricas	0.26
Giac	0.30
Rubi	0.41
Mathematica	0.63
Sympy	0.69
Mupad	1.18

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	118.50	0.99	115.00	0.97
Sympy	139.15	1.28	127.50	1.23
Maxima	183.64	1.66	170.00	1.71
Mathematica	192.48	0.81	140.00	0.78
Rubi	268.55	1.00	178.00	1.00
Giac	398.31	2.63	199.00	1.66
Maple	515.00	1.72	259.00	1.60
Fricas	652.41	1.82	200.00	1.68

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

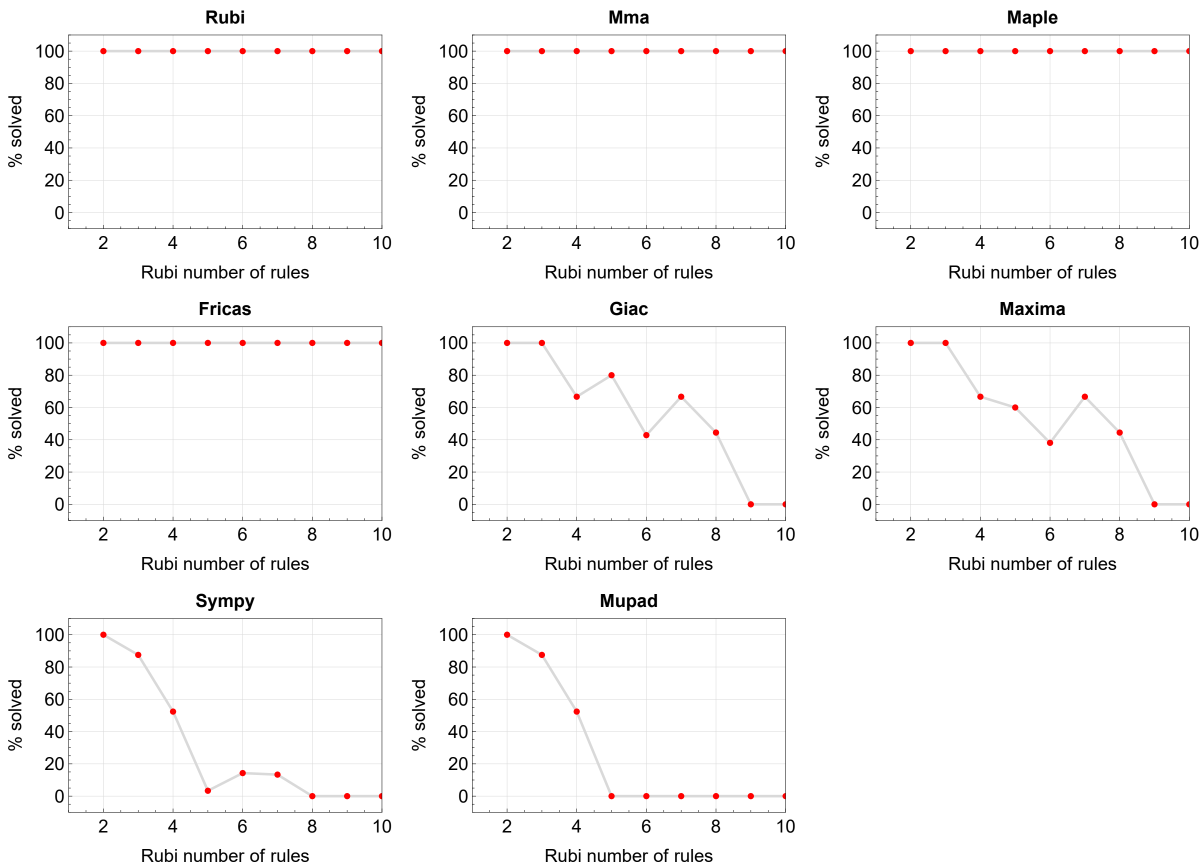


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

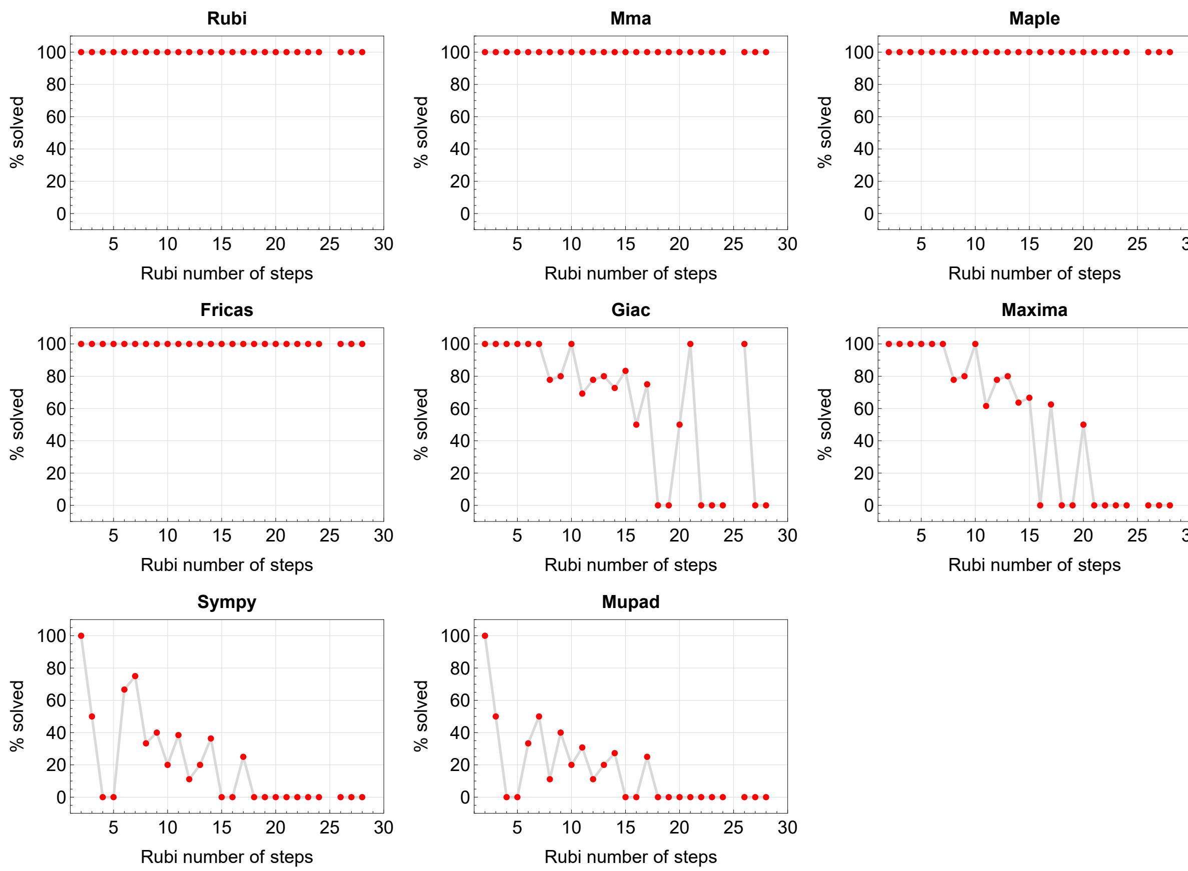


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

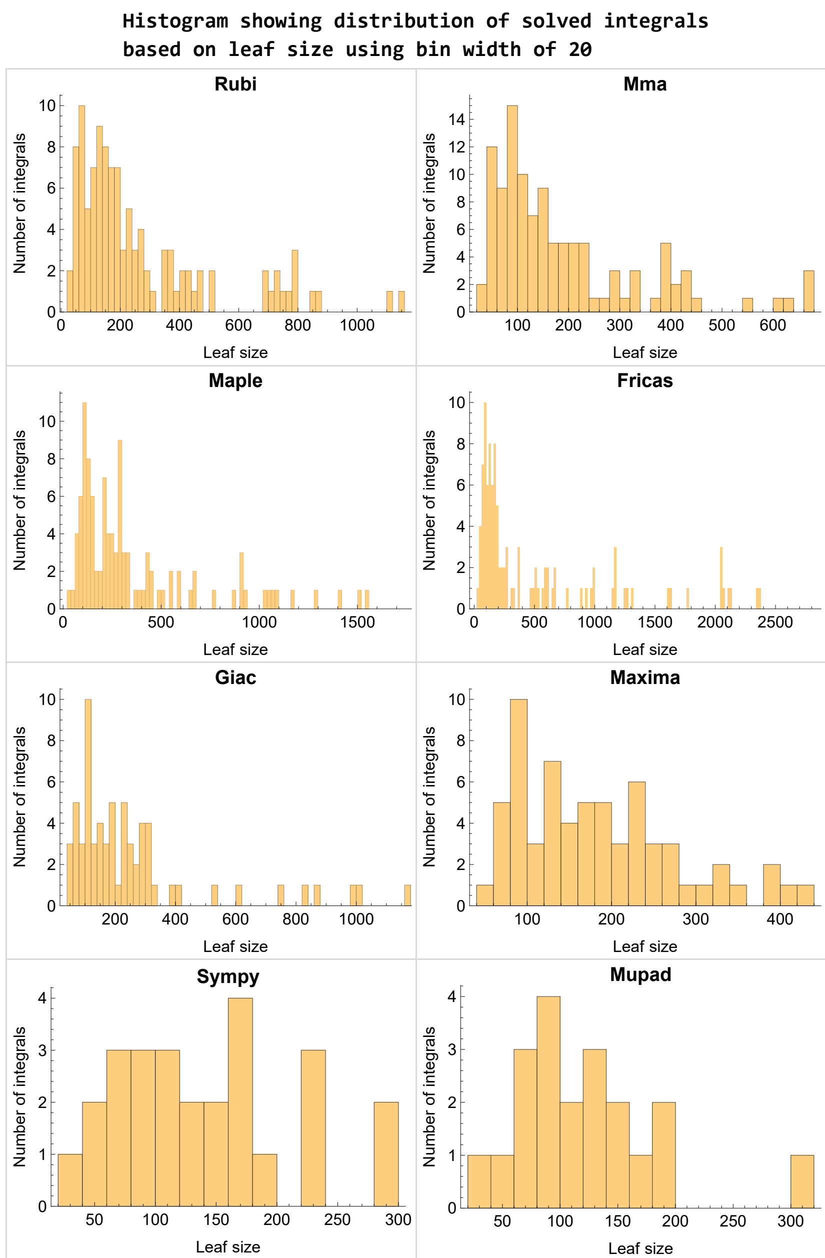


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

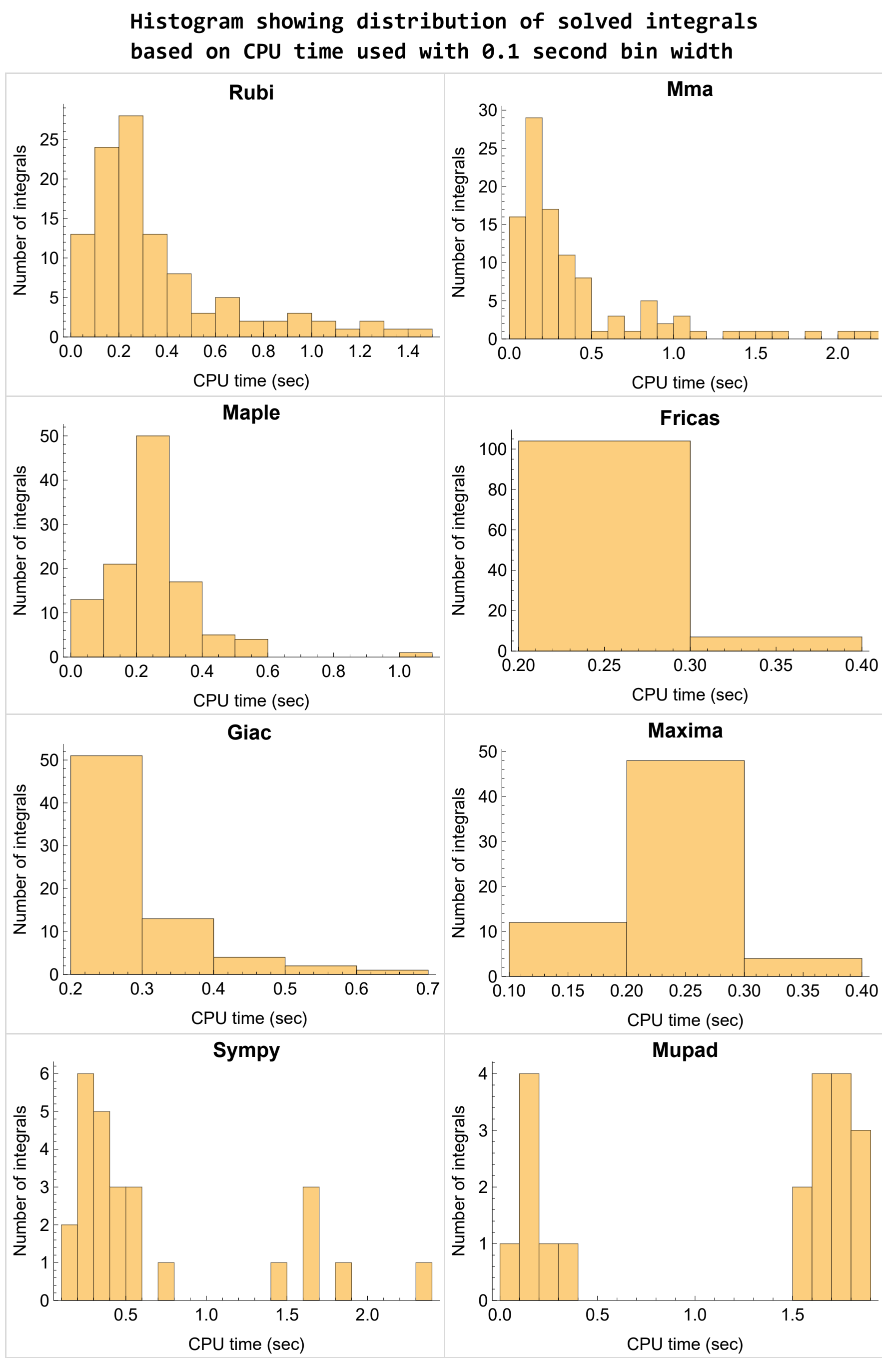


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

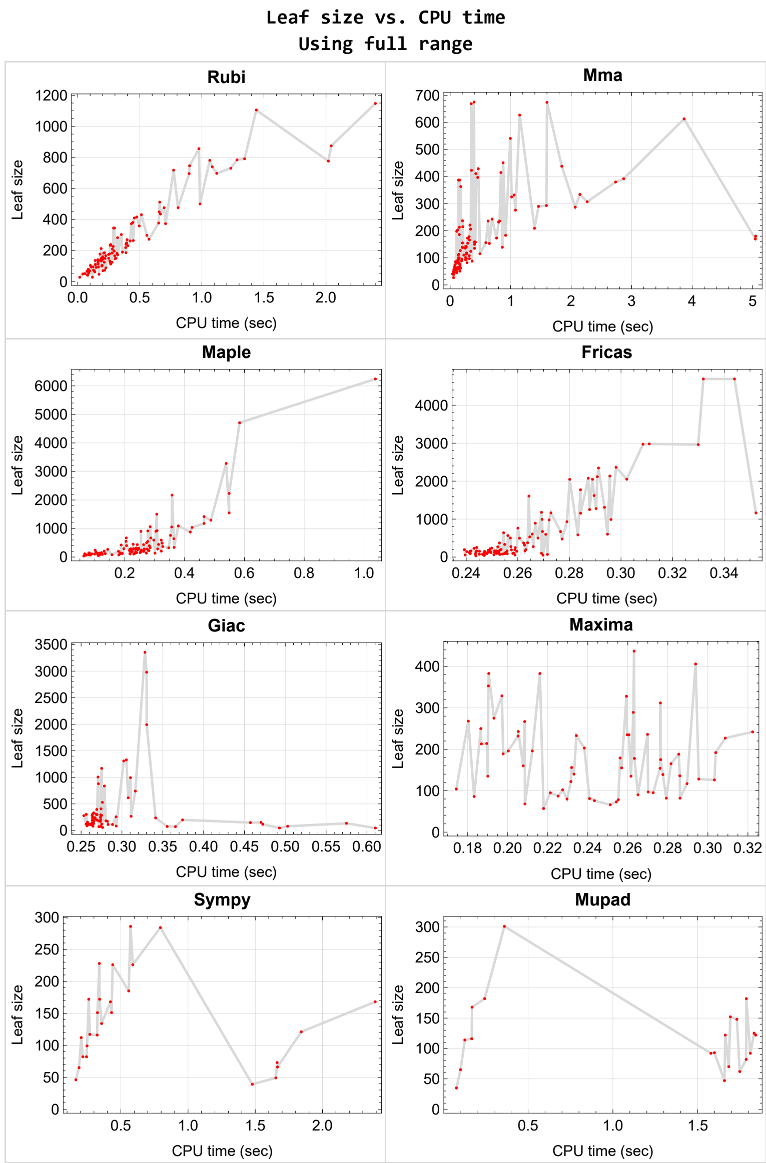


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

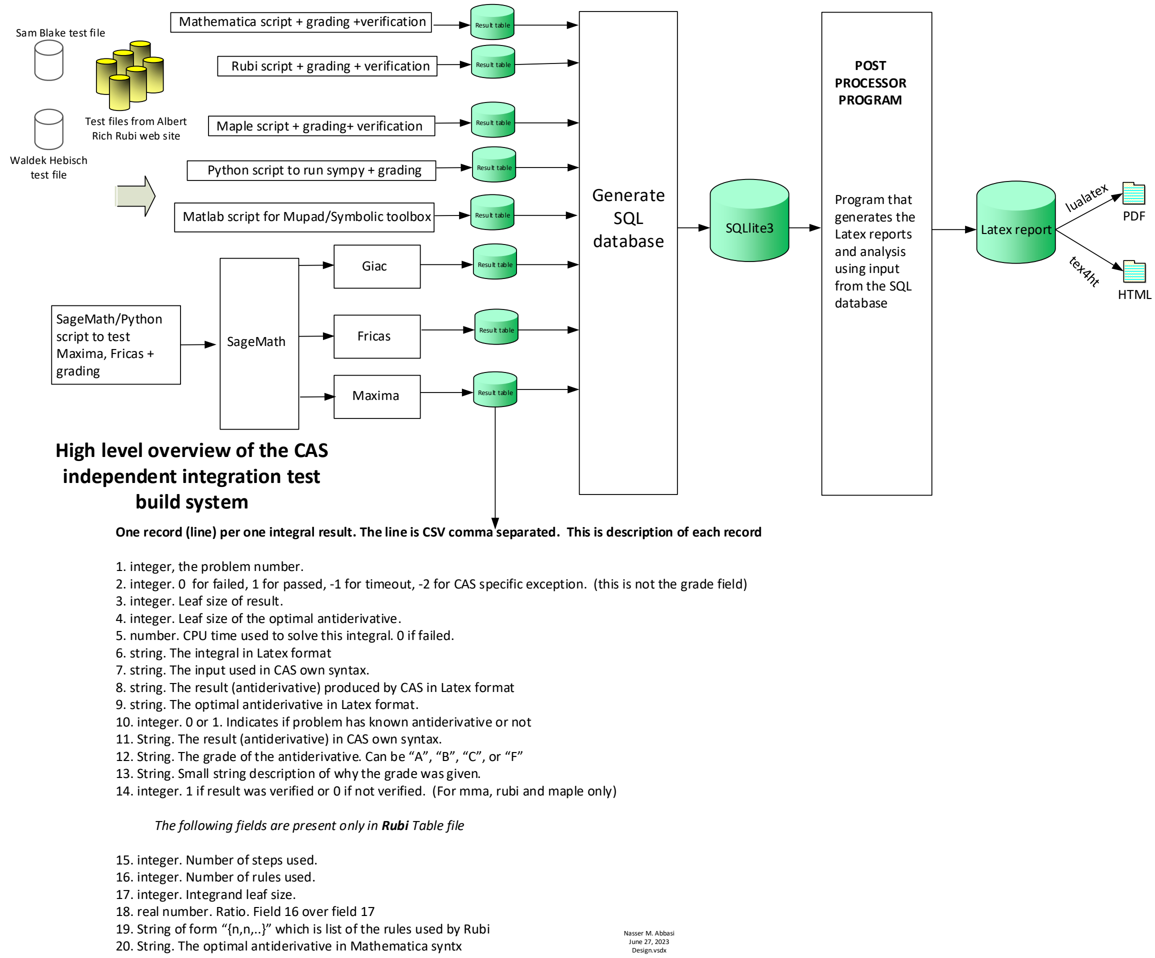
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { }

C grade { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 31, 32, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 71, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88 }

B grade { 26, 27, 28, 29, 33, 34, 35, 36, 38, 39, 52, 53, 65, 66, 67, 68, 72, 73, 74, 78, 84, 91, 92, 93 }

C grade { 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 62, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { 30, 33, 35, 36, 37, 38, 39, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 22, 24, 25, 26, 27, 28, 29, 30, 31, 36, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { 2, 3, 4, 5, 11, 12, 13, 19, 20, 21, 23, 42, 44, 52, 80, 81, 83 }

C grade { }

F normal fail { 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 96, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-1) timedout fail { 94, 95, 97, 98, 100, 101 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 79, 80, 81, 82, 83, 85, 86, 87, 88, 92 }

B grade { 12, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 84, 89, 90, 91, 93 }

C grade { }

F normal fail { 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111 }

F(-1) timedout fail { }

F(-2) exception fail { 65, 70, 106 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 10, 11, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade { 12 }

C grade { }

F normal fail { 6, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 33, 34, 37, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101 }

F(-1) timedout fail { 7, 8, 9, 30, 32, 35, 36, 38, 39, 72, 73, 74, 75, 76, 77, 78, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	82	119	232	85	151	152	122
N.S.	1	1.00	0.66	0.96	1.87	0.69	1.22	1.23	0.98
time (sec)	N/A	0.276	0.143	0.135	0.205	0.255	0.326	0.470	1.842

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	65	94	196	67	117	116	92
N.S.	1	1.00	0.69	1.00	2.09	0.71	1.24	1.23	0.98
time (sec)	N/A	0.167	0.127	0.125	0.212	0.271	0.269	0.472	1.810

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	45	73	160	48	82	79	62
N.S.	1	1.00	0.70	1.14	2.50	0.75	1.28	1.23	0.97
time (sec)	N/A	0.085	0.106	0.085	0.208	0.270	0.217	0.503	1.747

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	37	68	30	46	46	35
N.S.	1	1.00	0.96	1.32	2.43	1.07	1.64	1.64	1.25
time (sec)	N/A	0.016	0.057	0.063	0.209	0.244	0.165	0.610	0.078

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	52	97	54	39	47	0
N.S.	1	1.00	1.39	1.86	3.46	1.93	1.39	1.68	0.00
time (sec)	N/A	0.118	0.060	0.111	0.270	0.259	1.478	0.493	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	59	75	82	76	0	72	0
N.S.	1	1.00	1.26	1.60	1.74	1.62	0.00	1.53	0.00
time (sec)	N/A	0.171	0.137	0.116	0.286	0.246	0.000	0.366	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	78	137	66	116	0	134	0
N.S.	1	1.00	0.89	1.56	0.75	1.32	0.00	1.52	0.00
time (sec)	N/A	0.227	0.156	0.115	0.251	0.250	0.000	0.575	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	204	78	143	0	199	0
N.S.	1	1.00	0.83	1.55	0.59	1.08	0.00	1.51	0.00
time (sec)	N/A	0.259	0.226	0.131	0.255	0.240	0.000	0.374	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	140	269	82	161	0	266	0
N.S.	1	1.00	0.84	1.62	0.49	0.97	0.00	1.60	0.00
time (sec)	N/A	0.298	0.266	0.143	0.279	0.252	0.000	0.311	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	100	134	329	127	228	236	168
N.S.	1	1.00	0.54	0.73	1.79	0.69	1.24	1.28	0.91
time (sec)	N/A	0.278	0.150	0.301	0.197	0.251	0.340	0.341	0.170

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	87	108	275	95	172	171	125
N.S.	1	1.00	0.65	0.81	2.05	0.71	1.28	1.28	0.93
time (sec)	N/A	0.167	0.127	0.218	0.193	0.253	0.261	0.282	1.832

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	56	77	135	64	112	112	82
N.S.	1	1.00	1.14	1.57	2.76	1.31	2.29	2.29	1.67
time (sec)	N/A	0.041	0.095	0.180	0.190	0.246	0.205	0.289	1.786

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	121	175	94	73	113	0
N.S.	1	1.00	0.82	1.95	2.82	1.52	1.18	1.82	0.00
time (sec)	N/A	0.138	0.162	0.217	0.276	0.252	1.662	0.283	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	124	136	122	0	119	0
N.S.	1	1.00	0.89	1.77	1.94	1.74	0.00	1.70	0.00
time (sec)	N/A	0.203	0.171	0.192	0.286	0.255	0.000	0.275	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	93	186	126	153	0	181	0
N.S.	1	1.00	0.77	1.54	1.04	1.26	0.00	1.50	0.00
time (sec)	N/A	0.264	0.255	0.237	0.303	0.245	0.000	0.280	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	154	292	117	194	0	285	0
N.S.	1	1.00	0.90	1.70	0.68	1.13	0.00	1.66	0.00
time (sec)	N/A	0.321	0.290	0.229	0.290	0.259	0.000	0.275	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	400	128	231	0	395	0
N.S.	1	1.00	0.83	1.61	0.52	0.93	0.00	1.59	0.00
time (sec)	N/A	0.394	0.338	0.206	0.295	0.243	0.000	0.269	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	159	442	437	236	0	407	0
N.S.	1	1.00	0.73	2.02	2.00	1.08	0.00	1.86	0.00
time (sec)	N/A	0.386	0.402	0.313	0.263	0.250	0.000	0.274	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	118	292	328	190	0	256	0
N.S.	1	1.00	0.79	1.95	2.19	1.27	0.00	1.71	0.00
time (sec)	N/A	0.249	0.271	0.239	0.259	0.258	0.000	0.293	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	89	184	233	156	0	148	0
N.S.	1	1.00	0.89	1.84	2.33	1.56	0.00	1.48	0.00
time (sec)	N/A	0.194	0.190	0.223	0.234	0.252	0.000	0.457	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	64	114	156	118	0	83	0
N.S.	1	1.00	0.94	1.68	2.29	1.74	0.00	1.22	0.00
time (sec)	N/A	0.134	0.089	0.185	0.232	0.252	0.000	0.293	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	81	57	95	0	56	0
N.S.	1	1.00	0.96	1.59	1.12	1.86	0.00	1.10	0.00
time (sec)	N/A	0.059	0.060	0.157	0.218	0.255	0.000	0.276	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	108	155	123	0	75	0
N.S.	1	1.00	0.86	1.48	2.12	1.68	0.00	1.03	0.00
time (sec)	N/A	0.217	0.095	0.230	0.257	0.252	0.000	0.355	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	155	192	179	0	129	0
N.S.	1	1.00	0.89	1.37	1.70	1.58	0.00	1.14	0.00
time (sec)	N/A	0.265	0.257	0.251	0.304	0.249	0.000	0.265	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	178	281	242	275	0	248	0
N.S.	1	1.00	0.94	1.48	1.27	1.45	0.00	1.31	0.00
time (sec)	N/A	0.360	0.314	0.265	0.322	0.257	0.000	0.274	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	173	669	406	371	0	2979	0
N.S.	1	1.00	0.75	2.90	1.76	1.61	0.00	12.90	0.00
time (sec)	N/A	0.403	0.767	0.287	0.294	0.262	0.000	0.330	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	156	549	312	333	0	1991	0
N.S.	1	1.00	0.86	3.02	1.71	1.83	0.00	10.94	0.00
time (sec)	N/A	0.317	0.591	0.204	0.276	0.255	0.000	0.330	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	115	436	236	274	0	1308	0
N.S.	1	1.00	0.78	2.97	1.61	1.86	0.00	8.90	0.00
time (sec)	N/A	0.300	0.493	0.242	0.270	0.266	0.000	0.302	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	97	294	178	200	0	994	0
N.S.	1	1.00	0.78	2.35	1.42	1.60	0.00	7.95	0.00
time (sec)	N/A	0.214	0.318	0.197	0.263	0.254	0.000	0.310	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	132	81	149	0	615	0
N.S.	1	1.00	0.92	1.86	1.14	2.10	0.00	8.66	0.00
time (sec)	N/A	0.082	0.155	0.234	0.241	0.247	0.000	0.308	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	139	254	227	270	0	1329	0
N.S.	1	1.00	0.93	1.69	1.51	1.80	0.00	8.86	0.00
time (sec)	N/A	0.309	0.862	0.265	0.309	0.252	0.000	0.305	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	183	312	0	377	0	3353	0
N.S.	1	1.00	0.98	1.68	0.00	2.03	0.00	18.03	0.00
time (sec)	N/A	0.389	0.916	0.239	0.000	0.264	0.000	0.328	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	236	1055	0	566	0	879	0
N.S.	1	1.00	0.89	4.00	0.00	2.14	0.00	3.33	0.00
time (sec)	N/A	0.448	0.627	0.357	0.000	0.256	0.000	0.271	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	917	0	475	0	741	0
N.S.	1	1.00	0.63	3.80	0.00	1.97	0.00	3.07	0.00
time (sec)	N/A	0.388	0.648	0.277	0.000	0.277	0.000	0.316	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	158	435	0	373	0	529	0
N.S.	1	1.00	0.89	2.44	0.00	2.10	0.00	2.97	0.00
time (sec)	N/A	0.257	0.417	0.263	0.000	0.253	0.000	0.276	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	276	95	253	0	298	0
N.S.	1	1.00	0.85	2.65	0.91	2.43	0.00	2.87	0.00
time (sec)	N/A	0.102	0.363	0.219	0.221	0.244	0.000	0.270	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	451	488	0	601	0	837	0
N.S.	1	1.00	1.72	1.86	0.00	2.29	0.00	3.19	0.00
time (sec)	N/A	0.422	0.875	0.325	0.000	0.295	0.000	0.279	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	541	643	0	762	0	1006	0
N.S.	1	1.00	1.82	2.16	0.00	2.56	0.00	3.38	0.00
time (sec)	N/A	0.557	0.995	0.364	0.000	0.260	0.000	0.271	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	627	760	0	892	0	1169	0
N.S.	1	1.00	1.66	2.02	0.00	2.37	0.00	3.10	0.00
time (sec)	N/A	0.652	1.150	0.353	0.000	0.267	0.000	0.275	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	92	135	250	95	168	174	116
N.S.	1	1.00	0.66	0.97	1.80	0.68	1.21	1.25	0.83
time (sec)	N/A	0.185	0.127	0.119	0.187	0.258	0.423	0.266	0.168

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	74	117	214	78	134	138	93
N.S.	1	1.00	0.68	1.07	1.96	0.72	1.23	1.27	0.85
time (sec)	N/A	0.143	0.079	0.111	0.190	0.254	0.357	0.258	1.598

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	86	213	60	99	101	70
N.S.	1	1.00	0.72	1.09	2.70	0.76	1.25	1.28	0.89
time (sec)	N/A	0.085	0.061	0.081	0.187	0.251	0.249	0.263	1.683

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	63	86	42	65	70	47
N.S.	1	1.00	0.78	1.24	1.69	0.82	1.27	1.37	0.92
time (sec)	N/A	0.054	0.039	0.070	0.183	0.247	0.189	0.272	1.658

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	81	122	73	49	76	0
N.S.	1	1.00	1.34	1.98	2.98	1.78	1.20	1.85	0.00
time (sec)	N/A	0.080	0.069	0.080	0.232	0.245	1.653	0.274	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	83	80	82	0	80	0
N.S.	1	1.00	1.00	1.98	1.90	1.95	0.00	1.90	0.00
time (sec)	N/A	0.078	0.051	0.084	0.230	0.244	0.000	0.265	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	80	112	90	107	0	109	0
N.S.	1	1.00	1.08	1.51	1.22	1.45	0.00	1.47	0.00
time (sec)	N/A	0.127	0.101	0.090	0.265	0.248	0.000	0.264	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	95	175	73	127	0	170	0
N.S.	1	1.00	0.90	1.67	0.70	1.21	0.00	1.62	0.00
time (sec)	N/A	0.197	0.141	0.104	0.254	0.249	0.000	0.274	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	127	240	76	152	0	237	0
N.S.	1	1.00	0.85	1.61	0.51	1.02	0.00	1.59	0.00
time (sec)	N/A	0.220	0.167	0.101	0.243	0.260	0.000	0.265	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	138	202	383	155	286	304	182
N.S.	1	1.00	0.59	0.86	1.64	0.66	1.22	1.30	0.78
time (sec)	N/A	0.261	0.207	0.263	0.191	0.242	0.572	0.268	1.786

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	113	155	353	126	226	239	148
N.S.	1	1.00	0.61	0.84	1.92	0.68	1.23	1.30	0.80
time (sec)	N/A	0.209	0.143	0.240	0.190	0.250	0.440	0.267	1.731

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	85	117	189	98	172	180	114
N.S.	1	1.00	0.62	0.86	1.39	0.72	1.26	1.32	0.84
time (sec)	N/A	0.141	0.122	0.231	0.198	0.269	0.342	0.271	0.127

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	226	235	130	121	222	0
N.S.	1	1.00	0.75	2.05	2.14	1.18	1.10	2.02	0.00
time (sec)	N/A	0.142	0.279	0.216	0.260	0.252	1.842	0.267	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	200	179	127	0	197	0
N.S.	1	1.00	1.00	2.11	1.88	1.34	0.00	2.07	0.00
time (sec)	N/A	0.123	0.194	0.263	0.256	0.251	0.000	0.269	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	97	209	165	164	0	206	0
N.S.	1	1.00	0.85	1.83	1.45	1.44	0.00	1.81	0.00
time (sec)	N/A	0.164	0.266	0.225	0.282	0.251	0.000	0.274	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	241	135	171	0	236	0
N.S.	1	1.00	0.86	1.81	1.02	1.29	0.00	1.77	0.00
time (sec)	N/A	0.190	0.268	0.255	0.262	0.249	0.000	0.264	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	124	299	139	194	0	294	0
N.S.	1	1.00	0.71	1.71	0.79	1.11	0.00	1.68	0.00
time (sec)	N/A	0.249	0.341	0.238	0.278	0.239	0.000	0.274	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	276	369	0	605	0	0	0
N.S.	1	1.00	1.01	1.35	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.574	1.079	0.327	0.000	0.266	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	209	268	0	502	0	0	0
N.S.	1	1.00	1.00	1.28	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.279	1.395	0.217	0.000	0.257	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	221	259	0	496	0	0	0
N.S.	1	1.00	0.98	1.15	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.286	0.328	0.244	0.000	0.261	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	155	200	0	219	0	0	0
N.S.	1	1.00	0.88	1.13	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.179	0.203	0.179	0.000	0.249	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	166	212	0	316	0	0	0
N.S.	1	1.00	0.78	1.00	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.189	0.318	0.261	0.000	0.263	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	173	227	0	249	0	0	0
N.S.	1	1.00	0.88	1.15	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.294	0.288	0.256	0.000	0.259	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	243	288	0	599	0	0	0
N.S.	1	1.00	0.98	1.16	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.394	0.693	0.220	0.000	0.271	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	232	330	0	583	0	0	0
N.S.	1	1.00	0.86	1.22	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.398	0.801	0.285	0.000	0.283	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	307	1038	0	1179	0	0	0
N.S.	1	1.00	0.68	2.31	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.656	2.265	0.425	0.000	0.269	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	290	903	0	931	0	0	0
N.S.	1	1.00	0.67	2.10	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.513	1.460	0.304	0.000	0.279	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	287	901	0	1162	0	0	0
N.S.	1	1.00	0.69	2.17	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.476	2.067	0.254	0.000	0.273	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	236	449	0	641	0	0	0
N.S.	1	1.00	0.99	1.88	0.00	2.68	0.00	0.00	0.00
time (sec)	N/A	0.266	0.817	0.228	0.000	0.254	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	293	503	0	1162	0	0	0
N.S.	1	1.00	0.62	1.06	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.696	1.591	0.273	0.000	0.352	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	415	546	0	992	0	0	0
N.S.	1	1.00	0.95	1.26	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.667	0.840	0.277	0.000	0.269	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	500	500	334	595	0	1310	0	0	0
N.S.	1	1.00	0.67	1.19	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.985	2.145	0.322	0.000	0.294	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	332	1547	0	1620	0	0	0
N.S.	1	1.00	0.70	3.25	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.808	1.057	0.549	0.000	0.290	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	380	2172	0	2047	0	0	0
N.S.	1	1.00	0.51	2.91	0.00	2.74	0.00	0.00	0.00
time (sec)	N/A	0.901	2.734	0.358	0.000	0.280	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	512	325	1503	0	1607	0	0	0
N.S.	1	1.00	0.63	2.94	0.00	3.14	0.00	0.00	0.00
time (sec)	N/A	0.660	1.017	0.307	0.000	0.264	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	856	856	392	1064	0	2116	0	0	0
N.S.	1	1.00	0.46	1.24	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.976	2.868	0.285	0.000	0.291	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	674	1090	0	2076	0	0	0
N.S.	1	1.00	0.92	1.49	0.00	2.84	0.00	0.00	0.00
time (sec)	N/A	1.233	1.601	0.379	0.000	0.287	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	874	874	613	1178	0	2346	0	0	0
N.S.	1	1.00	0.70	1.35	0.00	2.68	0.00	0.00	0.00
time (sec)	N/A	2.042	3.867	0.465	0.000	0.291	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	438	1294	0	2363	0	0	0
N.S.	1	1.00	0.55	1.64	0.00	2.99	0.00	0.00	0.00
time (sec)	N/A	1.344	1.845	0.488	0.000	0.298	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	100	153	268	105	185	192	152
N.S.	1	1.00	0.65	0.99	1.74	0.68	1.20	1.25	0.99
time (sec)	N/A	0.205	0.136	0.122	0.180	0.247	0.559	0.264	1.692

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	84	129	267	87	151	156	122
N.S.	1	1.00	0.68	1.04	2.15	0.70	1.22	1.26	0.98
time (sec)	N/A	0.163	0.096	0.107	0.208	0.248	0.431	0.258	1.662

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	66	104	196	68	116	119	92
N.S.	1	1.00	0.70	1.11	2.09	0.72	1.23	1.27	0.98
time (sec)	N/A	0.109	0.073	0.099	0.200	0.253	0.324	0.256	1.578

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	81	104	53	82	88	65
N.S.	1	1.00	0.74	1.23	1.58	0.80	1.24	1.33	0.98
time (sec)	N/A	0.075	0.055	0.072	0.174	0.240	0.245	0.257	0.102

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	49	113	140	86	66	109	0
N.S.	1	1.00	0.88	2.02	2.50	1.54	1.18	1.95	0.00
time (sec)	N/A	0.090	0.112	0.066	0.233	0.256	1.666	0.258	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	114	102	90	0	111	0
N.S.	1	1.00	1.00	2.07	1.85	1.64	0.00	2.02	0.00
time (sec)	N/A	0.090	0.082	0.085	0.227	0.242	0.000	0.261	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	119	87	101	0	118	0
N.S.	1	1.00	1.25	1.72	1.26	1.46	0.00	1.71	0.00
time (sec)	N/A	0.097	0.091	0.083	0.225	0.258	0.000	0.260	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	73	146	95	118	0	141	0
N.S.	1	1.00	0.80	1.60	1.04	1.30	0.00	1.55	0.00
time (sec)	N/A	0.148	0.162	0.080	0.273	0.241	0.000	0.263	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	139	206	383	161	284	303	301
N.S.	1	1.00	0.59	0.88	1.64	0.69	1.21	1.29	1.29
time (sec)	N/A	0.268	0.201	0.322	0.216	0.263	0.794	0.256	0.361

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	111	159	243	130	226	244	182
N.S.	1	1.00	0.60	0.85	1.31	0.70	1.22	1.31	0.98
time (sec)	N/A	0.216	0.135	0.236	0.205	0.244	0.589	0.265	0.244

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	303	289	161	168	331	0
N.S.	1	1.00	0.68	1.89	1.81	1.01	1.05	2.07	0.00
time (sec)	N/A	0.201	0.315	0.236	0.263	0.244	2.392	0.266	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	143	306	235	160	0	308	0
N.S.	1	1.00	1.00	2.14	1.64	1.12	0.00	2.15	0.00
time (sec)	N/A	0.201	0.259	0.247	0.261	0.242	0.000	0.264	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	136	281	203	161	0	280	0
N.S.	1	1.00	0.96	1.99	1.44	1.14	0.00	1.99	0.00
time (sec)	N/A	0.161	0.249	0.275	0.238	0.256	0.000	0.263	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	135	284	188	187	0	279	0
N.S.	1	1.00	0.90	1.89	1.25	1.25	0.00	1.86	0.00
time (sec)	N/A	0.190	0.403	0.310	0.285	0.253	0.000	0.253	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	150	322	154	196	0	315	0
N.S.	1	1.00	0.90	1.93	0.92	1.17	0.00	1.89	0.00
time (sec)	N/A	0.226	0.403	0.347	0.276	0.263	0.000	0.264	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	373	373	213	925	0	989	0	0	0
N.S.	1	1.00	0.57	2.48	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	0.708	0.152	0.308	0.000	0.296	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	358	358	198	671	0	977	0	0	0
N.S.	1	1.00	0.55	1.87	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	0.495	0.109	0.205	0.000	0.272	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	283	283	170	423	0	500	0	0	0
N.S.	1	1.00	0.60	1.49	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.321	5.045	0.188	0.000	0.268	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	345	345	180	280	0	671	0	0	0
N.S.	1	1.00	0.52	0.81	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.296	5.052	0.234	0.000	0.277	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	345	345	180	143	0	673	0	0	0
N.S.	1	1.00	0.52	0.41	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.289	5.049	0.186	0.000	0.270	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	303	186	138	0	530	0	0	0
N.S.	1	1.00	0.61	0.46	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.351	0.151	0.272	0.000	0.265	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	381	381	215	175	0	1154	0	0	0
N.S.	1	1.00	0.56	0.46	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.444	0.211	0.254	0.000	0.284	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	410	410	237	226	0	1251	0	0	0
N.S.	1	1.00	0.58	0.55	0.00	3.05	0.00	0.00	0.00
time (sec)	N/A	0.456	0.204	0.281	0.000	0.288	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	718	718	363	877	0	2050	0	0	0
N.S.	1	1.00	0.51	1.22	0.00	2.86	0.00	0.00	0.00
time (sec)	N/A	0.773	0.175	0.419	0.000	0.302	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	373	373	203	594	0	1276	0	0	0
N.S.	1	1.00	0.54	1.59	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.432	0.125	0.297	0.000	0.290	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	695	695	387	395	0	2135	0	0	0
N.S.	1	1.00	0.56	0.57	0.00	3.07	0.00	0.00	0.00
time (sec)	N/A	0.897	0.138	0.282	0.000	0.296	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	739	739	387	226	0	2048	0	0	0
N.S.	1	1.00	0.52	0.31	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	1.083	0.154	0.279	0.000	0.289	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	697	697	411	338	0	1773	0	0	0
N.S.	1	1.00	0.59	0.48	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	1.120	0.425	0.365	0.000	0.284	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	784	784	397	6243	0	2980	0	0	0
N.S.	1	1.00	0.51	7.96	0.00	3.80	0.00	0.00	0.00
time (sec)	N/A	1.283	0.456	1.038	0.000	0.311	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1105	1105	675	4708	0	4691	0	0	0
N.S.	1	1.00	0.61	4.26	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	1.439	0.395	0.584	0.000	0.344	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	776	776	429	3281	0	2962	0	0	0
N.S.	1	1.00	0.55	4.23	0.00	3.82	0.00	0.00	0.00
time (sec)	N/A	2.019	0.464	0.539	0.000	0.330	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	781	781	423	2230	0	2972	0	0	0
N.S.	1	1.00	0.54	2.86	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	1.064	0.353	0.549	0.000	0.309	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1147	1147	669	1416	0	4691	0	0	0
N.S.	1	1.00	0.58	1.23	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	2.397	0.348	0.465	0.000	0.332	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [111] had the largest ratio of [.5293999999999999982]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.00	15	0.267
2	A	9	4	1.00	15	0.267
3	A	7	4	1.00	13	0.308
4	A	2	2	1.00	12	0.167
5	A	6	5	1.00	15	0.333
6	A	9	5	1.00	15	0.333
7	A	11	5	1.00	15	0.333
8	A	13	5	1.00	15	0.333
9	A	15	5	1.00	15	0.333
10	A	14	4	1.00	17	0.235
11	A	11	4	1.00	15	0.267
12	A	3	2	1.00	14	0.143
13	A	8	7	1.00	17	0.412
14	A	10	6	1.00	17	0.353
15	A	14	5	1.00	17	0.294
16	A	17	5	1.00	17	0.294
17	A	20	5	1.00	17	0.294
18	A	15	7	1.00	17	0.412
19	A	11	7	1.00	17	0.412
20	A	8	7	1.00	17	0.412
21	A	6	5	1.00	15	0.333
22	A	3	3	1.00	14	0.214
23	A	8	4	1.00	17	0.235
24	A	12	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	17	5	1.00	17	0.294
26	A	15	8	1.00	17	0.471
27	A	12	8	1.00	17	0.471
28	A	10	6	1.00	17	0.353
29	A	9	5	1.00	15	0.333
30	A	4	4	1.00	14	0.286
31	A	12	5	1.00	17	0.294
32	A	16	5	1.00	17	0.294
33	A	15	6	1.00	17	0.353
34	A	14	5	1.00	17	0.294
35	A	11	5	1.00	15	0.333
36	A	5	4	1.00	14	0.286
37	A	17	5	1.00	17	0.294
38	A	21	5	1.00	17	0.294
39	A	26	5	1.00	17	0.294
40	A	12	3	1.00	17	0.176
41	A	10	3	1.00	17	0.176
42	A	8	3	1.00	15	0.200
43	A	6	3	1.00	14	0.214
44	A	7	6	1.00	17	0.353
45	A	7	6	1.00	17	0.353
46	A	10	5	1.00	17	0.294
47	A	12	5	1.00	17	0.294
48	A	14	5	1.00	17	0.294
49	A	17	3	1.00	19	0.158
50	A	14	3	1.00	17	0.176
51	A	11	3	1.00	16	0.188
52	A	11	6	1.00	19	0.316
53	A	10	7	1.00	19	0.368
54	A	12	7	1.00	19	0.368
55	A	13	6	1.00	19	0.316
56	A	17	5	1.00	19	0.263
57	A	14	7	1.00	19	0.368
58	A	12	6	1.00	19	0.316
59	A	11	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	8	4	1.00	17	0.235
61	A	8	4	1.00	16	0.250
62	A	13	4	1.00	19	0.210
63	A	14	6	1.00	19	0.316
64	A	18	5	1.00	19	0.263
65	A	24	9	1.00	19	0.474
66	A	20	8	1.00	19	0.421
67	A	17	6	1.00	19	0.316
68	A	9	5	1.00	17	0.294
69	A	18	5	1.00	16	0.312
70	A	22	6	1.00	19	0.316
71	A	32	6	1.00	19	0.316
72	A	27	8	1.00	19	0.421
73	A	28	7	1.00	19	0.368
74	A	19	6	1.00	17	0.353
75	A	28	5	1.00	16	0.312
76	A	41	7	1.00	19	0.368
77	A	60	6	1.00	19	0.316
78	A	46	7	1.00	19	0.368
79	A	13	4	1.00	17	0.235
80	A	11	4	1.00	17	0.235
81	A	9	4	1.00	15	0.267
82	A	7	4	1.00	14	0.286
83	A	8	6	1.00	17	0.353
84	A	8	7	1.00	17	0.412
85	A	8	6	1.00	17	0.353
86	A	11	5	1.00	17	0.294
87	A	17	4	1.00	17	0.235
88	A	14	4	1.00	16	0.250
89	A	14	7	1.00	19	0.368
90	A	13	8	1.00	19	0.421
91	A	12	8	1.00	19	0.421
92	A	14	7	1.00	19	0.368
93	A	15	7	1.00	19	0.368
94	A	15	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	14	6	1.00	19	0.316
96	A	11	4	1.00	19	0.210
97	A	11	4	1.00	17	0.235
98	A	11	4	1.00	16	0.250
99	A	16	4	1.00	19	0.210
100	A	17	5	1.00	19	0.263
101	A	18	6	1.00	19	0.316
102	A	23	6	1.00	19	0.316
103	A	12	5	1.00	19	0.263
104	A	34	7	1.00	17	0.412
105	A	36	8	1.00	16	0.500
106	A	41	8	1.00	19	0.421
107	A	36	8	1.00	19	0.421
108	A	47	9	1.00	19	0.474
109	A	71	10	1.00	19	0.526
110	A	37	9	1.00	19	0.474
111	A	89	9	1.00	17	0.529

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int x^3(a+bx)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}$$

```
[Out] -6*a*cosh(d*x+c)/d^4-24*b*x*cosh(d*x+c)/d^4-3*a*x^2*cosh(d*x+c)/d^2-4*b*x^3
*cosh(d*x+c)/d^2+24*b*sinh(d*x+c)/d^5+6*a*x*sinh(d*x+c)/d^3+12*b*x^2*sinh(d
*x+c)/d^3+a*x^3*sinh(d*x+c)/d+b*x^4*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6874, 3377, 2718, 2717}

$$\int x^3(a+bx)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

```
[In] Int[x^3*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d
*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*x*
```

```
Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d
 + (b*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^3\cosh(c+dx)+bx^4\cosh(c+dx)\right)\,dx \\
&= a\int x^3\cosh(c+dx)\,dx+b\int x^4\cosh(c+dx)\,dx \\
&= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(4b)\int x^3\sinh(c+dx)\,dx}{d} \\
&= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d} \\
&\quad +\frac{bx^4\sinh(c+dx)}{d}+\frac{(6a)\int x\cosh(c+dx)\,dx}{d^2}+\frac{(12b)\int x^2\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3} \\
&\quad +\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(6a)\int \sinh(c+dx)\,dx}{d^3}-\frac{(24b)\int x\sinh(c+dx)\,dx}{d^3} \\
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2} \\
&\quad -\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3} \\
&\quad +\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}+\frac{(24b)\int \cosh(c+dx)\,dx}{d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6a\cosh(c+dx)}{d^4} - \frac{24bx\cosh(c+dx)}{d^4} - \frac{3ax^2\cosh(c+dx)}{d^2} \\
&\quad - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c+dx)}{d^5} + \frac{6ax\sinh(c+dx)}{d^3} \\
&\quad + \frac{12bx^2\sinh(c+dx)}{d^3} + \frac{ax^3\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$
\int x^3(a+bx)\cosh(c+dx)\,dx
$$

$$
= \frac{-d(3a(2+d^2x^2)+4bx(6+d^2x^2))\cosh(c+dx)+(ad^2x(6+d^2x^2)+b(24+12d^2x^2+d^4x^4))\sinh(c+dx}{d^5}
$$

```
[In] Integrate[x^3*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(3*a*(2 + d^2*x^2) + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^2*x*(6
 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{3dx\left(x\left(\frac{4bx}{3}+a\right)d^2+8b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(\left(-b\,x^4-a\,x^3\right)d^4-6x(2bx+a)d^2-24b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+3\left(x^2\left(\frac{4bx}{3}+a\right)d^2+8bx \cdots\right.}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,x^4d^4+a\,d^4x^3-4b\,d^3x^3-3a\,d^3x^2+12b\,d^2x^2+6a\,d^2x-24dxb-6da+24b\right)\mathrm{e}^{dx+c}}{2d^5}-\frac{\left(b\,x^4d^4+a\,d^4x^3+4b\,d^3x^3+3a\,d^3x^2+\cdots\right.}{}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\frac{9}{2}\cdots\right.}{}\cdots\right.}{}$
parts	$\frac{b\,x^4\sinh(dx+c)}{d}+\frac{a\,x^3\sinh(dx+c)}{d}-\frac{-\frac{4b\,c^3\cosh(dx+c)}{d^3}+\frac{12b\,c^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^3}-\frac{12bc\left((dx+c)^2\cosh(dx+\cdots\right.}{}}{}$
derivativedivides	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{4bc\left((dx+c)^3\sinh(dx+c\cdots\right.}{}}{}$
default	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{4bc\left((dx+c)^3\sinh(dx+c\cdots\right.}{}}{}$

```
[In] int(x^3*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (3*d*x*(x*(4/3*b*x+a)*d^2+8*b)*tanh(1/2*d*x+1/2*c)^2+2*((-b*x^4-a*x^3)*d^4-
6*x*(2*b*x+a)*d^2-24*b)*tanh(1/2*d*x+1/2*c)+3*(x^2*(4/3*b*x+a)*d^2+8*b*x+4*
```

```
a)*d)/d^5/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int x^3(a+bx)\cosh(c+dx)\,dx = -\frac{(4\,bd^3x^3+3\,ad^3x^2+24\,bdx+6\,ad)\cosh(dx+c)-(bd^4x^4+ad^4x^3+12\,bd^2x^2+6\,ad^2x+24\,b)\sinh(dx}{d^5}$$

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
[Out] -((4*b*d^3*x^3 + 3*a*d^3*x^2 + 24*b*d*x + 6*a*d)*cosh(d*x + c) - (b*d^4*x^4
 + a*d^4*x^3 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^3\sinh(c+dx)}{d} - \frac{3ax^2\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} - \frac{6a\cosh(c+dx)}{d^4} + \frac{bx^4\sinh(c+dx)}{d} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{12bx^2\sinh(c+dx)}{d^3} \\ \left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)\cosh(c) \end{cases}$$

```
[In] integrate(x**3*(b*x+a)*cosh(d*x+c),x)
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**4*sinh(c + d*x)/d - 4*b*x**
3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/
d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*cosh(c),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.87

$$\int x^3(a+bx)\cosh(c+dx)\,dx = -\frac{1}{40}\,d\left(\frac{5\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ae^{(dx)}}{d^5}+\frac{5\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+2}{d^5}\right. + \frac{1}{20}\,(4\,bx^5+5\,ax^4)\cosh(dx+c)$$

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/40*d*(5*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 5*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 4*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*
x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 4*(d^5*x^5 + 5*d^4*x^4 + 2
0*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) + 1/20*(4*b*x^5
 + 5*a*x^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \frac{(bd^4x^4 + ad^4x^3 - 4\,bd^3x^3 - 3\,ad^3x^2 + 12\,bd^2x^2 + 6\,ad^2x - 24\,bdx - 6\,ad + 24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4 + ad^4x^3 + 4\,bd^3x^3 + 3\,ad^3x^2 + 12\,bd^2x^2 + 6\,ad^2x + 24\,bdx + 6\,ad + 24\,b)e^{(-dx-c)}}{2\,d^5}$$

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^3 - 4*b*d^3*x^3 - 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a
*d^2*x - 24*b*d*x - 6*a*d + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*
x^3 + 4*b*d^3*x^3 + 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b*d*x + 6*a
*d + 24*b)*e^(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \frac{12\,b\,x^2\sinh(c+d\,x) + 6\,a\,x\sinh(c+d\,x)}{d^3} - \frac{6\,a\cosh(c+d\,x) + 24\,b\,x\cosh(c+d\,x)}{d^4} - \frac{3\,a\,x^2\cosh(c+d\,x) + 4\,b\,x^3\cosh(c+d\,x)}{d^2} + \frac{a\,x^3\sinh(c+d\,x) + b\,x^4\sinh(c+d\,x)}{d} + \frac{24\,b\sinh(c+d\,x)}{d^5}$$

```
[In] int(x^3*cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (12*b*x^2*sinh(c + d*x) + 6*a*x*sinh(c + d*x))/d^3 - (6*a*cosh(c + d*x) + 2
4*b*x*cosh(c + d*x))/d^4 - (3*a*x^2*cosh(c + d*x) + 4*b*x^3*cosh(c + d*x))/
d^2 + (a*x^3*sinh(c + d*x) + b*x^4*sinh(c + d*x))/d + (24*b*sinh(c + d*x))/
d^5
```

3.2 $\int x^2(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x^2(a+bx)\cosh(c+dx)\,dx = -\frac{6b\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{6bx\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}$$

```
[Out] -6*b*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-3*b*x^2*cosh(d*x+c)/d^2+2*a*sinh
(d*x+c)/d^3+6*b*x*sinh(d*x+c)/d^3+a*x^2*sinh(d*x+c)/d+b*x^3*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6874, 3377, 2717, 2718}

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{6b\cosh(c+dx)}{d^4}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*
x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Sinh
[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2\cosh(c+dx)+bx^3\cosh(c+dx)\right)\,dx \\
&= a\int x^2\cosh(c+dx)\,dx+b\int x^3\cosh(c+dx)\,dx \\
&= \frac{ax^2\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}-\frac{(2a)\int x\sinh(c+dx)\,dx}{d}-\frac{(3b)\int x^2\sinh(c+dx)\,dx}{d} \\
&= -\frac{2ax\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d} \\
&\quad+\frac{bx^3\sinh(c+dx)}{d}+\frac{(2a)\int\cosh(c+dx)\,dx}{d^2}+\frac{(6b)\int x\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{2ax\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{6bx\sinh(c+dx)}{d^3} \\
&\quad+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}-\frac{(6b)\int\sinh(c+dx)\,dx}{d^3} \\
&= -\frac{6b\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3} \\
&\quad+\frac{6bx\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{-((2ad^2x+3b(2+d^2x^2))\cosh(c+dx))+d(a(2+d^2x^2)+bx(6+d^2x^2))\sinh(c+dx)}{d^4}$$

[In] Integrate[x^2*(a + b*x)*Cosh[c + d*x],x]

[Out] (-((2*a*d^2*x + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a*(2 + d^2*x^2) + b*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{2d^2x\left(\frac{3bx}{2}+a\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2d(x^2(bx+a)d^2+6bx+2a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+(3b\,x^2+2ax)d^2+12b}{d^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b\,d^3x^3+a\,d^3x^2-3b\,d^2x^2-2a\,d^2x+6dxb+2da-6b)\mathrm{e}^{dx+c}}{2d^4}-\frac{(b\,d^3x^3+a\,d^3x^2+3b\,d^2x^2+2a\,d^2x+6dxb+2da+6b)\mathrm{e}^{-dx-c}}{2d^4}$
parts	$\frac{b\,x^3\sinh(dx+c)}{d}+\frac{a\,x^2\sinh(dx+c)}{d}-\frac{\frac{3b\,c^2\cosh(dx+c)}{d^2}-\frac{6bc((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^2}+\frac{3b\left((dx+c)^2\cosh(dx+c)-2(dx \cdots\right)}{\cdots}}{d^2 \cdots}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i\left(\frac{15x \cdots}{2}\cdots\right)}{\cdots}\right)}{d^4}$
derivativedivides	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}+\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx \cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}+\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx \cdots\right)}{\cdots}}{\cdots}$

[In] int(x^2*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)

[Out] (2*d^2*x*(3/2*b*x+a)*tanh(1/2*d*x+1/2*c)^2-2*d*(x^2*(b*x+a)*d^2+6*b*x+2*a)*tanh(1/2*d*x+1/2*c)+(3*b*x^2+2*a*x)*d^2+12*b)/d^4/(tanh(1/2*d*x+1/2*c)^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int x^2(a+bx)\cosh(c+dx)\,dx = -\frac{(3\,bd^2x^2+2\,ad^2x+6\,b)\cosh(dx+c)-(bd^3x^3+ad^3x^2+6\,bdx+2\,ad)\sinh(dx+c)}{d^4}$$

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b*d^2*x^2 + 2*a*d^2*x + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3*x^2 +
6*b*d*x + 2*a*d)*sinh(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^2\sinh(c+dx)}{d}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{bx^3\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\cosh(c) \end{cases}$$

```
[In] integrate(x**2*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*s
inh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/
4)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(94) = 188.

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.09

$$\int x^2(a+bx)\cosh(c+dx)\,dx = -\frac{1}{24}\,d\left(\frac{4\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)ae^{(dx)}}{d^4}+\frac{4\,(d^3x^3+3\,d^2x^2+6\,dx+6)ae^{(-dx-c)}}{d^4}+\frac{3\,(d^4x^4e^c-\right. \\ +\frac{1}{12}\,(3\,bx^4+4\,ax^3)\cosh(dx+c)$$

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(4*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
+ 4*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^4*x^4*e^c -
 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b*e^(d*x)/d^5 + 3*(d
^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b*e^(-d*x - c)/d^5) + 1/12*(
3*b*x^4 + 4*a*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{(bd^3x^3+ad^3x^2-3\,bd^2x^2-2\,ad^2x+6\,bdx+2\,ad-6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3+ad^3x^2+3\,bd^2x^2+2\,ad^2x+6\,bdx+2\,ad+6\,b)e^{(-dx-c)}}{2\,d^4}$$

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 + a*d^3*x^2 - 3*b*d^2*x^2 - 2*a*d^2*x + 6*b*d*x + 2*a*d - 6*
b)*e^(d*x + c)/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x^2 + 3*b*d^2*x^2 + 2*a*d^2*x +
 6*b*d*x + 2*a*d + 6*b)*e^(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{2\,a\sinh(c+d\,x)+6\,b\,x\sinh(c+d\,x)}{d^3} - \frac{2\,a\,x\cosh(c+d\,x)+3\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{a\,x^2\sinh(c+d\,x)+b\,x^3\sinh(c+d\,x)}{d} - \frac{6\,b\cosh(c+d\,x)}{d^4}$$

```
[In] int(x^2*cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (2*a*sinh(c + d*x) + 6*b*x*sinh(c + d*x))/d^3 - (2*a*x*cosh(c + d*x) + 3*b*
x^2*cosh(c + d*x))/d^2 + (a*x^2*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d - (6
*b*cosh(c + d*x))/d^4
```

3.3 $\int x(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{a\cosh(c+dx)}{d^2} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] -a*cosh(d*x+c)/d^2-2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*x*sinh(d*x+c
)/d+b*x^2*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6874, 3377, 2718, 2717}

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{a\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{2b\sinh(c+dx)}{d^3} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{bx^2\sinh(c+dx)}{d}$$

```
[In] Int[x*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d^2) - (2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/
d^3 + (a*x*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax\cosh(c+dx)+bx^2\cosh(c+dx)\right)\,dx \\
&= a\int x\cosh(c+dx)\,dx+b\int x^2\cosh(c+dx)\,dx \\
&= \frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}-\frac{a\int\sinh(c+dx)\,dx}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d} \\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d} \\
&\quad+\frac{bx^2\sinh(c+dx)}{d}+\frac{(2b)\int\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x(a+bx)\cosh(c+dx)\,dx=\frac{-d(a+2bx)\cosh(c+dx)+(ad^2x+b(2+d^2x^2))\sinh(c+dx)}{d^3}$$

```
[In] Integrate[x*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(a + 2*b*x)*Cosh[c + d*x]) + (a*d^2*x + b*(2 + d^2*x^2))*Sinh[c + d*x]
)/d^3
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{2x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}bd+2\left(\left(-b\,x^{2}-ax\right)d^{2}-2b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2(bx+a)d}{d^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,d^{2}x^{2}+a\,d^{2}x-2dxb-da+2b\right)\mathrm{e}^{dx+c}}{2d^{3}}-\frac{\left(b\,d^{2}x^{2}+a\,d^{2}x+2dxb+da+2b\right)\mathrm{e}^{-dx-c}}{2d^{3}}$
parts	$\frac{b\,x^{2}\sinh(dx+c)}{d}+\frac{ax\sinh(dx+c)}{d}-\frac{\frac{2b((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d}-\frac{2bc\cosh(dx+c)}{d}+a\cosh(dx+c)}{d^{2}}$
derivativedivides	$\frac{\frac{b\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+a((dx+c)\sinh(dx+c)-\cosh}{d^{2}}$
default	$\frac{\frac{b\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+a((dx+c)\sinh(dx+c)-\cosh}{d^{2}}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^{2}d^{2}}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^{3}}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^{2}d^{2}}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{3}}-$

```
[In] int(x*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
[Out] 2*(x*tanh(1/2*d*x+1/2*c)^2*b*d+((-b*x^2-a*x)*d^2-2*b)*tanh(1/2*d*x+1/2*c)+(
b*x+a)*d)/d^3/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{(2\,bdx+ad)\cosh\left(dx+c\right)-\left(bd^{2}x^{2}+ad^{2}x+2\,b\right)\sinh\left(dx+c\right)}{d^{3}}$$

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
[Out] -((2*b*d*x + a*d)*cosh(d*x + c) - (b*d^2*x^2 + a*d^2*x + 2*b)*sinh(d*x + c)
)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int x(a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{ax\sinh(c+dx)}{d} - \frac{a\cosh(c+dx)}{d^2} + \frac{bx^2\sinh(c+dx)}{d} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\cosh(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**2*sinh(c + d*x
)/d - 2*b*x*cosh(c + d*x)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x**
2/2 + b*x**3/3)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(64) = 128.

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.50

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{1}{12}d\left(\frac{3\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)ae^{(dx)}}{d^3} + \frac{3\,(d^2x^2 + 2\,dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - \cdots}{d^4}\right. + \frac{1}{6}\left(2\,bx^3 + 3\,ax^2\right)\cosh(dx+c)$$

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/12*d*(3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 3*(d^2*x^2 + 2
*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c -
 6*e^c)*b*e^(d*x)/d^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^(-d*x - c)/
d^4) + 1/6*(2*b*x^3 + 3*a*x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x(a+bx)\cosh(c+dx)\,dx = \frac{(bd^2x^2 + ad^2x - 2\,bdx - ad + 2\,b)e^{(dx+c)}}{2\,d^3} - \frac{(bd^2x^2 + ad^2x + 2\,bdx + ad + 2\,b)e^{(-dx-c)}}{2\,d^3}$$

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + a*d^2*x - 2*b*d*x - a*d + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^
2*x^2 + a*d^2*x + 2*b*d*x + a*d + 2*b)*e^(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x(a+bx)\cosh(c+dx)\,dx = \frac{b\,x^2\sinh(c+d\,x)+a\,x\sinh(c+d\,x)}{d} - \frac{a\cosh(c+d\,x)+2\,b\,x\cosh(c+d\,x)}{d^2} + \frac{2\,b\sinh(c+d\,x)}{d^3}$$

```
[In] int(x*cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (b*x^2*sinh(c + d*x) + a*x*sinh(c + d*x))/d - (a*cosh(c + d*x) + 2*b*x*cosh
(c + d*x))/d^2 + (2*b*sinh(c + d*x))/d^3
```

3.4 $\int (a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (a+bx)\cosh(c+dx)\,dx=-\frac{b\cosh(c+dx)}{d^2}+\frac{(a+bx)\sinh(c+dx)}{d}$$

```
[Out] -b*cosh(d*x+c)/d^2+(b*x+a)*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3377, 2718}

$$\int (a+bx)\cosh(c+dx)\,dx=\frac{(a+bx)\sinh(c+dx)}{d}-\frac{b\cosh(c+dx)}{d^2}$$

```
[In] Int[(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) + ((a + b*x)*Sinh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(a+bx)\sinh(c+dx)}{d} - \frac{b\int \sinh(c+dx)\,dx}{d}\\ &= -\frac{b\cosh(c+dx)}{d^2} + \frac{(a+bx)\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{-b\cosh(c+dx)+d(a+bx)\sinh(c+dx)}{d^2}$$

```
[In] Integrate[(a + b*x)*Cosh[c + d*x],x]
[Out] (-(b*Cosh[c + d*x]) + d*(a + b*x)*Sinh[c + d*x])/d^2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

method	result
parts	$\frac{bx\sinh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}-\frac{b\cosh(dx+c)}{d^2}$
parallelrisch	$\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)d(bx+a)+2b}{d^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(dxb+da-b)\mathrm{e}^{dx+c}}{2d^2}-\frac{(dxb+da+b)\mathrm{e}^{-dx-c}}{2d^2}$
derivativedivides	$\frac{\frac{b((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{bc\sinh(dx+c)}{d}+a\sinh(dx+c)}{d}$
default	$\frac{\frac{b((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{bc\sinh(dx+c)}{d}+a\sinh(dx+c)}{d}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{a\cosh(c)\sinh(dx)}{d}-\frac{a\sinh($

```
[In] int((b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
[Out] b*x*sinh(d*x+c)/d+a*sinh(d*x+c)/d-b*cosh(d*x+c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (a+bx)\cosh(c+dx)\,dx = -\frac{b\cosh(dx+c) - (bdx+ad)\sinh(dx+c)}{d^2}$$

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="fricas")
[Out] -(b*cosh(d*x + c) - (b*d*x + a*d)*sinh(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{a\sinh(c+dx)}{d} + \frac{bx\sinh(c+dx)}{d} - \frac{b\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right)\cosh(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)*cosh(d*x+c),x)
[Out] Piecewise((a*sinh(c + d*x)/d + b*x*sinh(c + d*x)/d - b*cosh(c + d*x)/d**2,
Ne(d, 0)), ((a*x + b*x**2/2)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{ae^{(dx+c)}}{2d} + \frac{(dxe^c - e^c)be^{(dx)}}{2d^2} - \frac{(dx+1)be^{(-dx-c)}}{2d^2} - \frac{ae^{(-dx-c)}}{2d}$$

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="maxima")
[Out] 1/2*a*e^(d*x + c)/d + 1/2*(d*x*e^c - e^c)*b*e^(d*x)/d^2 - 1/2*(d*x + 1)*b*e
^(-d*x - c)/d^2 - 1/2*a*e^(-d*x - c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{(bdx+ad-b)e^{(dx+c)}}{2\,d^2} - \frac{(bdx+ad+b)e^{(-dx-c)}}{2\,d^2}$$

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + a*d - b)*e^(d*x + c)/d^2 - 1/2*(b*d*x + a*d + b)*e^(-d*x - c)/
d^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{a\sinh(c+d\,x)+b\,x\sinh(c+d\,x)}{d} - \frac{b\cosh(c+d\,x)}{d^2}$$

```
[In] int(cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (a*sinh(c + d*x) + b*x*sinh(c + d*x))/d - (b*cosh(c + d*x))/d^2
```

3.5 $\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx)+\frac{b\sinh(c+dx)}{d}+a\sinh(c)\text{Shi}(dx)$$

```
[Out] a*Chi(d*x)*cosh(c)+a*Shi(d*x)*sinh(c)+b*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 2717, 3384, 3379, 3382}

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx)+a\sinh(c)\text{Shi}(dx)+\frac{b\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[
d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(b\cosh(c+dx) + \frac{a\cosh(c+dx)}{x}\right) dx \\
&= a\int \frac{\cosh(c+dx)}{x}\,dx + b\int \cosh(c+dx)\,dx \\
&= \frac{b\sinh(c+dx)}{d} + (a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + (a\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= a\cosh(c)\text{Chi}(dx) + \frac{b\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$
\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx) + \frac{b\cosh(dx)\sinh(c)}{d} + \frac{b\cosh(c)\sinh(dx)}{d} + a\sinh(c)\text{Shi}(dx)
$$

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x
])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{a\,e^{c}\,\mathrm{Ei}_1(-dx)}{2}-\frac{a\,e^{-c}\,\mathrm{Ei}_1(dx)}{2}-\frac{e^{-dx-c}b}{2d}+\frac{e^{dx+c}b}{2d}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d}-\frac{b\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}+\frac{2\,\mathrm{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)}{2}+a\,\mathrm{Shi}\,(dx)\,\mathrm{si}$

[In] int((b*x+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)

[Out] -1/2*a*exp(c)*Ei(1,-d*x)-1/2*a*exp(-c)*Ei(1,d*x)-1/2/d*exp(-d*x-c)*b+1/2/d*exp(d*x+c)*b

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = \frac{(ad\mathrm{Ei}(dx)+ad\mathrm{Ei}(-dx))\cosh{(c)}+2\,b\sinh{(dx+c)}+(ad\mathrm{Ei}(dx)-ad\mathrm{Ei}(-dx))\sinh{(c)}}{2\,d}$$

[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="fricas")

[Out] 1/2*((a*d*Ei(d*x) + a*d*Ei(-d*x))*cosh(c) + 2*b*sinh(d*x + c) + (a*d*Ei(d*x) - a*d*Ei(-d*x))*sinh(c))/d

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = -a(-\sinh{(c)}\,\mathrm{Shi}\,(dx)-\cosh{(c)}\,\mathrm{Chi}\,(dx)) - b\left(\begin{cases}-x\cosh{(c)} & \text{for}\ d=0\\ -\frac{\sinh{(c+dx)}}{d} & \text{otherwise}\end{cases}\right)$$

[In] integrate((b*x+a)*cosh(d*x+c)/x,x)

[Out] -a*(-sinh(c)*Shi(d*x) - cosh(c)*Chi(d*x)) - b*Piecewise((-x*cosh(c), Eq(d, 0)), (-sinh(c + d*x)/d, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.46

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = -\frac{1}{2}\left(b\left(\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^{2}}+\frac{(dx+1)e^{(-dx-c)}}{d^{2}}\right)+\frac{2\,a\cosh\left(dx+c\right)\log\left(x\right)}{d}-\frac{\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)a}{d}\right. + (bx+a\log\left(x\right))\cosh\left(dx+c\right)$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + 2*a*co
sh(d*x + c)*log(x)/d - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + (b*x + a*lo
g(x))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = \frac{ad\mathrm{Ei}(-dx)\,e^{(-c)}+ad\mathrm{Ei}(dx)\,e^{c}+be^{(dx+c)}-be^{(-dx-c)}}{2\,d}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a*d*Ei(-d*x)*e^(-c) + a*d*Ei(d*x)*e^c + b*e^(d*x + c) - b*e^(-d*x - c)
)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = a\,\mathrm{coshint}(d\,x)\,\cosh(c)+a\,\mathrm{sinhint}(d\,x)\,\sinh(c)+\frac{b\sinh(c+d\,x)}{d}$$

```
[In] int((cosh(c + d*x)*(a + b*x))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) + (b*sinh(c + d*x))/d
```

3.6 $\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c+dx)}{x} + b\cosh(c)\mathrm{Chi}(dx) + ad\mathrm{Chi}(dx)\sinh(c) + ad\cosh(c)\mathrm{Shi}(dx) + b\sinh(c)\mathrm{Shi}(dx)$$

```
[Out] b*Chi(d*x)*cosh(c)-a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c
)+b*Shi(d*x)*sinh(c)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = ad\sinh(c)\mathrm{Chi}(dx) + ad\cosh(c)\mathrm{Shi}(dx) - \frac{a\cosh(c+dx)}{x} + b\cosh(c)\mathrm{Chi}(dx) + b\sinh(c)\mathrm{Shi}(dx)$$

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] + a*d*CoshIntegral[d*x
]*Sinh[c] + a*d*Cosh[c]*SinhIntegral[d*x] + b*Sinh[c]*SinhIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^2}+\frac{b\cosh(c+dx)}{x}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^2}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{x}+(ad)\int \frac{\sinh(c+dx)}{x}\,dx \\
&\quad +(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{x}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx) \\
&\quad +(ad\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(ad\sinh(c))\int \frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{x}+b\cosh(c)\text{Chi}(dx)+ad\text{Chi}(dx)\sinh(c) \\
&\quad +ad\cosh(c)\text{Shi}(dx)+b\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c)\cosh(dx)}{x} + b\cosh(c)\text{Chi}(dx) - \frac{a\sinh(c)\sinh(dx)}{x} + b\sinh(c)\text{Shi}(dx) + ad(\text{Chi}(dx)\sinh(c) + \cosh(c)\text{Shi}(dx))$$

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c]*Cosh[d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] - (a*Sinh[c]*Sinh[
d*x])/x + b*Sinh[c]*SinhIntegral[d*x] + a*d*(CoshIntegral[d*x]*Sinh[c] + Co
sh[c]*SinhIntegral[d*x])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)adx - e^{-c}\,\text{Ei}_1(dx)adx + e^{c}\,\text{Ei}_1(-dx)bx + e^{-c}\,\text{Ei}_1(dx)bx + e^{-dx-c}a + a\,e^{dx+c}}{2x}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}} + \frac{2\,\text{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)}{2} + b\,\text{Shi}\,(dx)\sinh\,(c) + \frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}} - \frac{4i\,\text{Shi}(dx)}{\sqrt{\pi}}\right)}{4} +$

```
[In] int((b*x+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(exp(c)*Ei(1,-d*x)*a*d*x-exp(-c)*Ei(1,d*x)*a*d*x+exp(c)*Ei(1,-d*x)*b*x
+exp(-c)*Ei(1,d*x)*b*x+exp(-d*x-c)*a+a*exp(d*x+c))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{2\,a\cosh\,(dx+c) - ((ad+b)x\text{Ei}(dx) - (ad-b)x\text{Ei}(-dx))\cosh\,(c) - ((ad+b)x\text{Ei}(dx) + (ad-b)x\text{Ei}(-}{2\,x}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c) - ((a*d + b)*x*Ei(d*x) - (a*d - b)*x*Ei(-d*x))*cosh
(c) - ((a*d + b)*x*Ei(d*x) + (a*d - b)*x*Ei(-d*x))*sinh(c))/x
```

Sympy [F]

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx)\cosh{(c+dx)}}{x^2}\,dx$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**2,x)
[Out] Integral((a + b*x)*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{1}{2}\left((\mathrm{Ei}(-dx)\,e^{(-c)} - \mathrm{Ei}(dx)\,e^{c})a + \frac{2\,b\cosh{(dx+c)}\log{(x)}}{d} - \frac{(\mathrm{Ei}(-dx)\,e^{(-c)} + \mathrm{Ei}(dx)\,e^{c})b}{d}\right)d + \left(b\log{(x)} - \frac{a}{x}\right)\cosh{(dx+c)}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
[Out] -1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a + 2*b*cosh(d*x + c)*log(x)/d - (Ei(
-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + (b*log(x) - a/x)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{adx\mathrm{Ei}(-dx)\,e^{(-c)} - adx\mathrm{Ei}(dx)\,e^{c} - bx\mathrm{Ei}(-dx)\,e^{(-c)} - bx\mathrm{Ei}(dx)\,e^{c} + ae^{(dx+c)} + ae^{(-dx-c)}}{2\,x}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c - b*x*Ei(-d*x)*e^(-c) - b*x
*Ei(d*x)*e^c + a*e^(d*x + c) + a*e^(-d*x - c))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx=\int \frac{\cosh(c+d\,x)\,\,(a+b\,x)}{x^2}\,dx$$

[In] int((cosh(c + d*x)*(a + b*x))/x^2,x)

[Out] int((cosh(c + d*x)*(a + b*x))/x^2, x)

3.7 $\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\begin{aligned}\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = & -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}+\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)\\ & +bd\text{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{2x}\\ & +bd\cosh(c)\text{Shi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)\end{aligned}$$

```
[Out] 1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2-b*cosh(d*x+c)/x+b*d*cosh(c
)*Shi(d*x)+b*d*Chi(d*x)*sinh(c)+1/2*a*d^2*Shi(d*x)*sinh(c)-1/2*a*d*sinh(d*x
+c)/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = & \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)\\ & -\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+bd\sinh(c)\text{Chi}(dx)\\ & +bd\cosh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{x}\end{aligned}$$

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x])/x^2 - (b*Cosh[c + d*x])/x + (a*d^2*Cosh[c]*CoshInteg
ral[d*x])/2 + b*d*CoshIntegral[d*x]*Sinh[c] - (a*d*Sinh[c + d*x])/(2*x) + b
*d*Cosh[c]*SinhIntegral[d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x^2}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^3}\,dx+b\int \frac{\cosh(c+dx)}{x^2}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx+(bd)\int \frac{\sinh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x}\,dx \\
&\quad +(bd\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(bd\sinh(c))\int \frac{\cosh(dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} + bd\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{2x} + bd\cosh(c)\text{Shi}(dx) + \frac{1}{2}\left(ad^2\cosh(c)\right)\int\frac{\cosh(dx)}{x}\,dx + \frac{1}{2}\left(ad^2\sinh(c)\right)\int\frac{\sinh(dx)}{x}\,dx$$

$$= -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + bd\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{2x} + bd\cosh(c)\text{Shi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int\frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \frac{-a\cosh(c+dx) - 2bx\cosh(c+dx) + dx^2\text{Chi}(dx)(ad\cosh(c) + 2b\sinh(c)) - adx\sinh(c+dx) + dx^2(2b}{2x^2}$$

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

```
[Out] (-(a*Cosh[c + d*x]) - 2*b*x*Cosh[c + d*x] + d*x^2*CoshIntegral[d*x]*(a*d*Co
sh[c] + 2*b*Sinh[c]) - a*d*x*Sinh[c + d*x] + d*x^2*(2*b*Cosh[c] + a*d*Sinh[
c])*SinhIntegral[d*x])/(2*x^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)a\,d^2x^2+e^{-c}\,\text{Ei}_1(dx)a\,d^2x^2+2\,e^{c}\,\text{Ei}_1(-dx)bd\,x^2-2\,e^{-c}\,\text{Ei}_1(dx)bd\,x^2-e^{-dx-c}adx+e^{dx+c}adx+2\,e^{-dx-c}bx+2\,e^{dx+c}bx}{4x^2}$
meijerg	$\frac{idb\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\text{Shi}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{db\sinh(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)+4\ln(id)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}+\frac{4\,\text{Chi}(dx)-4\ln(dx)-4\gamma}{\sqrt{\pi}}\right)}{4}-\frac{a\cosh}{}$

```
[In] int((b*x+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(exp(c)*Ei(1,-d*x)*a*d^2*x^2+exp(-c)*Ei(1,d*x)*a*d^2*x^2+2*exp(c)*Ei(1
,-d*x)*b*d*x^2-2*exp(-c)*Ei(1,d*x)*b*d*x^2-exp(-d*x-c)*a*d*x+exp(d*x+c)*a*d
*x+2*exp(-d*x-c)*b*x+2*exp(d*x+c)*b*x+exp(-d*x-c)*a+a*exp(d*x+c))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = -\frac{2\,adx\sinh\left(dx+c\right)+2\left(2\,bx+a\right)\cosh\left(dx+c\right)-\left((ad^2+2\,bd)x^2\mathrm{Ei}(dx)+(ad^2-2\,bd)x^2\mathrm{Ei}(-dx)\right)\cosh (}{4\,x^2}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*(2*b*x + a)*cosh(d*x + c) - ((a*d^2 + 2*b*d
)*x^2*Ei(d*x) + (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b*d)*x^
2*Ei(d*x) - (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx &= \frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx)+ade^{c}\Gamma(-1,-dx)-2\,b\mathrm{Ei}(-dx)\,e^{(-c)}+2\,b\mathrm{Ei}(dx)\,e^{c}\right)d \\ &\quad -\frac{(2\,bx+a)\cosh\left(dx+c\right)}{2\,x^2}\end{aligned}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*b*Ei(-d*x)*e^(
-c) + 2*b*Ei(d*x)*e^c)*d - 1/2*(2*b*x + a)*cosh(d*x + c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \frac{ad^2x^2\text{Ei}(-dx)\,e^{(-c)} + ad^2x^2\text{Ei}(dx)\,e^{c} - 2\,bdx^2\text{Ei}(-dx)\,e^{(-c)} + 2\,bdx^2\text{Ei}(dx)\,e^{c} - adxe^{(dx+c)} + adxe^{(-dx-c)}}{4\,x^2}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c - 2*b*d*x^2*Ei(-d*x)
*e^(-c) + 2*b*d*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) -
2*b*x*e^(d*x + c) - 2*b*x*e^(-d*x - c) - a*e^(d*x + c) - a*e^(-d*x - c))/x^
2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^3}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x))/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^3, x)
```

3.8 $\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 132

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) + \frac{1}{6}ad^3\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{6x^2} - \frac{bd\sinh(c+dx)}{2x} + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx) + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)$$

```
[Out] 1/2*b*d^2*Chi(d*x)*cosh(c)-1/3*a*cosh(d*x+c)/x^3-1/2*b*cosh(d*x+c)/x^2-1/6*
a*d^2*cosh(d*x+c)/x+1/6*a*d^3*cosh(c)*Shi(d*x)+1/6*a*d^3*Chi(d*x)*sinh(c)+1
/2*b*d^2*Shi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2-1/2*b*d*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}ad^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx) - \frac{ad^2\cosh(c+dx)}{6x} - \frac{a\cosh(c+dx)}{3x^3} - \frac{ad\sinh(c+dx)}{6x^2} + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx) - \frac{b\cosh(c+dx)}{2x^2} - \frac{bd\sinh(c+dx)}{2x}$$

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a*Cosh[c + d*x])/x^3 - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(6*x) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^3*CoshIntegral[d*x]*
Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) - (b*d*Sinh[c + d*x])/(2*x) + (a*d
^3*Cosh[c]*SinhIntegral[d*x])/6 + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^3}\right)\,dx\\
&= a\int \frac{\cosh(c+dx)}{x^4}\,dx+b\int \frac{\cosh(c+dx)}{x^3}\,dx\\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx+\frac{1}{2}(bd)\int \frac{\sinh(c+dx)}{x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{6x^2} - \frac{bd\sinh(c+dx)}{2x} \\
&\quad + \frac{1}{6}(ad^2)\int \frac{\cosh(c+dx)}{x^2}\,dx + \frac{1}{2}(bd^2)\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} \\
&\quad - \frac{ad\sinh(c+dx)}{6x^2} - \frac{bd\sinh(c+dx)}{2x} + \frac{1}{6}(ad^3)\int \frac{\sinh(c+dx)}{x}\,dx \\
&\quad + \frac{1}{2}(bd^2\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + \frac{1}{2}(bd^2\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) \\
&\quad - \frac{ad\sinh(c+dx)}{6x^2} - \frac{bd\sinh(c+dx)}{2x} + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx) \\
&\quad + \frac{1}{6}(ad^3\cosh(c))\int \frac{\sinh(dx)}{x}\,dx + \frac{1}{6}(ad^3\sinh(c))\int \frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} \\
&\quad + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) + \frac{1}{6}ad^3\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{6x^2} \\
&\quad - \frac{bd\sinh(c+dx)}{2x} + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx) + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$
\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{2a\cosh(c+dx) + 3bx\cosh(c+dx) + ad^2x^2\cosh(c+dx) - d^2x^3\text{Chi}(dx)(3b\cosh(c) + ad\sinh(c)) + adx}{6x^3}
$$

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a*Cosh[c + d*x] + 3*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d
^2*x^3*CoshIntegral[d*x]*(3*b*Cosh[c] + a*d*Sinh[c]) + a*d*x*Sinh[c + d*x]
+ 3*b*d*x^2*Sinh[c + d*x] - d^2*x^3*(a*d*Cosh[c] + 3*b*Sinh[c])*SinhIntegra
l[d*x])/x^3
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{e^{c} \operatorname{Ei}_{1}(-dx) a\, d^{3} x^{3}-e^{-c} \operatorname{Ei}_{1}(dx) a\, d^{3} x^{3}+3\, e^{c} \operatorname{Ei}_{1}(-dx) b\, d^{2} x^{3}+3\, e^{-c} \operatorname{Ei}_{1}(dx) b\, d^{2} x^{3}+e^{dx+c} a\, d^{2} x^{2}+e^{-dx-c} a\, d^{2} x^{2}+3\, e^{dx+c} b d\, x^{2}-3}{12 x^{3}}$
meijerg	$-\frac{d^{2} b \cosh(c) \sqrt{\pi}\left(\frac{4}{\sqrt{\pi}\, x^{2} d^{2}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}-\frac{4\left(\frac{9x^{2}d^{2}}{2}+3\right)}{3\sqrt{\pi}\, x^{2} d^{2}}+\frac{4\cosh(dx)}{\sqrt{\pi}\, x^{2} d^{2}}+\frac{4\sinh(dx)}{\sqrt{\pi}\, x d}-\frac{4(\operatorname{Chi}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}\right)}{8}+\frac{i d^{2} b \sinh(c)}{}$

[In] int((b*x+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/12*(exp(c)*Ei(1,-d*x)*a*d^3*x^3-exp(-c)*Ei(1,d*x)*a*d^3*x^3+3*exp(c)*Ei(
1,-d*x)*b*d^2*x^3+3*exp(-c)*Ei(1,d*x)*b*d^2*x^3+exp(d*x+c)*a*d^2*x^2+exp(-d
*x-c)*a*d^2*x^2+3*exp(d*x+c)*b*d*x^2-3*exp(-d*x-c)*b*d*x^2+exp(d*x+c)*a*d*x
-exp(-d*x-c)*a*d*x+3*exp(d*x+c)*b*x+3*exp(-d*x-c)*b*x+2*a*exp(d*x+c)+2*exp(
-d*x-c)*a)/x^3

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{2\,(ad^2x^2+3\,bx+2\,a)\cosh\left(dx+c\right)-((ad^3+3\,bd^2)x^3\mathrm{Ei}(dx)-(ad^3-3\,bd^2)x^3\mathrm{Ei}(-dx))\cosh\left(c\right)+2\,(3}{12\,x^3}$$

[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*(a*d^2*x^2 + 3*b*x + 2*a)*cosh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*E
i(d*x) - (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(3*b*d*x^2 + a*d*x)*si
nh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) + (a*d^3 - 3*b*d^2)*x^3*Ei(-d*
x))*sinh(c))/x^3

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \text{Timed out}$$

[In] integrate((b*x+a)*cosh(d*x+c)/x**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \frac{1}{12}\left(2\,ad^2e^{(-c)}\Gamma(-2,dx) - 2\,ad^2e^{c}\Gamma(-2,-dx) + 3\,bde^{(-c)}\Gamma(-1,dx) + 3\,bde^{c}\Gamma(-1,-dx)\right)d - \frac{(3\,bx+2\,a)\cosh(dx+c)}{6\,x^3}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(2*a*d^2*e^(-c)*gamma(-2, d*x) - 2*a*d^2*e^c*gamma(-2, -d*x) + 3*b*d*e
^(-c)*gamma(-1, d*x) + 3*b*d*e^c*gamma(-1, -d*x))*d - 1/6*(3*b*x + 2*a)*cos
h(d*x + c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^{c} - 3\,bd^2x^3\mathrm{Ei}(-dx)\,e^{(-c)} - 3\,bd^2x^3\mathrm{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2x^2e}{}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c - 3*b*d^2*x^3*Ei(-
d*x)*e^(-c) - 3*b*d^2*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e
^(-d*x - c) + 3*b*d*x^2*e^(d*x + c) - 3*b*d*x^2*e^(-d*x - c) + a*d*x*e^(d*x
 + c) - a*d*x*e^(-d*x - c) + 3*b*x*e^(d*x + c) + 3*b*x*e^(-d*x - c) + 2*a*e
^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^4}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x))/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^4, x)
```

3.9 $\int \frac{(a+bx)\cosh(c+dx)}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 166

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5} dx = -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{24x^2} - \frac{bd^2\cosh(c+dx)}{6x} + \frac{1}{24}ad^4\cosh(c)\mathrm{Chi}(dx) + \frac{1}{6}bd^3\mathrm{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{6x^2} - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{6}bd^3\cosh(c)\mathrm{Shi}(dx) + \frac{1}{24}ad^4\sinh(c)\mathrm{Shi}(dx)$$

```
[Out] 1/24*a*d^4*Chi(d*x)*cosh(c)-1/4*a*cosh(d*x+c)/x^4-1/3*b*cosh(d*x+c)/x^3-1/2
4*a*d^2*cosh(d*x+c)/x^2-1/6*b*d^2*cosh(d*x+c)/x+1/6*b*d^3*cosh(c)*Shi(d*x)+
1/6*b*d^3*Chi(d*x)*sinh(c)+1/24*a*d^4*Shi(d*x)*sinh(c)-1/12*a*d*sinh(d*x+c)
/x^3-1/6*b*d*sinh(d*x+c)/x^2-1/24*a*d^3*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{6874, 3378, 3384, 3379, 3382\}$

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = \frac{1}{24}ad^4\cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4\sinh(c)\text{Shi}(dx) - \frac{ad^3\sinh(c+dx)}{24x} - \frac{ad^2\cosh(c+dx)}{24x^2} - \frac{a\cosh(c+dx)}{4x^4} - \frac{ad\sinh(c+dx)}{12x^3} + \frac{1}{6}bd^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}bd^3\cosh(c)\text{Shi}(dx) - \frac{bd^2\cosh(c+dx)}{6x} - \frac{b\cosh(c+dx)}{3x^3} - \frac{bd\sinh(c+dx)}{6x^2}$$

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a*Cosh[c + d*x])/x^4 - (b*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*
x])/(24*x^2) - (b*d^2*Cosh[c + d*x])/(6*x) + (a*d^4*Cosh[c]*CoshIntegral[d*
x])/24 + (b*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(12*x^3)
 - (b*d*Sinh[c + d*x])/(6*x^2) - (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^3*Cosh
[c]*SinhIntegral[d*x])/6 + (a*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^4}\right)dx\\
&= a\int \frac{\cosh(c+dx)}{x^5}\,dx+b\int \frac{\cosh(c+dx)}{x^4}\,dx\\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}+\frac{1}{4}(ad)\int \frac{\sinh(c+dx)}{x^4}\,dx+\frac{1}{3}(bd)\int \frac{\sinh(c+dx)}{x^3}\,dx\\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{6x^2}\\
&\quad+\frac{1}{12}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x^3}\,dx+\frac{1}{6}\left(bd^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}\\
&\quad-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{6x^2}+\frac{1}{24}\left(ad^3\right)\int \frac{\sinh(c+dx)}{x^2}\,dx\\
&\quad+\frac{1}{6}\left(bd^3\right)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}\\
&\quad-\frac{bd^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{6x^2}\\
&\quad-\frac{ad^3\sinh(c+dx)}{24x}+\frac{1}{24}\left(ad^4\right)\int \frac{\cosh(c+dx)}{x}\,dx\\
&\quad+\frac{1}{6}\left(bd^3\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx+\frac{1}{6}\left(bd^3\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}\\
&\quad-\frac{bd^2\cosh(c+dx)}{6x}+\frac{1}{6}bd^3\mathrm{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{12x^3}\\
&\quad-\frac{bd\sinh(c+dx)}{6x^2}-\frac{ad^3\sinh(c+dx)}{24x}+\frac{1}{6}bd^3\cosh(c)\mathrm{Shi}(dx)\\
&\quad+\frac{1}{24}\left(ad^4\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx+\frac{1}{24}\left(ad^4\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{24x^2} - \frac{bd^2\cosh(c+dx)}{6x} + \frac{1}{24}ad^4\cosh(c)\text{Chi}(dx) + \frac{1}{6}bd^3\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{6x^2} - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{6}bd^3\cosh(c)\text{Shi}(dx) + \frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = -\frac{6a\cosh(c+dx) + 8bx\cosh(c+dx) + ad^2x^2\cosh(c+dx) + 4bd^2x^3\cosh(c+dx) - d^3x^4\text{Chi}(dx)(ad\cosh(}{}$$

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/24*(6*a*Cosh[c + d*x] + 8*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] +
4*b*d^2*x^3*Cosh[c + d*x] - d^3*x^4*CoshIntegral[d*x]*(a*d*Cosh[c] + 4*b*Si
nh[c]) + 2*a*d*x*Sinh[c + d*x] + 4*b*d*x^2*Sinh[c + d*x] + a*d^3*x^3*Sinh[c
 + d*x] - d^3*x^4*(4*b*Cosh[c] + a*d*Sinh[c])*SinhIntegral[d*x])/x^4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)a\,d^4x^4+e^{-c}\,\text{Ei}_1(dx)a\,d^4x^4+4\,e^{c}\,\text{Ei}_1(-dx)b\,d^3x^4-4\,e^{-c}\,\text{Ei}_1(dx)b\,d^3x^4+e^{dx+c}a\,d^3x^3-e^{-dx-c}a\,d^3x^3+4\,e^{dx+c}b\,d^2x^3+4}{}$
meijerg	$-\frac{id^3b\cosh(c)\sqrt{\pi}\left(-\frac{8i\left(x^2d^2+2\right)\cosh(dx)}{3d^3x^3\sqrt{\pi}}-\frac{8i\sinh(dx)}{3x^2d^2\sqrt{\pi}}+\frac{8i\,\text{Shi}(dx)}{3\sqrt{\pi}}\right)}{16} - \frac{d^3b\sinh(c)\sqrt{\pi}\left(\frac{8}{\sqrt{\pi}\,x^2d^2}-\frac{4\left(2\gamma-\frac{11}{3}+2\ln(x)+2\ln(id)\right)}{3\sqrt{\pi}}-\frac{8\left(\frac{55x^2}{2}\right.}{45\sqrt{\pi}}\right.}{}$

```
[In] int((b*x+a)*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(exp(c)*Ei(1,-d*x)*a*d^4*x^4+exp(-c)*Ei(1,d*x)*a*d^4*x^4+4*exp(c)*Ei(
1,-d*x)*b*d^3*x^4-4*exp(-c)*Ei(1,d*x)*b*d^3*x^4+exp(d*x+c)*a*d^3*x^3-exp(-d
*x-c)*a*d^3*x^3+4*exp(d*x+c)*b*d^2*x^3+4*exp(-d*x-c)*b*d^2*x^3+exp(d*x+c)*a
*d^2*x^2+exp(-d*x-c)*a*d^2*x^2+4*exp(d*x+c)*b*d*x^2-4*exp(-d*x-c)*b*d*x^2+2
*exp(d*x+c)*a*d*x-2*exp(-d*x-c)*a*d*x+8*exp(d*x+c)*b*x+8*exp(-d*x-c)*b*x+6*
a*exp(d*x+c)+6*exp(-d*x-c)*a)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx =$$
$$-\frac{2\,(4\,bd^2x^3 + ad^2x^2 + 8\,bx + 6\,a)\cosh\left(dx+c\right) - ((ad^4 + 4\,bd^3)x^4\mathrm{Ei}(dx) + (ad^4 - 4\,bd^3)x^4\mathrm{Ei}(-dx))\cosh}{}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(4*b*d^2*x^3 + a*d^2*x^2 + 8*b*x + 6*a)*cosh(d*x + c) - ((a*d^4 +
4*b*d^3)*x^4*Ei(d*x) + (a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*cosh(c) + 2*(a*d^3*x
^3 + 4*b*d*x^2 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 4*b*d^3)*x^4*Ei(d*x) -
(a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{1}{24}\left(3\,ad^3e^{(-c)}\Gamma(-3,dx) + 3\,ad^3e^c\Gamma(-3,-dx) + 4\,bd^2e^{(-c)}\Gamma(-2,dx) - 4\,bd^2e^c\Gamma(-2,-dx)\right)d$$
$$-\frac{(4\,bx + 3\,a)\cosh\left(dx+c\right)}{12\,x^4}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a*d^3*e^(-c)*gamma(-3, d*x) + 3*a*d^3*e^c*gamma(-3, -d*x) + 4*b*d^2
*e^(-c)*gamma(-2, d*x) - 4*b*d^2*e^c*gamma(-2, -d*x))*d - 1/12*(4*b*x + 3*a
)*cosh(d*x + c)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{ad^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + ad^4x^4\mathrm{Ei}(dx)\,e^{c} - 4\,bd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,bd^3x^4\mathrm{Ei}(dx)\,e^{c} - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-c}}{}$$

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c - 4*b*d^3*x^4*Ei(-d
*x)*e^(-c) + 4*b*d^3*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*e^
(-d*x - c) - 4*b*d^2*x^3*e^(d*x + c) - 4*b*d^2*x^3*e^(-d*x - c) - a*d^2*x^2
*e^(d*x + c) - a*d^2*x^2*e^(-d*x - c) - 4*b*d*x^2*e^(d*x + c) + 4*b*d*x^2*e
^(-d*x - c) - 2*a*d*x*e^(d*x + c) + 2*a*d*x*e^(-d*x - c) - 8*b*x*e^(d*x + c
) - 8*b*x*e^(-d*x - c) - 6*a*e^(d*x + c) - 6*a*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^5}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x))/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^5, x)
```

3.10 $\int x^2(a+bx)^2\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = -\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{24b^2\sinh(c+dx)}{d^5}+\frac{2a^2\sinh(c+dx)}{d^3}+\frac{12abx\sinh(c+dx)}{d^3}+\frac{12b^2x^2\sinh(c+dx)}{d^3}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^4\sinh(c+dx)}{d}$$

```
[Out] -12*a*b*cosh(d*x+c)/d^4-24*b^2*x*cosh(d*x+c)/d^4-2*a^2*x*cosh(d*x+c)/d^2-6*
a*b*x^2*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^2+24*b^2*sinh(d*x+c)/d^5+2*
a^2*sinh(d*x+c)/d^3+12*a*b*x*sinh(d*x+c)/d^3+12*b^2*x^2*sinh(d*x+c)/d^3+a^2
*x^2*sinh(d*x+c)/d+2*a*b*x^3*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used

$= \{6874, 3377, 2717, 2718\}$

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = \frac{2a^2\sinh(c+dx)}{d^3} - \frac{2a^2x\cosh(c+dx)}{d^2} + \frac{a^2x^2\sinh(c+dx)}{d} - \frac{12ab\cosh(c+dx)}{d^4} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{24b^2\sinh(c+dx)}{d^5} - \frac{24b^2x\cosh(c+dx)}{d^4} + \frac{12b^2x^2\sinh(c+dx)}{d^3} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{b^2x^4\sinh(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[
c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])/d
^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*Sin
h[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/
d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(a^2 x^2 \cosh(c+dx) + 2abx^3 \cosh(c+dx) + b^2 x^4 \cosh(c+dx)\right) dx$$

$$= a^2 \int x^2 \cosh(c+dx)\, dx + (2ab) \int x^3 \cosh(c+dx)\, dx + b^2 \int x^4 \cosh(c+dx)\, dx$$

$$= \frac{a^2 x^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2 x^4 \sinh(c+dx)}{d} - \frac{(2a^2) \int x \sinh(c+dx)\, dx}{d} - \frac{(6ab) \int x^2 \sinh(c+dx)\, dx}{d} - \frac{(4b^2) \int x^3 \sinh(c+dx)\, dx}{d}$$

$$= -\frac{2a^2 x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{4b^2 x^3 \cosh(c+dx)}{d^2} + \frac{a^2 x^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2 x^4 \sinh(c+dx)}{d} + \frac{(2a^2) \int \cosh(c+dx)\, dx}{d^2} + \frac{(12ab) \int x \cosh(c+dx)\, dx}{d^2} + \frac{(12b^2) \int x^2 \cosh(c+dx)\, dx}{d^2}$$

$$= -\frac{2a^2 x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{4b^2 x^3 \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{12abx \sinh(c+dx)}{d^3} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3} + \frac{a^2 x^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2 x^4 \sinh(c+dx)}{d} - \frac{(12ab) \int \sinh(c+dx)\, dx}{d^3} - \frac{(24b^2) \int x \sinh(c+dx)\, dx}{d^3}$$

$$= -\frac{12ab \cosh(c+dx)}{d^4} - \frac{24b^2 x \cosh(c+dx)}{d^4} - \frac{2a^2 x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{4b^2 x^3 \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{12abx \sinh(c+dx)}{d^3} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3} + \frac{a^2 x^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2 x^4 \sinh(c+dx)}{d} + \frac{(24b^2) \int \cosh(c+dx)\, dx}{d^4}$$

$$= -\frac{12ab \cosh(c+dx)}{d^4} - \frac{24b^2 x \cosh(c+dx)}{d^4} - \frac{2a^2 x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{4b^2 x^3 \cosh(c+dx)}{d^2} + \frac{24b^2 \sinh(c+dx)}{d^5} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{12abx \sinh(c+dx)}{d^3} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3} + \frac{a^2 x^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2 x^4 \sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.54

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$= \frac{-2d(a+2bx)\left(ad^2x+b(6+d^2x^2)\right)\cosh(c+dx)+\left(a^2d^2(2+d^2x^2)+2abd^2x(6+d^2x^2)+b^2(24+12d^2x^2\ -}{d^5}$$

```
[In] Integrate[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*(a + 2*b*x)*(a*d^2*x + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^2*(2 +
 d^2*x^2) + 2*a*b*d^2*x*(6 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Si
nh[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{2dx((2bx+a)(bx+a)d^2+12b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(-x^2(bx+a)^2d^4+2(-6x^2b^2-6abx-a^2)d^2-24b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2(bx+}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b^2x^4d^4+2ab\,d^4x^3+a^2d^4x^2-4b^2d^3x^3-6ab\,d^3x^2-2a^2d^3x+12x^2d^2b^2+12ab\,d^2x+2a^2d^2-24b^2dx-12dab+24b^2)e^{dx+c}}{2d^5} -$
parts	$\frac{b^2x^4\sinh(dx+c)}{d}+\frac{2ab\,x^3\sinh(dx+c)}{d}+\frac{a^2x^2\sinh(dx+c)}{d}-\frac{2\left(\frac{2b^2\left((dx+c)^3\cosh(dx+c)-3(dx+c)^2\sinh(dx+c)+6(dx+c)\right.}{d^3}\right.}{}$
meijerg	$-\frac{16ib^2\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b^2\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\right.}{}\right.}{}$
derivativedivides	$\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4b^2c\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+\right.}{d^2}$
default	$\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4b^2c\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+\right.}{d^2}$

```
[In] int(x^2*(b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x*((2*b*x+a)*(b*x+a)*d^2+12*b^2)*tanh(1/2*d*x+1/2*c)^2+(-x^2*(b*x+a)^2
*d^4+2*(-6*b^2*x^2-6*a*b*x-a^2)*d^2-24*b^2)*tanh(1/2*d*x+1/2*c)+(b*x+a)*d*(
x*(2*b*x+a)*d^2+12*b))/d^5/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx =$$
$$-\frac{2\left(2\,b^2d^3x^3+3\,abd^3x^2+6\,abd+(a^2d^3+12\,b^2d)x\right)\cosh\left(dx+c\right)-\left(b^2d^4x^4+2\,abd^4x^3+12\,abd^2x+2\,a^2\right.}{d^5}$$

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 + 6*a*b*d + (a^2*d^3 + 12*b^2*d)*x)*cosh
(d*x + c) - (b^2*d^4*x^4 + 2*a*b*d^4*x^3 + 12*a*b*d^2*x + 2*a^2*d^2 + (a^2*
d^4 + 12*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.24

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$=\begin{cases}\frac{a^2x^2\sinh\left(c+dx\right)}{d}-\frac{2a^2x\cosh\left(c+dx\right)}{d^2}+\frac{2a^2\sinh\left(c+dx\right)}{d^3}+\frac{2abx^3\sinh\left(c+dx\right)}{d}-\frac{6abx^2\cosh\left(c+dx\right)}{d^2}+\frac{12abx\sinh\left(c+dx\right)}{d^3}-\frac{12ab\cosh}{d} \\ \left(\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}\right)\cosh\left(c\right)\end{cases}$$

```
[In] integrate(x**2*(b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2
*sinh(c + d*x)/d**3 + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)
/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**4
*sinh(c + d*x)/d - 4*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d
*x)/d**3 - 24*b**2*x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d,
 0)), ((a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.79

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx =$$
$$-\frac{1}{60}\,d\left(\frac{10\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)a^2e^{(dx)}}{d^4}+\frac{10\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)a^2e^{(-dx-c)}}{d^4}+\frac{15\left(d^4\right.}{}\right.$$
$$+\frac{1}{30}\left(6\,b^2x^5+15\,abx^4+10\,a^2x^3\right)\cosh\left(dx+c\right)$$

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/60*d*(10*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/d
^4 + 10*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 15*(d^4*x^
4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b*e^(d*x)/d
^5 + 15*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*b*e^(-d*x - c)/d
^5 + 6*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120
*d*x*e^c - 120*e^c)*b^2*e^(d*x)/d^6 + 6*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 +
 60*d^2*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6) + 1/30*(6*b^2*x^5 + 15*a
*b*x^4 + 10*a^2*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$= \frac{(b^2d^4x^4 + 2\,abd^4x^3 + a^2d^4x^2 - 4\,b^2d^3x^3 - 6\,abd^3x^2 - 2\,a^2d^3x + 12\,b^2d^2x^2 + 12\,abd^2x + 2\,a^2d^2 - 24\,b^2dx -}{2\,d^5}$$
$$- \frac{(b^2d^4x^4 + 2\,abd^4x^3 + a^2d^4x^2 + 4\,b^2d^3x^3 + 6\,abd^3x^2 + 2\,a^2d^3x + 12\,b^2d^2x^2 + 12\,abd^2x + 2\,a^2d^2 + 24\,b^2dx}{2\,d^5}$$

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 4*b^2*d^3*x^3 - 6*a*b*d^3*
x^2 - 2*a^2*d^3*x + 12*b^2*d^2*x^2 + 12*a*b*d^2*x + 2*a^2*d^2 - 24*b^2*d*x
- 12*a*b*d + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a
^2*d^4*x^2 + 4*b^2*d^3*x^3 + 6*a*b*d^3*x^2 + 2*a^2*d^3*x + 12*b^2*d^2*x^2 +
 12*a*b*d^2*x + 2*a^2*d^2 + 24*b^2*d*x + 12*a*b*d + 24*b^2)*e^(-d*x - c)/d^
5
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = \frac{2\sinh(c+d\,x)\,\left(a^2\,d^2+12\,b^2\right)}{d^5} - \frac{4\,b^2\,x^3\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^4\sinh(c+d\,x)}{d} - \frac{12\,a\,b\cosh(c+d\,x)}{d^4} - \frac{2\,x\cosh(c+d\,x)\,\left(a^2\,d^2+12\,b^2\right)}{d^4} + \frac{x^2\sinh(c+d\,x)\,\left(a^2\,d^2+12\,b^2\right)}{d^3} - \frac{6\,a\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x^3\sinh(c+d\,x)}{d} + \frac{12\,a\,b\,x\sinh(c+d\,x)}{d^3}$$

```
[In] int(x^2*cosh(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*x))/d^2 +
(b^2*x^4*sinh(c + d*x))/d - (12*a*b*cosh(c + d*x))/d^4 - (2*x*cosh(c + d*x)
*(12*b^2 + a^2*d^2))/d^4 + (x^2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^3 - (6*
a*b*x^2*cosh(c + d*x))/d^2 + (2*a*b*x^3*sinh(c + d*x))/d + (12*a*b*x*sinh(c
 + d*x))/d^3
```

3.11 $\int x(a+bx)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int x(a+bx)^2 \cosh(c+dx)\,dx = -\frac{6b^2\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{6b^2x\sinh(c+dx)}{d^3} + \frac{a^2x\sinh(c+dx)}{d} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d}$$

```
[Out] -6*b^2*cosh(d*x+c)/d^4-a^2*cosh(d*x+c)/d^2-4*a*b*x*cosh(d*x+c)/d^2-3*b^2*x^
2*cosh(d*x+c)/d^2+4*a*b*sinh(d*x+c)/d^3+6*b^2*x*sinh(d*x+c)/d^3+a^2*x*sinh(
d*x+c)/d+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^3*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2718, 2717}

$$\int x(a+bx)^2 \cosh(c+dx)\,dx = -\frac{a^2\cosh(c+dx)}{d^2} + \frac{a^2x\sinh(c+dx)}{d} + \frac{4ab\sinh(c+dx)}{d^3} - \frac{4abx\cosh(c+dx)}{d^2} + \frac{2abx^2\sinh(c+dx)}{d} - \frac{6b^2\cosh(c+dx)}{d^4} + \frac{6b^2x\sinh(c+dx)}{d^3} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + \frac{b^2x^3\sinh(c+dx)}{d}$$

```
[In] Int[x*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*
x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^
```

```
2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x]
)/d + (b^2*x^3*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2 x \cosh(c+dx) + 2abx^2 \cosh(c+dx) + b^2 x^3 \cosh(c+dx)\right) dx \\
&= a^2 \int x \cosh(c+dx)\, dx + (2ab) \int x^2 \cosh(c+dx)\, dx + b^2 \int x^3 \cosh(c+dx)\, dx \\
&= \frac{a^2 x \sinh(c+dx)}{d} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{b^2 x^3 \sinh(c+dx)}{d} \\
&\quad - \frac{a^2 \int \sinh(c+dx)\, dx}{d} - \frac{(4ab) \int x \sinh(c+dx)\, dx}{d} - \frac{(3b^2) \int x^2 \sinh(c+dx)\, dx}{d} \\
&= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{4abx \cosh(c+dx)}{d^2} - \frac{3b^2 x^2 \cosh(c+dx)}{d^2} \\
&\quad + \frac{a^2 x \sinh(c+dx)}{d} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{b^2 x^3 \sinh(c+dx)}{d} \\
&\quad + \frac{(4ab) \int \cosh(c+dx)\, dx}{d^2} + \frac{(6b^2) \int x \cosh(c+dx)\, dx}{d^2} \\
&= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{4abx \cosh(c+dx)}{d^2} - \frac{3b^2 x^2 \cosh(c+dx)}{d^2} \\
&\quad + \frac{4ab \sinh(c+dx)}{d^3} + \frac{6b^2 x \sinh(c+dx)}{d^3} + \frac{a^2 x \sinh(c+dx)}{d} \\
&\quad + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{b^2 x^3 \sinh(c+dx)}{d} - \frac{(6b^2) \int \sinh(c+dx)\, dx}{d^3}
\end{aligned}$$

$$= -\frac{6b^2\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{6b^2x\sinh(c+dx)}{d^3} + \frac{a^2x\sinh(c+dx)}{d} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int x(a+bx)^2\cosh(c+dx)\,dx$$
$$= \frac{-((a^2d^2+4abd^2x+3b^2(2+d^2x^2))\cosh(c+dx)) + d(a^2d^2x+2ab(2+d^2x^2)+b^2x(6+d^2x^2))\sinh(c+dx}{d^4}$$

```
[In] Integrate[x*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a^2*d^
2*x + 2*a*b*(2 + d^2*x^2) + b^2*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{4\left(\frac{3bx}{4}+a\right)d^2xb\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2d\left(x(bx+a)^2d^2+6b^2x+4ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+(3x^2b^2+4abx+2a^2)d^2+12b^2}{d^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b^2d^3x^3+2ab\,d^3x^2+a^2d^3x-3x^2d^2b^2-4ab\,d^2x-a^2d^2+6b^2dx+4dab-6b^2)e^{dx+c}}{2d^4} - \frac{(b^2d^3x^3+2ab\,d^3x^2+a^2d^3x+3x^2d^2b^2-}{}$
parts	$\frac{b^2x^3\sinh(dx+c)}{d} + \frac{2ab\,x^2\sinh(dx+c)}{d} + \frac{a^2x\sinh(dx+c)}{d} - \frac{\frac{3b^2\left((dx+c)^2\cosh(dx+c)-2(dx+c)\sinh(dx+c)+2\cosh(dx+c)\right)}{d^2}}{}$
meijerg	$\frac{8b^2\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4} - \frac{8ib^2\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i(1}{}\right.}{d^4}$
derivativedivides	$\frac{\frac{b^2\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d^2}-\frac{3b^2c\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)-\right.}{d^2}}{}$
default	$\frac{\frac{b^2\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d^2}-\frac{3b^2c\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)-\right.}{d^2}}{}$

```
[In] int(x*(b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (4*(3/4*b*x+a)*d^2*x*b*tanh(1/2*d*x+1/2*c)^2-2*d*(x*(b*x+a)^2*d^2+6*b^2*x+4
*a*b)*tanh(1/2*d*x+1/2*c)+(3*b^2*x^2+4*a*b*x+2*a^2)*d^2+12*b^2)/d^4/(tanh(1
/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int x(a+bx)^2\cosh(c+dx)\,dx = -\frac{(3\,b^2d^2x^2+4\,abd^2x+a^2d^2+6\,b^2)\cosh(dx+c)-(b^2d^3x^3+2\,abd^3x^2+4\,abd+(a^2d^3+6\,b^2d)x)\sinh(d}{d^4}$$

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2)*cosh(d*x + c) - (b^2*d^3*
x^3 + 2*a*b*d^3*x^2 + 4*a*b*d + (a^2*d^3 + 6*b^2*d)*x)*sinh(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2x\sinh(c+dx)}{d}-\frac{a^2\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{3b^2x^2\cosh(c+}{d^2} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}\right)\cosh(c) \end{cases}$$

```
[In] integrate(x*(b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**2*si
nh(c + d*x)/d - 4*a*b*x*cosh(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**
2*x**3*sinh(c + d*x)/d - 3*b**2*x**2*cosh(c + d*x)/d**2 + 6*b**2*x*sinh(c +
 d*x)/d**3 - 6*b**2*cosh(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x*
*3/3 + b**2*x**4/4)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(134) = 268.

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.05

$$\int x(a+bx)^2\cosh(c+dx)\,dx = -\frac{1}{24}\,d\left(\frac{6\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2e^{(dx)}}{d^3}+\frac{6\,(d^2x^2+2\,dx+2)a^2e^{(-dx-c)}}{d^3}+\frac{8\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c}{d^4}\right. + \frac{1}{12}\left(3\,b^2x^4+8\,abx^3+6\,a^2x^2\right)\cosh(dx+c)$$

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(6*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 6*(d^2*x^2 +
 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 8*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e
^c - 6*e^c)*a*b*e^(d*x)/d^4 + 8*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d
*x - c)/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c
+ 24*e^c)*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x +
24)*b^2*e^(-d*x - c)/d^5) + 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)*cosh(d
*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \frac{(b^2d^3x^3+2\,abd^3x^2+a^2d^3x-3\,b^2d^2x^2-4\,abd^2x-a^2d^2+6\,b^2dx+4\,abd-6\,b^2)e^{(dx+c)}}{2\,d^4} - \frac{(b^2d^3x^3+2\,abd^3x^2+a^2d^3x+3\,b^2d^2x^2+4\,abd^2x+a^2d^2+6\,b^2dx+4\,abd+6\,b^2)e^{(-dx-c)}}{2\,d^4}$$

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 3*b^2*d^2*x^2 - 4*a*b*d^2*x
- a^2*d^2 + 6*b^2*d*x + 4*a*b*d - 6*b^2)*e^(d*x + c)/d^4 - 1/2*(b^2*d^3*x^3
 + 2*a*b*d^3*x^2 + a^2*d^3*x + 3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^
2*d*x + 4*a*b*d + 6*b^2)*e^(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \frac{b^2\,x^3\,\sinh(c+d\,x)}{d} - \frac{3\,b^2\,x^2\cosh(c+d\,x)}{d^2} - \frac{\cosh(c+d\,x)\,\left(a^2\,d^2+6\,b^2\right)}{d^4} + \frac{4\,a\,b\sinh(c+d\,x)}{d^3} + \frac{x\sinh(c+d\,x)\,\left(a^2\,d^2+6\,b^2\right)}{d^3} + \frac{2\,a\,b\,x^2\sinh(c+d\,x)}{d} - \frac{4\,a\,b\,x\cosh(c+d\,x)}{d^2}$$

```
[In] int(x*cosh(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (b^2*x^3*sinh(c + d*x))/d - (3*b^2*x^2*cosh(c + d*x))/d^2 - (cosh(c + d*x)*
(6*b^2 + a^2*d^2))/d^4 + (4*a*b*sinh(c + d*x))/d^3 + (x*sinh(c + d*x)*(6*b^
2 + a^2*d^2))/d^3 + (2*a*b*x^2*sinh(c + d*x))/d - (4*a*b*x*cosh(c + d*x))/d
^2
```

3.12 $\int (a+bx)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (a+bx)^2 \cosh(c+dx)\,dx = -\frac{2b(a+bx)\cosh(c+dx)}{d^2} + \frac{2b^2\sinh(c+dx)}{d^3} + \frac{(a+bx)^2\sinh(c+dx)}{d}$$

```
[Out] -2*b*(b*x+a)*cosh(d*x+c)/d^2+2*b^2*sinh(d*x+c)/d^3+(b*x+a)^2*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3377, 2717}

$$\int (a+bx)^2 \cosh(c+dx)\,dx = -\frac{2b(a+bx)\cosh(c+dx)}{d^2} + \frac{(a+bx)^2\sinh(c+dx)}{d} + \frac{2b^2\sinh(c+dx)}{d^3}$$

```
[In] Int[(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*b*(a + b*x)*Cosh[c + d*x])/d^2 + (2*b^2*Sinh[c + d*x])/d^3 + ((a + b*x)
^2*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)^2\sinh(c+dx)}{d} - \frac{(2b)\int(a+bx)\sinh(c+dx)\,dx}{d} \\
&= -\frac{2b(a+bx)\cosh(c+dx)}{d^2} + \frac{(a+bx)^2\sinh(c+dx)}{d} + \frac{(2b^2)\int\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{2b(a+bx)\cosh(c+dx)}{d^2} + \frac{2b^2\sinh(c+dx)}{d^3} + \frac{(a+bx)^2\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int (a+bx)^2\cosh(c+dx)\,dx = \frac{-2bd(a+bx)\cosh(c+dx) + (a^2d^2 + 2abd^2x + b^2(2+d^2x^2))\sinh(c+dx)}{d^3}$$

```
[In] Integrate[(a + b*x)^2*Cosh[c + d*x],x]
[Out] (-2*b*d*(a + b*x)*Cosh[c + d*x] + (a^2*d^2 + 2*a*b*d^2*x + b^2*(2 + d^2*x^2
))*Sinh[c + d*x])/d^3
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

method	result
parallelrisch	$\frac{2x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{2}d+2\left(-(bx+a)^{2}d^{2}-2b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4d\left(\frac{bx}{2}+a\right)b}{d^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
parts	$\frac{b^{2}x^{2}\sinh(dx+c)}{d}+\frac{2abx\sinh(dx+c)}{d}+\frac{a^{2}\sinh(dx+c)}{d}-\frac{2b\left(\frac{b((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d}-\frac{bc\cosh(dx+c)}{d}+a\cosh\right.}{d^{2}}$
risch	$\frac{(x^{2}d^{2}b^{2}+2ab\,d^{2}x+a^{2}d^{2}-2b^{2}dx-2dab+2b^{2})\mathrm{e}^{dx+c}}{2d^{3}}-\frac{(x^{2}d^{2}b^{2}+2ab\,d^{2}x+a^{2}d^{2}+2b^{2}dx+2dab+2b^{2})\mathrm{e}^{-dx-c}}{2d^{3}}$
derivativedivides	$\frac{\frac{b^{2}\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}-\frac{2b^{2}c((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}+\frac{2ba((dx+c)\sinh(dx+c)-\cos}{d}}{d}$
default	$\frac{\frac{b^{2}\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}-\frac{2b^{2}c((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}+\frac{2ba((dx+c)\sinh(dx+c)-\cos}{d}}{d}$
meijerg	$\frac{4ib^{2}\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^{2}d^{2}}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^{3}}+\frac{4b^{2}\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^{2}d^{2}}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{3}}$

```
[In] int((b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x*tanh(1/2*d*x+1/2*c)^2*b^2*d+(-(b*x+a)^2*d^2-2*b^2)*tanh(1/2*d*x+1/2*c)
+2*d*(1/2*b*x+a)*b)/d^3/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int (a+bx)^2\cosh(c+dx)\,dx = -\frac{2\,(b^2dx+abd)\cosh\left(dx+c\right)-(b^2d^2x^2+2\,abd^2x+a^2d^2+2\,b^2)\sinh\left(dx+c\right)}{d^3}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(b^2*d*x + a*b*d)*cosh(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
+ 2*b^2)*sinh(d*x + c))/d^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (a+bx)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2\sinh{(c+dx)}}{d} + \frac{2abx\sinh{(c+dx)}}{d} - \frac{2ab\cosh{(c+dx)}}{d^2} + \frac{b^2x^2\sinh{(c+dx)}}{d} - \frac{2b^2x\cosh{(c+dx)}}{d^2} + \frac{2b^2\sinh{(c+dx)}}{d^3} & \text{for } d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\cosh{(c)} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x*sinh(c + d*x)/d - 2*a*b*cosh(c +
d*x)/d**2 + b**2*x**2*sinh(c + d*x)/d - 2*b**2*x*cosh(c + d*x)/d**2 + 2*b**
2*sinh(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*cosh(c)
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.76

$$\int (a+bx)^2\cosh(c+dx)\,dx = \frac{a^2e^{(dx+c)}}{2\,d} + \frac{(dxe^c - e^c)abe^{(dx)}}{d^2} - \frac{(dx+1)abe^{(-dx-c)}}{d^2} - \frac{a^2e^{(-dx-c)}}{2\,d} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)b^2e^{(dx)}}{2\,d^3} - \frac{(d^2x^2 + 2\,dx + 2)b^2e^{(-dx-c)}}{2\,d^3}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d + (d*x*e^c - e^c)*a*b*e^(d*x)/d^2 - (d*x + 1)*a*b*e^(
-d*x - c)/d^2 - 1/2*a^2*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e
^c)*b^2*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (a+bx)^2\cosh(c+dx)\,dx = \frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 - 2\,b^2dx - 2\,abd + 2\,b^2)e^{(dx+c)}}{2\,d^3} - \frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 + 2\,b^2dx + 2\,abd + 2\,b^2)e^{(-dx-c)}}{2\,d^3}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2*d*x - 2*a*b*d + 2*b^2)*e^(
d*x + c)/d^3 - 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2*d*x + 2*a*b
*d + 2*b^2)*e^(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int (a+bx)^2 \cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a^2\,d^2+2\,b^2\right)}{d^3} + \frac{b^2\,x^2\,\sinh(c+d\,x)}{d} - \frac{2\,a\,b\cosh(c+d\,x)}{d^2} - \frac{2\,b^2\,x\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x\sinh(c+d\,x)}{d}$$

```
[In] int(cosh(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (sinh(c + d*x)*(2*b^2 + a^2*d^2))/d^3 + (b^2*x^2*sinh(c + d*x))/d - (2*a*b*
cosh(c + d*x))/d^2 - (2*b^2*x*cosh(c + d*x))/d^2 + (2*a*b*x*sinh(c + d*x))/
d
```

3.13 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = -\frac{b^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\text{Chi}(dx) + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x\sinh(c+dx)}{d} + a^2\sinh(c)\text{Shi}(dx)$$

```
[Out] a^2*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+2*a*b*sinh(d*
x+c)/d+b^2*x*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6874, 2717, 3384, 3379, 3382, 3377, 2718}

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = a^2\cosh(c)\text{Chi}(dx) + a^2\sinh(c)\text{Shi}(dx) + \frac{2ab\sinh(c+dx)}{d} - \frac{b^2\cosh(c+dx)}{d^2} + \frac{b^2x\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) + a^2*Cosh[c]*CoshIntegral[d*x] + (2*a*b*Sinh[c
+ d*x])/d + (b^2*x*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x}+b^2x\cosh(c+dx)\right)\,dx\\
&= a^2\int \frac{\cosh(c+dx)}{x}\,dx+(2ab)\int \cosh(c+dx)\,dx+b^2\int x\cosh(c+dx)\,dx\\
&= \frac{2ab\sinh(c+dx)}{d}+\frac{b^2x\sinh(c+dx)}{d}-\frac{b^2\int \sinh(c+dx)\,dx}{d}\\
&\quad+\left(a^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx+\left(a^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{b^2 \cosh(c+dx)}{d^2} + a^2 \cosh(c)\text{Chi}(dx) + \frac{2ab \sinh(c+dx)}{d} + \frac{b^2 x \sinh(c+dx)}{d} + a^2 \sinh(c)\text{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x}\,dx = a^2 \cosh(c)\text{Chi}(dx) + \frac{b(-b\cosh(c+dx) + d(2a+bx)\sinh(c+dx))}{d^2} + a^2 \sinh(c)\text{Shi}(dx)$$

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

```
[Out] a^2*Cosh[c]*CoshIntegral[d*x] + (b*(-(b*Cosh[c + d*x]) + d*(2*a + b*x)*Sinh
[c + d*x]))/d^2 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{a^2 e^c \text{Ei}_1(-dx)}{2} - \frac{a^2 e^{-c} \text{Ei}_1(dx)}{2} - \frac{e^{-dx-c} b^2 x}{2d} + \frac{e^{dx+c} b^2 x}{2d} - \frac{e^{-dx-c} ab}{d} + \frac{e^{dx+c} ab}{d} - \frac{e^{-dx-c} b^2}{2d^2} - \frac{e^{dx+c} b^2}{2d^2}$
meijerg	$-\frac{2b^2 \cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cosh(dx)}{2\sqrt{\pi}} - \frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{b^2 \sinh(c)(\cosh(dx)xd - \sinh(dx))}{d^2} + \frac{2ab\cosh(c)\sinh(dx)}{d} - \frac{2ba\sinh(c)\sqrt{\pi}\,(\ldots}{d}$

```
[In] int((b*x+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*exp(c)*Ei(1,-d*x)-1/2*a^2*exp(-c)*Ei(1,d*x)-1/2/d*exp(-d*x-c)*b^2*
x+1/2/d*exp(d*x+c)*b^2*x-1/d*exp(-d*x-c)*a*b+1/d*exp(d*x+c)*a*b-1/2/d^2*exp
(-d*x-c)*b^2-1/2/d^2*exp(d*x+c)*b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = -\frac{2\,b^2\cosh\left(dx+c\right) - \left(a^2d^2\mathrm{Ei}(dx) + a^2d^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right) - 2\left(b^2dx + 2\,abd\right)\sinh\left(dx+c\right) - \left(a^2d^2\mathrm{Ei}(dx\right.}{2\,d^2}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei(d*x) + a^2*d^2*Ei(-d*x))*cosh(c) -
2*(b^2*d*x + 2*a*b*d)*sinh(d*x + c) - (a^2*d^2*Ei(d*x) - a^2*d^2*Ei(-d*x))*
sinh(c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = a^2\sinh\left(c\right)\mathrm{Shi}\left(dx\right) + a^2\cosh\left(c\right)\mathrm{Chi}\left(dx\right) + 2ab\left(\begin{cases} x\cosh\left(c\right) & \text{for}\ d=0 \\ \frac{\sinh\left(c+dx\right)}{d} & \text{otherwise} \end{cases}\right) + b^2\left(\begin{cases} \frac{x\sinh\left(c+dx\right)}{d} - \frac{\cosh\left(c+dx\right)}{d^2} & \text{for}\ d\neq 0 \\ \frac{x^2\cosh\left(c\right)}{2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x,x)
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*cosh(c),
 Eq(d, 0)), (sinh(c + d*x)/d, True)) + b**2*Piecewise((x*sinh(c + d*x)/d -
cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.82

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = -\frac{1}{4}\left(4\,ab\left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + b^2\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)}{d^3}\right.\right.$$
$$+ \frac{1}{2}\left(b^2x^2 + 4\,abx + 2\,a^2\log\left(x\right)\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + b^
2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^
(-d*x - c)/d^3) + 4*a^2*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x)*e^(-c) + Ei(d*
x)*e^c)*a^2/d)*d + 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = \frac{a^2d^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^2\mathrm{Ei}(dx)\,e^{c} + b^2dxe^{(dx+c)} - b^2dxe^{(-dx-c)} + 2\,abde^{(dx+c)} - 2\,abde^{(-dx-c)} - b^2e^{(dx+c)} -}{2\,d^2}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*Ei(-d*x)*e^(-c) + a^2*d^2*Ei(d*x)*e^c + b^2*d*x*e^(d*x + c) -
b^2*d*x*e^(-d*x - c) + 2*a*b*d*e^(d*x + c) - 2*a*b*d*e^(-d*x - c) - b^2*e^(
d*x + c) - b^2*e^(-d*x - c))/d^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x, x)
```

3.14 $\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = -\frac{a^2\cosh(c+dx)}{x} + 2ab\cosh(c)\text{Chi}(dx) + a^2 d\text{Chi}(dx)\sinh(c) + \frac{b^2\sinh(c+dx)}{d} + a^2 d\cosh(c)\text{Shi}(dx) + 2ab\sinh(c)\text{Shi}(dx)$$

```
[Out] 2*a*b*Chi(d*x)*cosh(c)-a^2*cosh(d*x+c)/x+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi(d
*x)*sinh(c)+2*a*b*Shi(d*x)*sinh(c)+b^2*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2717, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = a^2 d\sinh(c)\text{Chi}(dx) + a^2 d\cosh(c)\text{Shi}(dx) - \frac{a^2\cosh(c+dx)}{x} + 2ab\cosh(c)\text{Chi}(dx) + 2ab\sinh(c)\text{Shi}(dx) + \frac{b^2\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + a^2*d*CoshInte
gral[d*x]*Sinh[c] + (b^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
 + 2*a*b*Sinh[c]*SinhIntegral[d*x]
```

<u>Rule 2717</u>

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(b^2\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^2}+\frac{2ab\cosh(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x}\,dx+b^2\int \cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}+\left(a^2d\right)\int \frac{\sinh(c+dx)}{x}\,dx\\
&\quad+(2ab\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(2ab\sinh(c))\int \frac{\sinh(dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c)\text{Chi}(dx)+\frac{b^2\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)\\
&\quad+\left(a^2d\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx+\left(a^2d\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{a^2\cosh(c+dx)}{x} + 2ab\cosh(c)\mathrm{Chi}(dx) + a^2d\mathrm{Chi}(dx)\sinh(c) + \frac{b^2\sinh(c+dx)}{d} + a^2d\cosh(c)\mathrm{Shi}(dx) + 2ab\sinh(c)\mathrm{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = -\frac{a^2\cosh(c+dx)}{x} + a\mathrm{Chi}(dx)(2b\cosh(c)+ad\sinh(c)) + \frac{b^2\sinh(c+dx)}{d} + a(ad\cosh(c)+2b\sinh(c))\mathrm{Shi}(dx)$$

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + a*CoshIntegral[d*x]*(2*b*Cosh[c] + a*d*Sinh[c])
+ (b^2*Sinh[c + d*x])/d + a*(a*d*Cosh[c] + 2*b*Sinh[c])*SinhIntegral[d*x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

method	result
risch	$-\frac{e^{c}\operatorname{Ei}_1(-dx)a^2d^2x - e^{-c}\operatorname{Ei}_1(dx)a^2d^2x + 2e^{c}\operatorname{Ei}_1(-dx)abdx + 2e^{-c}\operatorname{Ei}_1(dx)abdx + e^{-dx-c}a^2d + e^{-dx-c}b^2x + e^{dx+c}a^2d - e^{dx+c}b^2x}{2dx}$
meijerg	$\frac{b^2\cosh(c)\sinh(dx)}{d} - \frac{b^2\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d} + ab\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}} + \frac{2\operatorname{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)$

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(exp(c)*Ei(1,-d*x)*a^2*d^2*x-exp(-c)*Ei(1,d*x)*a^2*d^2*x+2*exp(c)*Ei
(1,-d*x)*a*b*d*x+2*exp(-c)*Ei(1,d*x)*a*b*d*x+exp(-d*x-c)*a^2*d+exp(-d*x-c)*
b^2*x+exp(d*x+c)*a^2*d-exp(d*x+c)*b^2*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = -\frac{2\,a^2d\cosh\left(dx+c\right)-2\,b^2x\sinh\left(dx+c\right)-\left(\left(a^2d^2+2\,abd\right)x\mathrm{Ei}(dx)-\left(a^2d^2-2\,abd\right)x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-}{2\,dx}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
[Out] -1/2*(2*a^2*d*cosh(d*x + c) - 2*b^2*x*sinh(d*x + c) - ((a^2*d^2 + 2*a*b*d)*
x*Ei(d*x) - (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*cosh(c) - ((a^2*d^2 + 2*a*b*d)*
x*Ei(d*x) + (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = \int \frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^2}\,dx$$

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**2,x)
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = -\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)a^2+b^2\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)+\frac{4\,ab\cosh\left(dx+c\right)\log\left(x\right.}{d}\right. + \left(b^2x+2\,ab\log\left(x\right)-\frac{a^2}{x}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
[Out] -1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a^2 + b^2*((d*x*e^c - e^c)*e^(d*x)/d^
2 + (d*x + 1)*e^(-d*x - c)/d^2) + 4*a*b*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x
)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + (b^2*x + 2*a*b*log(x) - a^2/x)*cosh(d*x
+ c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = -\frac{a^2d^2x\mathrm{Ei}(-dx)\,e^{(-c)} - a^2d^2x\mathrm{Ei}(dx)\,e^{c} - 2\,abdx\mathrm{Ei}(-dx)\,e^{(-c)} - 2\,abdx\mathrm{Ei}(dx)\,e^{c} + a^2de^{(dx+c)} - b^2xe^{(dx+c}}{2\,dx}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*x*Ei(-d*x)*e^(-c) - a^2*d^2*x*Ei(d*x)*e^c - 2*a*b*d*x*Ei(-d*x
)*e^(-c) - 2*a*b*d*x*Ei(d*x)*e^c + a^2*d*e^(d*x + c) - b^2*x*e^(d*x + c) +
a^2*d*e^(-d*x - c) + b^2*x*e^(-d*x - c))/(d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^2}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^2, x)
```

3.15 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = -\frac{a^2\cosh(c+dx)}{2x^2} - \frac{2ab\cosh(c+dx)}{x} + b^2\cosh(c)\mathrm{Chi}(dx) + \frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx) + 2abd\mathrm{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{2x} + 2abd\cosh(c)\mathrm{Shi}(dx) + b^2\sinh(c)\mathrm{Shi}(dx) + \frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)$$

```
[Out] b^2*Chi(d*x)*cosh(c)+1/2*a^2*d^2*Chi(d*x)*cosh(c)-1/2*a^2*cosh(d*x+c)/x^2-2
*a*b*cosh(d*x+c)/x+2*a*b*d*cosh(c)*Shi(d*x)+2*a*b*d*Chi(d*x)*sinh(c)+b^2*Sh
i(d*x)*sinh(c)+1/2*a^2*d^2*Shi(d*x)*sinh(c)-1/2*a^2*d*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx) + \frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx) - \frac{a^2\cosh(c+dx)}{2x^2} - \frac{a^2d\sinh(c+dx)}{2x} + 2abd\sinh(c)\mathrm{Chi}(dx) + 2abd\cosh(c)\mathrm{Shi}(dx) - \frac{2ab\cosh(c+dx)}{x} + b^2\cosh(c)\mathrm{Chi}(dx) + b^2\sinh(c)\mathrm{Shi}(dx)$$

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a^2*Cosh[c + d*x])/x^2 - (2*a*b*Cosh[c + d*x])/x + b^2*Cosh[c]*CoshIn
tegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + 2*a*b*d*CoshIntegral[
d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(2*x) + 2*a*b*d*Cosh[c]*SinhIntegral[d
*x] + b^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^3}+\frac{2ab\cosh(c+dx)}{x^2}+\frac{b^2\cosh(c+dx)}{x}\right)dx\\ &= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int \frac{\cosh(c+dx)}{x}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^2\cosh(c+dx)}{2x^2} - \frac{2ab\cosh(c+dx)}{x} \\
&\quad + \frac{1}{2}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^2}\,dx + (2abd)\int \frac{\sinh(c+dx)}{x}\,dx \\
&\quad + \left(b^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx + \left(b^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{2x^2} - \frac{2ab\cosh(c+dx)}{x} + b^2\cosh(c)\mathrm{Chi}(dx) \\
&\quad - \frac{a^2d\sinh(c+dx)}{2x} + b^2\sinh(c)\mathrm{Shi}(dx) + \frac{1}{2}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x}\,dx \\
&\quad + (2abd\cosh(c))\int \frac{\sinh(dx)}{x}\,dx + (2abd\sinh(c))\int \frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{2x^2} - \frac{2ab\cosh(c+dx)}{x} + b^2\cosh(c)\mathrm{Chi}(dx) + 2abd\mathrm{Chi}(dx)\sinh(c) \\
&\quad - \frac{a^2d\sinh(c+dx)}{2x} + 2abd\cosh(c)\mathrm{Shi}(dx) + b^2\sinh(c)\mathrm{Shi}(dx) \\
&\quad + \frac{1}{2}\left(a^2d^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx + \frac{1}{2}\left(a^2d^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{2x^2} - \frac{2ab\cosh(c+dx)}{x} + b^2\cosh(c)\mathrm{Chi}(dx) \\
&\quad + \frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx) + 2abd\mathrm{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{2x} \\
&\quad + 2abd\cosh(c)\mathrm{Shi}(dx) + b^2\sinh(c)\mathrm{Shi}(dx) + \frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.77

$$
\begin{aligned}
\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}\Bigg(&\mathrm{Chi}(dx)\left(\left(2b^2+a^2d^2\right)\cosh(c)+4abd\sinh(c)\right) \\
&- \frac{a((a+4bx)\cosh(c+dx)+adx\sinh(c+dx))}{x^2} \\
&+ \left(4abd\cosh(c)+\left(2b^2+a^2d^2\right)\sinh(c)\right)\mathrm{Shi}(dx)\Bigg)
\end{aligned}
$$

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] (CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[c] + 4*a*b*d*Sinh[c]) - (a*((a +
 4*b*x)*Cosh[c + d*x] + a*d*x*Sinh[c + d*x]))/x^2 + (4*a*b*d*Cosh[c] + (2*b
^2 + a^2*d^2)*Sinh[c])*SinhIntegral[d*x])/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{e^{c} \mathrm{Ei}_{1}(-dx) a^{2} d^{2} x^{2}+e^{-c} \mathrm{Ei}_{1}(dx) a^{2} d^{2} x^{2}+4 e^{c} \mathrm{Ei}_{1}(-dx) abd\, x^{2}-4 e^{-c} \mathrm{Ei}_{1}(dx) abd\, x^{2}+2 e^{c} \mathrm{Ei}_{1}(-dx) b^{2} x^{2}+2 e^{-c} \mathrm{Ei}_{1}(dx) b^{2} x^{2}-e^{-d}}{4x^{2}}$
meijerg	$\frac{b^{2} \cosh(c) \sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}+\frac{2\ \mathrm{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)}{2}+b^{2} \mathrm{Shi}\left(dx\right) \sinh\left(c\right)+\frac{idab \cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}(dx)}{\sqrt{\pi}}\right)}{2}$

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(exp(c)*Ei(1,-d*x)*a^2*d^2*x^2+exp(-c)*Ei(1,d*x)*a^2*d^2*x^2+4*exp(c)*
Ei(1,-d*x)*a*b*d*x^2-4*exp(-c)*Ei(1,d*x)*a*b*d*x^2+2*exp(c)*Ei(1,-d*x)*b^2*
x^2+2*exp(-c)*Ei(1,d*x)*b^2*x^2-exp(-d*x-c)*a^2*d*x+exp(d*x+c)*a^2*d*x+4*ex
p(-d*x-c)*a*b*x+4*exp(d*x+c)*a*b*x+exp(-d*x-c)*a^2+exp(d*x+c)*a^2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = -\frac{2\,a^2dx\sinh\left(dx+c\right)+2\left(4\,abx+a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^2+4\,abd+2\,b^2\right)x^2\mathrm{Ei}(dx)+\left(a^2d^2-4\,abd+\right.\right.}{4\,x}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*sinh(d*x + c) + 2*(4*a*b*x + a^2)*cosh(d*x + c) - ((a^2*d^2
 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x))
*cosh(c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 - 4*a*b*d +
2*b^2)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \int \frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$$
$$=\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a^2-4\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)ab-\frac{4\,b^2\cosh\left(dx+c\right)\log\left(x\right)}{d}+\right.$$
$$+\frac{1}{2}\left(2\,b^2\log\left(x\right)-\frac{4\,abx+a^2}{x^2}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - 4*(Ei(-d*x)*e^
(-c) - Ei(d*x)*e^c)*a*b - 4*b^2*cosh(d*x + c)*log(x)/d + 2*(Ei(-d*x)*e^(-c)
 + Ei(d*x)*e^c)*b^2/d)*d + 1/2*(2*b^2*log(x) - (4*a*b*x + a^2)/x^2)*cosh(d*
x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$$
$$=\frac{a^2d^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^2x^2\mathrm{Ei}(dx)\,e^{c}-4\,abdx^2\mathrm{Ei}(-dx)\,e^{(-c)}+4\,abdx^2\mathrm{Ei}(dx)\,e^{c}+2\,b^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}+}{4\,x^2}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c - 4*a*b*d*x^2*Ei
(-d*x)*e^(-c) + 4*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) + 2*b^2
*x^2*Ei(d*x)*e^c - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x - c) - 4*a*b*x*e^(
d*x + c) - 4*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*e^(-d*x - c))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx=\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^3}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^3,x)
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^3, x)
```

3.16 $\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 172

$$\begin{aligned}\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = &-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}-\frac{b^2\cosh(c+dx)}{x}\\ &-\frac{a^2d^2\cosh(c+dx)}{6x}+abd^2\cosh(c)\mathrm{Chi}(dx)\\ &+b^2d\mathrm{Chi}(dx)\sinh(c)+\frac{1}{6}a^2d^3\mathrm{Chi}(dx)\sinh(c)\\ &-\frac{a^2d\sinh(c+dx)}{6x^2}-\frac{abd\sinh(c+dx)}{x}+b^2d\cosh(c)\mathrm{Shi}(dx)\\ &+\frac{1}{6}a^2d^3\cosh(c)\mathrm{Shi}(dx)+abd^2\sinh(c)\mathrm{Shi}(dx)\end{aligned}$$

```
[Out] a*b*d^2*Chi(d*x)*cosh(c)-1/3*a^2*cosh(d*x+c)/x^3-a*b*cosh(d*x+c)/x^2-b^2*co
sh(d*x+c)/x-1/6*a^2*d^2*cosh(d*x+c)/x+b^2*d*cosh(c)*Shi(d*x)+1/6*a^2*d^3*co
sh(c)*Shi(d*x)+b^2*d*Chi(d*x)*sinh(c)+1/6*a^2*d^3*Chi(d*x)*sinh(c)+a*b*d^2*
Shi(d*x)*sinh(c)-1/6*a^2*d*sinh(d*x+c)/x^2-a*b*d*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

= {6874, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{a^2\cosh(c+dx)}{3x^3} - \frac{a^2d\sinh(c+dx)}{6x^2} + abd^2\cosh(c)\text{Chi}(dx) + abd^2\sinh(c)\text{Shi}(dx) - \frac{ab\cosh(c+dx)}{x^2} - \frac{abd\sinh(c+dx)}{x} + b^2d\sinh(c)\text{Chi}(dx) + b^2d\cosh(c)\text{Shi}(dx) - \frac{b^2\cosh(c+dx)}{x}$$

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a^2*Cosh[c + d*x])/x^3 - (a*b*Cosh[c + d*x])/x^2 - (b^2*Cosh[c + d*x]
)/x - (a^2*d^2*Cosh[c + d*x])/(6*x) + a*b*d^2*Cosh[c]*CoshIntegral[d*x] + b
^2*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a
^2*d*Sinh[c + d*x])/(6*x^2) - (a*b*d*Sinh[c + d*x])/x + b^2*d*Cosh[c]*SinhI
ntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + a*b*d^2*Sinh[c]*Sinh
Integral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2\cosh(c+dx)}{x^4} + \frac{2ab\cosh(c+dx)}{x^3} + \frac{b^2\cosh(c+dx)}{x^2}\right) dx$$

$$= a^2 \int \frac{\cosh(c+dx)}{x^4} dx + (2ab) \int \frac{\cosh(c+dx)}{x^3} dx + b^2 \int \frac{\cosh(c+dx)}{x^2} dx$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} + \frac{1}{3}\left(a^2d\right) \int \frac{\sinh(c+dx)}{x^3} dx + (abd) \int \frac{\sinh(c+dx)}{x^2} dx + \left(b^2d\right) \int \frac{\sinh(c+dx)}{x} dx$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{abd\sinh(c+dx)}{x} + \frac{1}{6}\left(a^2d^2\right) \int \frac{\cosh(c+dx)}{x^2} dx + \left(abd^2\right) \int \frac{\cosh(c+dx)}{x} dx + \left(b^2d\cosh(c)\right) \int \frac{\sinh(dx)}{x} dx + \left(b^2d\sinh(c)\right) \int \frac{\cosh(dx)}{x} dx$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d^2\cosh(c+dx)}{6x} + b^2d\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{abd\sinh(c+dx)}{x} + b^2d\cosh(c)\text{Shi}(dx) + \frac{1}{6}\left(a^2d^3\right) \int \frac{\sinh(c+dx)}{x} dx + \left(abd^2\cosh(c)\right) \int \frac{\cosh(dx)}{x} dx + \left(abd^2\sinh(c)\right) \int \frac{\sinh(dx)}{x} dx$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d^2\cosh(c+dx)}{6x} + abd^2\cosh(c)\text{Chi}(dx) + b^2d\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{abd\sinh(c+dx)}{x} + b^2d\cosh(c)\text{Shi}(dx) + abd^2\sinh(c)\text{Shi}(dx) + \frac{1}{6}\left(a^2d^3\cosh(c)\right) \int \frac{\sinh(dx)}{x} dx + \frac{1}{6}\left(a^2d^3\sinh(c)\right) \int \frac{\cosh(dx)}{x} dx$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d^2\cosh(c+dx)}{6x} + abd^2\cosh(c)\text{Chi}(dx) + b^2d\text{Chi}(dx)\sinh(c) + \frac{1}{6}a^2d^3\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{abd\sinh(c+dx)}{x} + b^2d\cosh(c)\text{Shi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) + abd^2\sinh(c)\text{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = -\frac{2a^2\cosh(c+dx) + 6abx\cosh(c+dx) + 6b^2x^2\cosh(c+dx) + a^2d^2x^2\cosh(c+dx) - dx^3\text{Chi}(dx)\,(6abd}{}$$

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a^2*Cosh[c + d*x] + 6*a*b*x*Cosh[c + d*x] + 6*b^2*x^2*Cosh[c + d*x]
 + a^2*d^2*x^2*Cosh[c + d*x] - d*x^3*CoshIntegral[d*x]*(6*a*b*d*Cosh[c] + (
6*b^2 + a^2*d^2)*Sinh[c]) + a^2*d*x*Sinh[c + d*x] + 6*a*b*d*x^2*Sinh[c + d*
x] - d*x^3*(6*b^2*Cosh[c] + a^2*d^2*Cosh[c] + 6*a*b*d*Sinh[c])*SinhIntegral
[d*x])/x^3
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{-e^{-c}\,\text{Ei}_1(dx)a^2d^3x^3+e^{c}\,\text{Ei}_1(-dx)a^2d^3x^3+6\,e^{-c}\,\text{Ei}_1(dx)ab\,d^2x^3+6\,e^{c}\,\text{Ei}_1(-dx)ab\,d^2x^3-6\,e^{-c}\,\text{Ei}_1(dx)b^2d\,x^3+6\,e^{c}\,\text{Ei}_1(-dx)b^2d\,x}{}$
meijerg	$\frac{id\,b^2\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\text{Shi}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{d\,b^2\sinh(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)+4\ln(id)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}+\frac{4\,\text{Chi}(dx)-4\ln(dx)-4\gamma}{\sqrt{\pi}}\right)}{4}-\frac{d^2}{}$

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-exp(-c)*Ei(1,d*x)*a^2*d^3*x^3+exp(c)*Ei(1,-d*x)*a^2*d^3*x^3+6*exp(-
c)*Ei(1,d*x)*a*b*d^2*x^3+6*exp(c)*Ei(1,-d*x)*a*b*d^2*x^3-6*exp(-c)*Ei(1,d*x
)*b^2*d*x^3+6*exp(c)*Ei(1,-d*x)*b^2*d*x^3+exp(-d*x-c)*a^2*d^2*x^2+exp(d*x+c
)*a^2*d^2*x^2-6*exp(-d*x-c)*a*b*d*x^2+6*exp(d*x+c)*a*b*d*x^2-exp(-d*x-c)*a^
2*d*x+6*exp(-d*x-c)*b^2*x^2+exp(d*x+c)*a^2*d*x+6*exp(d*x+c)*b^2*x^2+6*exp(-
d*x-c)*a*b*x+6*exp(d*x+c)*a*b*x+2*exp(-d*x-c)*a^2+2*exp(d*x+c)*a^2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = -\frac{2\,(6\,abx+(a^2d^2+6\,b^2)x^2+2\,a^2)\cosh(dx+c)-((a^2d^3+6\,abd^2+6\,b^2d)x^3\mathrm{Ei}(dx)-(a^2d^3-6\,abd^2+6$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*a*b*x + (a^2*d^2 + 6*b^2)*x^2 + 2*a^2)*cosh(d*x + c) - ((a^2*d^
3 + 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) - (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*
Ei(-d*x))*cosh(c) + 2*(6*a*b*d*x^2 + a^2*d*x)*sinh(d*x + c) - ((a^2*d^3 + 6
*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) + (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(-d
*x))*sinh(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx)^2\cosh{(c+dx)}}{x^4}\,dx$$

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx$$
$$= \frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma(-2,dx)-a^2d^2e^c\Gamma(-2,-dx)+3\,abde^{(-c)}\Gamma(-1,dx)+3\,abde^c\Gamma(-1,-dx)-3\,b^2\mathrm{Ei}(-dx)\,e^{(-c}\right.$$
$$-\frac{(3\,b^2x^2+3\,abx+a^2)\cosh(dx+c)}{3\,x^3}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) + 3*a*b*d*
e^(-c)*gamma(-1, d*x) + 3*a*b*d*e^c*gamma(-1, -d*x) - 3*b^2*Ei(-d*x)*e^(-c)
 + 3*b^2*Ei(d*x)*e^c)*d - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*cosh(d*x + c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} \, dx = -\frac{a^2d^3x^3\text{Ei}(-dx)\,e^{(-c)} - a^2d^3x^3\text{Ei}(dx)\,e^c - 6\,abd^2x^3\text{Ei}(-dx)\,e^{(-c)} - 6\,abd^2x^3\text{Ei}(dx)\,e^c + 6\,b^2dx^3\text{Ei}(-dx)}{}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^3*x^3*Ei(-d*x)*e^(-c) - a^2*d^3*x^3*Ei(d*x)*e^c - 6*a*b*d^2*x^
3*Ei(-d*x)*e^(-c) - 6*a*b*d^2*x^3*Ei(d*x)*e^c + 6*b^2*d*x^3*Ei(-d*x)*e^(-c)
 - 6*b^2*d*x^3*Ei(d*x)*e^c + a^2*d^2*x^2*e^(d*x + c) + a^2*d^2*x^2*e^(-d*x
- c) + 6*a*b*d*x^2*e^(d*x + c) - 6*a*b*d*x^2*e^(-d*x - c) + a^2*d*x*e^(d*x
+ c) + 6*b^2*x^2*e^(d*x + c) - a^2*d*x*e^(-d*x - c) + 6*b^2*x^2*e^(-d*x - c
) + 6*a*b*x*e^(d*x + c) + 6*a*b*x*e^(-d*x - c) + 2*a^2*e^(d*x + c) + 2*a^2*
e^(-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} \, dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^4} \, dx$$

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^4, x)
```

3.17 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 248

$$\begin{aligned}\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx &= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}\\&\quad-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{abd^2\cosh(c+dx)}{3x}+\frac{1}{2}b^2d^2\cosh(c)\text{Chi}(dx)\\&\quad+\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{3}abd^3\text{Chi}(dx)\sinh(c)-\frac{a^2d\sinh(c+dx)}{12x^3}\\&\quad-\frac{abd\sinh(c+dx)}{3x^2}-\frac{b^2d\sinh(c+dx)}{2x}-\frac{a^2d^3\sinh(c+dx)}{24x}\\&\quad+\frac{1}{3}abd^3\cosh(c)\text{Shi}(dx)+\frac{1}{2}b^2d^2\sinh(c)\text{Shi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)\end{aligned}$$

```
[Out] 1/2*b^2*d^2*Chi(d*x)*cosh(c)+1/24*a^2*d^4*Chi(d*x)*cosh(c)-1/4*a^2*cosh(d*x
+c)/x^4-2/3*a*b*cosh(d*x+c)/x^3-1/2*b^2*cosh(d*x+c)/x^2-1/24*a^2*d^2*cosh(d
*x+c)/x^2-1/3*a*b*d^2*cosh(d*x+c)/x+1/3*a*b*d^3*cosh(c)*Shi(d*x)+1/3*a*b*d^
3*Chi(d*x)*sinh(c)+1/2*b^2*d^2*Shi(d*x)*sinh(c)+1/24*a^2*d^4*Shi(d*x)*sinh(
c)-1/12*a^2*d*sinh(d*x+c)/x^3-1/3*a*b*d*sinh(d*x+c)/x^2-1/2*b^2*d*sinh(d*x+
c)/x-1/24*a^2*d^3*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx = \frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx) - \frac{a^2d^3\sinh(c+dx)}{24x} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d\sinh(c+dx)}{12x^3} + \frac{1}{3}abd^3\sinh(c)\text{Chi}(dx) + \frac{1}{3}abd^3\cosh(c)\text{Shi}(dx) - \frac{abd^2\cosh(c+dx)}{3x} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{abd\sinh(c+dx)}{3x^2} + \frac{1}{2}b^2d^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}b^2d^2\sinh(c)\text{Shi}(dx) - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{b^2d\sinh(c+dx)}{2x}$$

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a^2*Cosh[c + d*x])/x^4 - (2*a*b*Cosh[c + d*x])/(3*x^3) - (b^2*Cosh[c
+ d*x])/(2*x^2) - (a^2*d^2*Cosh[c + d*x])/(24*x^2) - (a*b*d^2*Cosh[c + d*x]
)/(3*x) + (b^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a^2*d^4*Cosh[c]*CoshInte
gral[d*x])/24 + (a*b*d^3*CoshIntegral[d*x]*Sinh[c])/3 - (a^2*d*Sinh[c + d*x
])/(12*x^3) - (a*b*d*Sinh[c + d*x])/(3*x^2) - (b^2*d*Sinh[c + d*x])/(2*x) -
 (a^2*d^3*Sinh[c + d*x])/(24*x) + (a*b*d^3*Cosh[c]*SinhIntegral[d*x])/3 + (
b^2*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/
24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^4}+\frac{b^2\cosh(c+dx)}{x^3}\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^4}\,dx+b^2\int \frac{\cosh(c+dx)}{x^3}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}+\frac{1}{4}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^4}\,dx\\
&\quad+\frac{1}{3}(2abd)\int \frac{\sinh(c+dx)}{x^3}\,dx+\frac{1}{2}\left(b^2d\right)\int \frac{\sinh(c+dx)}{x^2}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{12x^3}\\
&\quad-\frac{abd\sinh(c+dx)}{3x^2}-\frac{b^2d\sinh(c+dx)}{2x}+\frac{1}{12}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x^3}\,dx\\
&\quad+\frac{1}{3}\left(abd^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx+\frac{1}{2}\left(b^2d^2\right)\int \frac{\cosh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}\\
&\quad-\frac{abd^2\cosh(c+dx)}{3x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{abd\sinh(c+dx)}{3x^2}-\frac{b^2d\sinh(c+dx)}{2x}\\
&\quad+\frac{1}{24}\left(a^2d^3\right)\int \frac{\sinh(c+dx)}{x^2}\,dx+\frac{1}{3}\left(abd^3\right)\int \frac{\sinh(c+dx)}{x}\,dx\\
&\quad+\frac{1}{2}\left(b^2d^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx+\frac{1}{2}\left(b^2d^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} \\
&\quad - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{abd^2\cosh(c+dx)}{3x} + \frac{1}{2}b^2d^2\cosh(c)\mathrm{Chi}(dx) \\
&\quad - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{abd\sinh(c+dx)}{3x^2} - \frac{b^2d\sinh(c+dx)}{2x} \\
&\quad - \frac{a^2d^3\sinh(c+dx)}{24x} + \frac{1}{2}b^2d^2\sinh(c)\mathrm{Shi}(dx) + \frac{1}{24}(a^2d^4)\int\frac{\cosh(c+dx)}{x}\,dx \\
&\quad + \frac{1}{3}(abd^3\cosh(c))\int\frac{\sinh(dx)}{x}\,dx + \frac{1}{3}(abd^3\sinh(c))\int\frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} \\
&\quad - \frac{abd^2\cosh(c+dx)}{3x} + \frac{1}{2}b^2d^2\cosh(c)\mathrm{Chi}(dx) + \frac{1}{3}abd^3\mathrm{Chi}(dx)\sinh(c) \\
&\quad - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{abd\sinh(c+dx)}{3x^2} - \frac{b^2d\sinh(c+dx)}{2x} \\
&\quad - \frac{a^2d^3\sinh(c+dx)}{24x} + \frac{1}{3}abd^3\cosh(c)\mathrm{Shi}(dx) + \frac{1}{2}b^2d^2\sinh(c)\mathrm{Shi}(dx) \\
&\quad + \frac{1}{24}(a^2d^4\cosh(c))\int\frac{\cosh(dx)}{x}\,dx + \frac{1}{24}(a^2d^4\sinh(c))\int\frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} \\
&\quad - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{abd^2\cosh(c+dx)}{3x} + \frac{1}{2}b^2d^2\cosh(c)\mathrm{Chi}(dx) \\
&\quad + \frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx) + \frac{1}{3}abd^3\mathrm{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{12x^3} \\
&\quad - \frac{abd\sinh(c+dx)}{3x^2} - \frac{b^2d\sinh(c+dx)}{2x} - \frac{a^2d^3\sinh(c+dx)}{24x} \\
&\quad + \frac{1}{3}abd^3\cosh(c)\mathrm{Shi}(dx) + \frac{1}{2}b^2d^2\sinh(c)\mathrm{Shi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.83

$$
\int\frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx =
$$

$$
-\frac{6a^2\cosh(c+dx) + 16abx\cosh(c+dx) + 12b^2x^2\cosh(c+dx) + a^2d^2x^2\cosh(c+dx) + 8abd^2x^3\cosh(c
$$

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/24*(6*a^2*Cosh[c + d*x] + 16*a*b*x*Cosh[c + d*x] + 12*b^2*x^2*Cosh[c + d
*x] + a^2*d^2*x^2*Cosh[c + d*x] + 8*a*b*d^2*x^3*Cosh[c + d*x] - d^2*x^4*Cos
```

```
hIntegral[d*x]*((12*b^2 + a^2*d^2)*Cosh[c] + 8*a*b*d*Sinh[c]) + 2*a^2*d*x*S
inh[c + d*x] + 8*a*b*d*x^2*Sinh[c + d*x] + 12*b^2*d*x^3*Sinh[c + d*x] + a^2
*d^3*x^3*Sinh[c + d*x] - d^2*x^4*(8*a*b*d*Cosh[c] + 12*b^2*Sinh[c] + a^2*d^
2*Sinh[c])*SinhIntegral[d*x])/x^4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{e^{c} \operatorname{Ei}_{1}(-dx)a^{2}d^{4}x^{4}+e^{-c} \operatorname{Ei}_{1}(dx)a^{2}d^{4}x^{4}+8\,e^{c} \operatorname{Ei}_{1}(-dx)ab\,d^{3}x^{4}-8\,e^{-c} \operatorname{Ei}_{1}(dx)ab\,d^{3}x^{4}+12\,e^{c} \operatorname{Ei}_{1}(-dx)b^{2}d^{2}x^{4}+12\,e^{-c} \operatorname{Ei}_{1}(dx)b^{2}d^{2}}{}$
meijerg	$-\frac{d^{2}b^{2}\cosh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}\,x^{2}d^{2}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}-\frac{4\left(\frac{9x^{2}d^{2}}{2}+3\right)}{3\sqrt{\pi}\,x^{2}d^{2}}+\frac{4\cosh(dx)}{\sqrt{\pi}\,x^{2}d^{2}}+\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}-\frac{4(\operatorname{Chi}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}\right)}{8}+\frac{id^{2}b^{2}\sinh(c)}{}$

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(exp(c)*Ei(1,-d*x)*a^2*d^4*x^4+exp(-c)*Ei(1,d*x)*a^2*d^4*x^4+8*exp(c)
*Ei(1,-d*x)*a*b*d^3*x^4-8*exp(-c)*Ei(1,d*x)*a*b*d^3*x^4+12*exp(c)*Ei(1,-d*x
)*b^2*d^2*x^4+12*exp(-c)*Ei(1,d*x)*b^2*d^2*x^4-exp(-d*x-c)*a^2*d^3*x^3+exp(
d*x+c)*a^2*d^3*x^3+8*exp(-d*x-c)*a*b*d^2*x^3+8*exp(d*x+c)*a*b*d^2*x^3+exp(-
d*x-c)*a^2*d^2*x^2-12*exp(-d*x-c)*b^2*d*x^3+exp(d*x+c)*a^2*d^2*x^2+12*exp(d
*x+c)*b^2*d*x^3-8*exp(-d*x-c)*a*b*d*x^2+8*exp(d*x+c)*a*b*d*x^2-2*exp(-d*x-c
)*a^2*d*x+12*exp(-d*x-c)*b^2*x^2+2*exp(d*x+c)*a^2*d*x+12*exp(d*x+c)*b^2*x^2
+16*exp(-d*x-c)*a*b*x+16*exp(d*x+c)*a*b*x+6*exp(-d*x-c)*a^2+6*exp(d*x+c)*a^
2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx = -\frac{2\left(8\,abd^2x^3+16\,abx+(a^2d^2+12\,b^2)x^2+6\,a^2\right)\cosh\left(dx+c\right)-\left((a^2d^4+8\,abd^3+12\,b^2d^2)x^4\operatorname{Ei}(dx)+(a^2\right.}{}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(8*a*b*d^2*x^3 + 16*a*b*x + (a^2*d^2 + 12*b^2)*x^2 + 6*a^2)*cosh(d
*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(d*x) + (a^2*d^4 - 8*a*
b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*(8*a*b*d*x^2 + 2*a^2*d*x + (a
^2*d^3 + 12*b^2*d)*x^3)*sinh(d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)
*x^4*Ei(d*x) - (a^2*d^4 - 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*sinh(c))/x^
4
```

Sympy [F]

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx)^2 \cosh(c+dx)}{x^5}\,dx$$

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5}\,dx$$
$$= \frac{1}{24}\left(3\,a^2d^3e^{(-c)}\Gamma(-3,dx) + 3\,a^2d^3e^c\Gamma(-3,-dx) + 8\,abd^2e^{(-c)}\Gamma(-2,dx) - 8\,abd^2e^c\Gamma(-2,-dx) + 6\,b^2de^{(-c}\right.$$
$$- \frac{(6\,b^2x^2 + 8\,abx + 3\,a^2)\cosh(dx+c)}{12\,x^4}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*d^3*e^(-c)*gamma(-3, d*x) + 3*a^2*d^3*e^c*gamma(-3, -d*x) + 8*a
*b*d^2*e^(-c)*gamma(-2, d*x) - 8*a*b*d^2*e^c*gamma(-2, -d*x) + 6*b^2*d*e^(-
c)*gamma(-1, d*x) + 6*b^2*d*e^c*gamma(-1, -d*x))*d - 1/12*(6*b^2*x^2 + 8*a*
b*x + 3*a^2)*cosh(d*x + c)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5}\,dx$$
$$= \frac{a^2d^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^4x^4\mathrm{Ei}(dx)\,e^c - 8\,abd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 8\,abd^3x^4\mathrm{Ei}(dx)\,e^c + 12\,b^2d^2x^4\mathrm{Ei}(-dx)\,e}{}$$

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c - 8*a*b*d^3*x^4
*Ei(-d*x)*e^(-c) + 8*a*b*d^3*x^4*Ei(d*x)*e^c + 12*b^2*d^2*x^4*Ei(-d*x)*e^(-
c) + 12*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) + a^2*d^3*x^3*e^(
```

```
-d*x - c) - 8*a*b*d^2*x^3*e^(d*x + c) - 8*a*b*d^2*x^3*e^(-d*x - c) - a^2*d^
2*x^2*e^(d*x + c) - 12*b^2*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c) + 1
2*b^2*d*x^3*e^(-d*x - c) - 8*a*b*d*x^2*e^(d*x + c) + 8*a*b*d*x^2*e^(-d*x -
c) - 2*a^2*d*x*e^(d*x + c) - 12*b^2*x^2*e^(d*x + c) + 2*a^2*d*x*e^(-d*x - c
) - 12*b^2*x^2*e^(-d*x - c) - 16*a*b*x*e^(d*x + c) - 16*a*b*x*e^(-d*x - c)
- 6*a^2*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^5}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^5,x)
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^5, x)
```

3.18 $\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 219

$$\begin{aligned}\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = &-\frac{6\cosh(c+dx)}{bd^4}-\frac{a^2\cosh(c+dx)}{b^3d^2}+\frac{2ax\cosh(c+dx)}{b^2d^2}\\&-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{a^4\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^5}\\&-\frac{2a\sinh(c+dx)}{b^2d^3}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{6x\sinh(c+dx)}{bd^3}\\&+\frac{a^2x\sinh(c+dx)}{b^3d}-\frac{ax^2\sinh(c+dx)}{b^2d}\\&+\frac{x^3\sinh(c+dx)}{bd}+\frac{a^4\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}\end{aligned}$$

```
[Out] a^4*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^5-6*cosh(d*x+c)/b/d^4-a^2*cosh(d*x+c)/b
^3/d^2+2*a*x*cosh(d*x+c)/b^2/d^2-3*x^2*cosh(d*x+c)/b/d^2-a^4*Shi(a*d/b+d*x)
*sinh(-c+a*d/b)/b^5-2*a*sinh(d*x+c)/b^2/d^3-a^3*sinh(d*x+c)/b^4/d+6*x*sinh(
d*x+c)/b/d^3+a^2*x*sinh(d*x+c)/b^3/d-a*x^2*sinh(d*x+c)/b^2/d+x^3*sinh(d*x+c
)/b/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{6874, 2717, 3377, 2718, 3384, 3379, 3382\}$

$$\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx = \frac{a^4 \cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^4 \sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{a^3\sinh(c+dx)}{b^4 d} - \frac{a^2\cosh(c+dx)}{b^3 d^2} + \frac{a^2 x\sinh(c+dx)}{b^3 d} - \frac{2a\sinh(c+dx)}{b^2 d^3} + \frac{2ax\cosh(c+dx)}{b^2 d^2} - \frac{ax^2\sinh(c+dx)}{b^2 d} - \frac{6\cosh(c+dx)}{bd^4} + \frac{6x\sinh(c+dx)}{bd^3} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{x^3\sinh(c+dx)}{bd}$$

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-6*Cosh[c + d*x])/(b*d^4) - (a^2*Cosh[c + d*x])/(b^3*d^2) + (2*a*x*Cosh[c
+ d*x])/(b^2*d^2) - (3*x^2*Cosh[c + d*x])/(b*d^2) + (a^4*Cosh[c - (a*d)/b]*
CoshIntegral[(a*d)/b + d*x])/b^5 - (2*a*Sinh[c + d*x])/(b^2*d^3) - (a^3*Sin
h[c + d*x])/(b^4*d) + (6*x*Sinh[c + d*x])/(b*d^3) + (a^2*x*Sinh[c + d*x])/(
b^3*d) - (a*x^2*Sinh[c + d*x])/(b^2*d) + (x^3*Sinh[c + d*x])/(b*d) + (a^4*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a^3\cosh(c+dx)}{b^4}+\frac{a^2x\cosh(c+dx)}{b^3}-\frac{ax^2\cosh(c+dx)}{b^2}+\frac{x^3\cosh(c+dx)}{b}+\frac{a^4\cosh(c+dx)}{b^4(a+bx)}\right)dx\\
&= -\frac{a^3\int\cosh(c+dx)\,dx}{b^4}+\frac{a^4\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^4}+\frac{a^2\int x\cosh(c+dx)\,dx}{b^3}-\frac{a\int x^2\cosh(c+dx)\,dx}{b^2}+\frac{\int x^3\cosh(c+dx)\,dx}{b}\\
&= -\frac{a^3\sinh(c+dx)}{b^4d}+\frac{a^2x\sinh(c+dx)}{b^3d}-\frac{ax^2\sinh(c+dx)}{b^2d}+\frac{x^3\sinh(c+dx)}{bd}-\frac{a^2\int\sinh(c+dx)\,dx}{b^3d}+\frac{(2a)\int x\sinh(c+dx)\,dx}{b^2d}-\frac{3\int x^2\sinh(c+dx)\,dx}{bd}\\
&\quad+\frac{\left(a^4\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}+\frac{\left(a^4\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}\\
&= -\frac{a^2\cosh(c+dx)}{b^3d^2}+\frac{2ax\cosh(c+dx)}{b^2d^2}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5}\\
&\quad-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{a^2x\sinh(c+dx)}{b^3d}-\frac{ax^2\sinh(c+dx)}{b^2d}+\frac{x^3\sinh(c+dx)}{bd}\\
&\quad+\frac{a^4\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}-\frac{(2a)\int\cosh(c+dx)\,dx}{b^2d^2}+\frac{6\int x\cosh(c+dx)\,dx}{bd^2}
\end{aligned}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{2a\sinh(c+dx)}{b^2d^3} - \frac{a^3\sinh(c+dx)}{b^4d} + \frac{6x\sinh(c+dx)}{bd^3} + \frac{a^2x\sinh(c+dx)}{b^3d} - \frac{ax^2\sinh(c+dx)}{b^2d} + \frac{x^3\sinh(c+dx)}{bd} + \frac{a^4\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{6\int\sinh(c+dx)\,dx}{bd^3}$$

$$= -\frac{6\cosh(c+dx)}{bd^4} - \frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{2a\sinh(c+dx)}{b^2d^3} - \frac{a^3\sinh(c+dx)}{b^4d} + \frac{6x\sinh(c+dx)}{bd^3} + \frac{a^2x\sinh(c+dx)}{b^3d} - \frac{ax^2\sinh(c+dx)}{b^2d} + \frac{x^3\sinh(c+dx)}{bd} + \frac{a^4\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.73

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = \frac{a^4d^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) - b(b(a^2d^2-2abd^2x+3b^2(2+d^2x^2))\cosh(c+dx)+d(a^3d^2-a^2bd^2x+}{b^5d^4}$$

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a^4*d^4*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - b*(b*(a^2*d^2 - 2*a*
b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x] + d*(a^3*d^2 - a^2*b*d^2*x + a
*b^2*(2 + d^2*x^2) - b^3*x*(6 + d^2*x^2))*Sinh[c + d*x]) + a^4*d^4*Sinh[c -
 (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^5*d^4)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.02

method	result
risch	$\frac{e^{dx+c}x^3}{2db} - \frac{e^{-dx-c}x^3}{2db} - \frac{e^{dx+c}a\,x^2}{2d\,b^2} - \frac{e^{-\frac{da-cb}{b}}\text{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^4}{2b^5} + \frac{e^{-dx-c}a\,x^2}{2d\,b^2} - \frac{e^{\frac{da-cb}{b}}\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^4}{2b^5} + e^{d}$

```
[In] int(x^4*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/b*exp(d*x+c)*x^3-1/2/d/b*exp(-d*x-c)*x^3-1/2/d/b^2*exp(d*x+c)*a*x^2-1
/2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4+1/2/d/b^2*exp(-d*x-c)
*a*x^2-1/2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4+1/2/d/b^3*exp(d
*x+c)*a^2*x-3/2/d^2/b*exp(d*x+c)*x^2-1/2/d/b^3*exp(-d*x-c)*a^2*x-3/2/d^2/b*
exp(-d*x-c)*x^2-1/2/d/b^4*a^3*exp(d*x+c)+1/d^2/b^2*exp(d*x+c)*a*x+1/2/d/b^4
*exp(-d*x-c)*a^3+1/d^2/b^2*exp(-d*x-c)*a*x-1/2/d^2/b^3*a^2*exp(d*x+c)+3/d^3
/b*exp(d*x+c)*x-1/2/d^2/b^3*exp(-d*x-c)*a^2-3/d^3/b*exp(-d*x-c)*x-1/d^3/b^2
*a*exp(d*x+c)+1/d^3/b^2*exp(-d*x-c)*a-3/d^4/b*exp(d*x+c)-3/d^4/b*exp(-d*x-c
)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.08

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = -\frac{2\left(3\,b^4d^2x^2-2\,ab^3d^2x+a^2b^2d^2+6\,b^4\right)\cosh\left(dx+c\right)-\left(a^4d^4\text{Ei}\left(\frac{bdx+ad}{b}\right)+a^4d^4\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc}{}\right.}{}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 + 6*b^4)*cosh(d*x + c)
 - (a^4*d^4*Ei((b*d*x + a*d)/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c
- a*d)/b) - 2*(b^4*d^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 - 2*a*b^3*d + (a^2*b
^2*d^3 + 6*b^4*d)*x)*sinh(d*x + c) + (a^4*d^4*Ei((b*d*x + a*d)/b) - a^4*d^4
*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*d^4)
```

Sympy [F]

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = \int \frac{x^4\cosh\left(c+dx\right)}{a+bx}\,dx$$

```
[In] integrate(x**4*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**4*cosh(c + dx)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.00

$$\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx = -\frac{1}{24}d\left(\frac{12\,a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^4d}-\frac{12\,a^3\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^4}+\frac{6\,a^2\left(\frac{(d^2}{}\right.}{}\right. + \frac{1}{12}\left(\frac{12\,a^4\log(bx+a)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{b^4}\right)\cosh(dx+c)$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*d*(12*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c -
 a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^4*d) - 12*a^3*((d*x*e^c - e
^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^4 + 6*a^2*((d^2*x^2*e^c - 2
*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3
 - 4*a*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^
3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b^2 + 3*((d^4*x^4*e^c - 4*
d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4
+ 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^(-d*x - c)/d^5)/b + 24*a^4*cosh(d
*x + c)*log(b*x + a)/(b^5*d)) + 1/12*(12*a^4*log(b*x + a)/b^5 + (3*b^3*x^4
- 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.86

$$\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx = \frac{b^4d^3x^3e^{(dx+c)}-b^4d^3x^3e^{(-dx-c)}-ab^3d^3x^2e^{(dx+c)}+ab^3d^3x^2e^{(-dx-c)}+a^4d^4\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^4d^4\mathrm{Ei}\left(-\frac{bdx}{}\right.}{}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^3*x^3*e^(d*x + c) - b^4*d^3*x^3*e^(-d*x - c) - a*b^3*d^3*x^2*e^(
d*x + c) + a*b^3*d^3*x^2*e^(-d*x - c) + a^4*d^4*Ei((b*d*x + a*d)/b)*e^(c -
a*d/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*b^2*d^3*x*e^(d*x
```

```
 + c) - 3*b^4*d^2*x^2*e^(d*x + c) - a^2*b^2*d^3*x*e^(-d*x - c) - 3*b^4*d^2*
x^2*e^(-d*x - c) - a^3*b*d^3*e^(d*x + c) + 2*a*b^3*d^2*x*e^(d*x + c) + a^3*
b*d^3*e^(-d*x - c) + 2*a*b^3*d^2*x*e^(-d*x - c) - a^2*b^2*d^2*e^(d*x + c) +
 6*b^4*d*x*e^(d*x + c) - a^2*b^2*d^2*e^(-d*x - c) - 6*b^4*d*x*e^(-d*x - c)
- 2*a*b^3*d*e^(d*x + c) + 2*a*b^3*d*e^(-d*x - c) - 6*b^4*e^(d*x + c) - 6*b^
4*e^(-d*x - c))/(b^5*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx} \, dx = \int \frac{x^4 \cosh(c+d\,x)}{a+b\,x} \, dx$$

```
[In] int((x^4*cosh(c + d*x))/(a + b*x),x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x), x)
```

3.19 $\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = \frac{a\cosh(c+dx)}{b^2d^2} - \frac{2x\cosh(c+dx)}{bd^2} - \frac{a^3\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{2\sinh(c+dx)}{bd^3} + \frac{a^2\sinh(c+dx)}{b^3d} - \frac{ax\sinh(c+dx)}{b^2d} + \frac{x^2\sinh(c+dx)}{bd} - \frac{a^3\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}$$

```
[Out] -a^3*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4+a*cosh(d*x+c)/b^2/d^2-2*x*cosh(d*x+c
)/b/d^2+a^3*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+2*sinh(d*x+c)/b/d^3+a^2*sinh(
d*x+c)/b^3/d-a*x*sinh(d*x+c)/b^2/d+x^2*sinh(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6874, 2717, 3377, 2718, 3384, 3379, 3382}

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = -\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^3\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{a^2\sinh(c+dx)}{b^3d} + \frac{a\cosh(c+dx)}{b^2d^2} - \frac{ax\sinh(c+dx)}{b^2d} + \frac{2\sinh(c+dx)}{bd^3} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{x^2\sinh(c+dx)}{bd}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a*Cosh[c + d*x])/(b^2*d^2) - (2*x*Cosh[c + d*x])/(b*d^2) - (a^3*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 + (2*Sinh[c + d*x])/(b*d^3) + (a^2
*Sinh[c + d*x])/(b^3*d) - (a*x*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d*x])
/(b*d) - (a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral}=\int\left(\frac{a^2\cosh(c+dx)}{b^3}-\frac{ax\cosh(c+dx)}{b^2}+\frac{x^2\cosh(c+dx)}{b}-\frac{a^3\cosh(c+dx)}{b^3(a+bx)}\right)dx$$

$$=\frac{a^2\int\cosh(c+dx)\,dx}{b^3}-\frac{a^3\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^3}-\frac{a\int x\cosh(c+dx)\,dx}{b^2}+\frac{\int x^2\cosh(c+dx)\,dx}{b}$$

$$=\frac{a^2\sinh(c+dx)}{b^3d}-\frac{ax\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}+\frac{a\int\sinh(c+dx)\,dx}{b^2d}-\frac{2\int x\sinh(c+dx)\,dx}{bd}-\frac{\left(a^3\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}-\frac{\left(a^3\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}$$

$$=\frac{a\cosh(c+dx)}{b^2d^2}-\frac{2x\cosh(c+dx)}{bd^2}-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{ax\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}-\frac{a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{2\int\cosh(c+dx)\,dx}{bd^2}$$

$$=\frac{a\cosh(c+dx)}{b^2d^2}-\frac{2x\cosh(c+dx)}{bd^2}-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{2\sinh(c+dx)}{bd^3}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{ax\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}-\frac{a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int\frac{x^3\cosh(c+dx)}{a+bx}\,dx$$
$$=\frac{-a^3d^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+b(bd(a-2bx)\cosh(c+dx)+(a^2d^2-abd^2x+b^2(2+d^2x^2))\sinh(c+}{b^4d^3}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-(a^3*d^3*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*(b*d*(a - 2*b*x
)*Cosh[c + d*x] + (a^2*d^2 - a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])
- a^3*d^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^4*d^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.95

method	result
risch	$\frac{\mathrm{e}^{-\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^3}{2b^4}-\frac{\mathrm{e}^{-dx-c}x^2}{2db}+\frac{\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^3}{2b^4}+\frac{\mathrm{e}^{dx+c}x^2}{2db}+\frac{\mathrm{e}^{-dx-c}ax}{2d\,b^2}-\frac{\mathrm{e}^{dx+c}ax}{2d\,b^2}-$

```
[In] int(x^3*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3-1/2/d/b*exp(-d*x-c)*
x^2+1/2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3+1/2/d/b*exp(d*x+c)
*x^2+1/2/d/b^2*exp(-d*x-c)*a*x-1/2/d/b^2*exp(d*x+c)*a*x-1/2/d/b^3*exp(-d*x-
c)*a^2-1/d^2/b*exp(-d*x-c)*x+1/2/d/b^3*a^2*exp(d*x+c)-1/d^2/b*exp(d*x+c)*x+
1/2/d^2/b^2*exp(-d*x-c)*a+1/2/d^2/b^2*a*exp(d*x+c)-1/d^3/b*exp(-d*x-c)+1/d^
3/b*exp(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.27

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = -\frac{2\left(2\,b^3dx-ab^2d\right)\cosh\left(dx+c\right)+\left(a^3d^3\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^3d^3\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^3d^2x^2-ab^2\right.}{2\,b^4d^3}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b^3*d*x - a*b^2*d)*cosh(d*x + c) + (a^3*d^3*Ei((b*d*x + a*d)/b)
+ a^3*d^3*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*d^2*x^2 - a*b
^2*d^2*x + a^2*b*d^2 + 2*b^3)*sinh(d*x + c) - (a^3*d^3*Ei((b*d*x + a*d)/b)
- a^3*d^3*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*d^3)
```

Sympy [F]

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = \int \frac{x^3\cosh\left(c+dx\right)}{a+bx}\,dx$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(151) = 302.

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.19

$$\int \frac{x^3 \cosh(c+dx)}{a+bx} dx$$
$$= \frac{1}{12} d \left(\frac{6 a^3 \left(\frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^3 d} - \frac{6 a^2 \left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2} \right)}{b^3} + \frac{3 a \left(\frac{(d^2x^2e^c - 2}{} \right.}{} \right.$$
$$- \frac{1}{6} \left(\frac{6 a^3 \log(bx+a)}{b^4} - \frac{2 b^2 x^3 - 3 abx^2 + 6 a^2 x}{b^3} \right) \cosh(dx+c)$$

[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")

[Out] 1/12*d*(6*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^3*d) - 6*a^2*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^3 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^2 - 2*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b + 12*a^3*cosh(d*x + c)*log(b*x + a)/(b^4*d)) - 1/6*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3)*cosh(d*x + c)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.71

$$\int \frac{x^3 \cosh(c+dx)}{a+bx} dx$$
$$= \frac{b^3 d^2 x^2 e^{(dx+c)} - b^3 d^2 x^2 e^{(-dx-c)} - a^3 d^3 \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} - a^3 d^3 \text{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} - ab^2 d^2 x e^{(dx+c)} + ab^2}{}$$

[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="giac")

[Out] 1/2*(b^3*d^2*x^2*e^(d*x + c) - b^3*d^2*x^2*e^(-d*x - c) - a^3*d^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^3*d^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b^2*d^2*x*e^(d*x + c) + a*b^2*d^2*x*e^(-d*x - c) + a^2*b*d^2*e^(d*x + c) - 2*b^3*d*x*e^(d*x + c) - a^2*b*d^2*e^(-d*x - c) - 2*b^3*d*x*e^(-d*x - c) + a*b^2*d*e^(d*x + c) + a*b^2*d*e^(-d*x - c) + 2*b^3*e^(d*x + c) - 2*b^3*e^(-d*x - c))/(b^4*d^3)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c + dx)}{a + bx} \, dx = \int \frac{x^3 \cosh(c + d\,x)}{a + b\,x} \, dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x),x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x), x)
```

3.20 $\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx = -\frac{\cosh(c+dx)}{bd^2} + \frac{a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} + \frac{x \sinh(c+dx)}{bd} + \frac{a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}$$

```
[Out] a^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-cosh(d*x+c)/b/d^2-a^2*Shi(a*d/b+d*x)*
sinh(-c+a*d/b)/b^3-a*sinh(d*x+c)/b^2/d+x*sinh(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6874, 2717, 3377, 2718, 3384, 3379, 3382}

$$\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx = \frac{a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} - \frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + (a^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*
x])/b^3 - (a*Sinh[c + d*x])/(b^2*d) + (x*Sinh[c + d*x])/(b*d) + (a^2*Sinh[c
 - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{a\cosh(c+dx)}{b^2}+\frac{x\cosh(c+dx)}{b}+\frac{a^2\cosh(c+dx)}{b^2(a+bx)}\right)dx\\ &= -\frac{a\int\cosh(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^2}+\frac{\int x\cosh(c+dx)\,dx}{b}\end{aligned}$$

$$= -\frac{a\sinh(c+dx)}{b^2d} + \frac{x\sinh(c+dx)}{bd} - \frac{\int \sinh(c+dx)\,dx}{bd} + \frac{\left(a^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} + \frac{\left(a^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a\sinh(c+dx)}{b^2d} + \frac{x\sinh(c+dx)}{bd} + \frac{a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = \frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + b(-b\cosh(c+dx)+d(-a+bx)\sinh(c+dx)) + a^2d^2\sinh\left(c-\frac{ad}{b}\right)\text{Sh}}{b^3d^2}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a^2*d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + b*(-(b*Cosh[c + d*x]
) + d*(-a + b*x)*Sinh[c + d*x]) + a^2*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*
(a/b + x)])/(b^3*d^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{e^{-\frac{da-cb}{b}}\text{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^2}{2b^3} - \frac{e^{\frac{da-cb}{b}}\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^2}{2b^3} - \frac{e^{-dx-c}x}{2db} + \frac{e^{dx+c}x}{2db} + \frac{e^{-dx-c}a}{2d\,b^2} - \frac{a\,e^{dx+c}}{2d\,b^2} - \frac{e^{-dx-c}}{2d^2b}$

```
[In] int(x^2*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2-1/2/b^3*exp((a*d-b*
c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2-1/2/d/b*exp(-d*x-c)*x+1/2/d/b*exp(d*x+c)*
x+1/2/d/b^2*exp(-d*x-c)*a-1/2/d/b^2*a*exp(d*x+c)-1/2/d^2/b*exp(-d*x-c)-1/2/
d^2/b*exp(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.56

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = -\frac{2\,b^2\cosh\left(dx+c\right) - \left(a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) + a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right) - 2\left(b^2dx-abd\right)\sinh\left(dx+c\right)}{2\,b^3d^2}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei((b*d*x + a*d)/b) + a^2*d^2*Ei(-(b*d
*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^2*d*x - a*b*d)*sinh(d*x + c) + (a
^2*d^2*Ei((b*d*x + a*d)/b) - a^2*d^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d
)/b))/(b^3*d^2)
```

Sympy [F]

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = \int \frac{x^2\cosh\left(c+dx\right)}{a+bx}\,dx$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a),x)
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(103) = 206.

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.33

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = -\frac{1}{4}\,d\left(\frac{2\,a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d} - \frac{2\,a\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^2} + \frac{(d^2x^2e^c-2\,dx\ldots}{d\ldots}\right.$$
$$+\frac{1}{2}\left(\frac{2\,a^2\log\left(bx+a\right)}{b^3} + \frac{bx^2-2\,ax}{b^2}\right)\cosh\left(dx+c\right)$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*d*(2*a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^2*d) - 2*a*((d*x*e^c - e^c)*e
^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2 + ((d^2*x^2*e^c - 2*d*x*e^c +
2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b + 4*a^2*cosh
(d*x + c)*log(b*x + a)/(b^3*d)) + 1/2*(2*a^2*log(b*x + a)/b^3 + (b*x^2 - 2*
a*x)/b^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int \frac{x^2 \cosh(c+dx)}{a+bx}\,dx$$
$$= \frac{a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + b^2dxe^{(dx+c)} - b^2dxe^{(-dx-c)} - abde^{(dx+c)} + abde^{(-dx-}}{2\,b^3d^2}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/
b)*e^(-c + a*d/b) + b^2*d*x*e^(d*x + c) - b^2*d*x*e^(-d*x - c) - a*b*d*e^(d
*x + c) + a*b*d*e^(-d*x - c) - b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^3*d^2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{a+bx}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{a+b\,x}\,dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x),x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x), x)
```

3.21 $\int \frac{x\cosh(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{\sinh(c+dx)}{bd} - \frac{a\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] -a*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^2+a*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^2+si
nh(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 2717, 3384, 3379, 3382}

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{a\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{\sinh(c+dx)}{bd}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] -((a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^2) + Sinh[c + d*x]/(b
*d) - (a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b(a+bx)}\right) dx$$

$$= \frac{\int \cosh(c+dx)\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b}$$

$$= \frac{\sinh(c+dx)}{bd} - \frac{\left(a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} - \frac{\left(a\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}$$

$$= -\frac{a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{\sinh(c+dx)}{bd} - \frac{a\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \frac{-ad\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+b\sinh(c+dx)-ad\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2d}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-(a*d*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*Sinh[c + d*x] - a*d
*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^2*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

method	result	size
risch	$\frac{\mathrm{e}^{-\frac{da-cb}{b}}\operatorname{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a}{2b^2}+\frac{\mathrm{e}^{\frac{da-cb}{b}}\operatorname{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a}{2b^2}-\frac{\mathrm{e}^{-dx-c}}{2db}+\frac{\mathrm{e}^{dx+c}}{2db}$	114

```
[In] int(x*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a+1/2/b^2*exp((a*d-b*c)/
b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-1/2/d/b*exp(-d*x-c)+1/2/d/b*exp(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.74

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{\left(ad\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)+ad\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\,b\sinh\left(dx+c\right)-\left(ad\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)-ad\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)}{2\,b^2d}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*Ei((b*d*x + a*d)/b) + a*d*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - 2*b*sinh(d*x + c) - (a*d*Ei((b*d*x + a*d)/b) - a*d*Ei(-(b*d*x + a*d)
/b))*sinh(-(b*c - a*d)/b))/(b^2*d)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \int \frac{x\cosh{(c+dx)}}{a+bx}\,dx$$

[In] integrate(x*cosh(d*x+c)/(b*x+a),x)

[Out] Integral(x*cosh(c + d*x)/(a + b*x), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(69) = 138.

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.29

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \frac{1}{2}\,d\left(\frac{a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{bd}-\frac{\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}}{b}+\frac{2\,a\cosh{(dx+c)}\log{(bx}}{b^2d}\right. \\ +\left(\frac{x}{b}-\frac{a\log{(bx+a)}}{b^2}\right)\cosh{(dx+c)}$$

[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")

[Out] 1/2*d*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b + 2*a*cosh(d*x + c)*log(b*x + a)/(b^2*d)) + (x/b - a*log(b*x + a)/b^2)*cosh(d*x + c)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-be^{(dx+c)}+be^{(-dx-c)}}{2\,b^2d}$$

[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="giac")

[Out] -1/2*(a*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - b*e^(d*x + c) + b*e^(-d*x - c))/(b^2*d)

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \int \frac{x\cosh(c+d\,x)}{a+b\,x}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x),x)
[Out] int((x*cosh(c + d*x))/(a + b*x), x)
```

3.22 $\int \frac{\cosh(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx=\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

```
[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b-Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx=\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x),x]
```

```
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh\left(c-\frac{ad}{b}\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx+\sinh\left(c-\frac{ad}{b}\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx\\ &= \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int\frac{\cosh(c+dx)}{a+bx}\,dx=\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)+\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x),x]
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x] + Sinh[c - (a*d)/b]*SinhInte
gral[(a*d)/b + d*x])/b
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{e^{\frac{da-cb}{b}}\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2b}-\frac{e^{-\frac{da-cb}{b}}\text{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)}{2b}$	81

```
[In] int(cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b*exp(-(a*d-b*c)/b)*Ei(
1,-d*x-c-(a*d-b*c)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \frac{\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*((Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - (E
i((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/b
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \int \frac{\cosh(c+dx)}{a+bx}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x+a),x)
[Out] Integral(cosh(c + d*x)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = -\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{2\,b} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{2\,b}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
[Out] -1/2*e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - 1/2*e^(c - a*d/b)*
exp_integral_e(1, -(b*x + a)*d/b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \frac{\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + \mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,b}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="giac")
[Out] 1/2*(Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \int \frac{\cosh(c+d\,x)}{a+b\,x}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*x),x)
[Out] int(cosh(c + d*x)/(a + b*x), x)
```

3.23 $\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \frac{\cosh(c)\mathrm{Chi}(dx)}{a} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a}$$

```
[Out] Chi(d*x)*cosh(c)/a-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a+Shi(d*x)*sinh(c)/a+Shi(a
*d/b+d*x)*sinh(-c+a*d/b)/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6874, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = -\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d
*x])/a + (Sinh[c]*SinhIntegral[d*x])/a - (Sinh[c - (a*d)/b]*SinhIntegral[(a
*d)/b + d*x])/a
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c+dx)}{a(a+bx)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a} \\
&= \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a} - \frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} \\
&\quad + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a} - \frac{\left(b\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} \\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$
\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx
= \frac{\cosh(c)\mathrm{Chi}(dx) - \cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + \sinh(c)\mathrm{Shi}(dx) - \sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{a}
$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x] - Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] +
Sinh[c]*SinhIntegral[d*x] - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

method	result	size
risch	$-\frac{\mathrm{e}^{-c} \operatorname{Ei}_{1}(d x)}{2 a}+\frac{\mathrm{e}^{\frac{d a-c b}{b}} \operatorname{Ei}_{1}\left(d x+c+\frac{d a-c b}{b}\right)}{2 a}-\frac{\mathrm{e}^{c} \operatorname{Ei}_{1}(-d x)}{2 a}+\frac{\mathrm{e}^{-\frac{d a-c b}{b}} \operatorname{Ei}_{1}\left(-d x-c-\frac{d a-c b}{b}\right)}{2 a}$	108

```
[In] int(cosh(d*x+c)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/2/a*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2
/a*exp(c)*Ei(1,-d*x)+1/2/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.68

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \frac{(\operatorname{Ei}(dx)+\operatorname{Ei}(-dx))\cosh(c)-\left(\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)+\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+(\operatorname{Ei}(dx)-\operatorname{Ei}(-dx))\sinh(c)+}{2a}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei(d*x) + Ei(-d*x))*cosh(c) - (Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d
)/b))*cosh(-(b*c - a*d)/b) + (Ei(d*x) - Ei(-d*x))*sinh(c) + (Ei((b*d*x + a*
d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/a
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(74) = 148$.

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$
$$=\frac{1}{2}d\left(\frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{ad}+\frac{2\cosh(dx+c)\log(bx+a)}{ad}-\frac{2\cosh(dx+c)\log(x)}{ad}\right.$$
$$-\left(\frac{\log(bx+a)}{a}-\frac{\log(x)}{a}\right)\cosh(dx+c)$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*(b*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)
*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a*d) + 2*cosh(d*x + c)*log(b*x + a)/
(a*d) - 2*cosh(d*x + c)*log(x)/(a*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a*d
)) - (log(b*x + a)/a - log(x)/a)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \frac{\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+\mathrm{Ei}(dx)\,e^{c}-\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,a}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(Ei(-d*x)*e^(-c) - Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(d*x)*e^c - Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \frac{\cosh(c+d\,x)}{x\,\left(a+b\,x\right)}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x)),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x)), x)
```

3.24 $\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = &-\frac{\cosh(c+dx)}{ax}-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}\\ &+\frac{d\text{Chi}(dx)\sinh(c)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}\\ &-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2}\end{aligned}$$

```
[Out] -b*Chi(d*x)*cosh(c)/a^2+b*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^2-cosh(d*x+c)/a/x
+d*cosh(c)*Shi(d*x)/a+d*Chi(d*x)*sinh(c)/a-b*Shi(d*x)*sinh(c)/a^2-b*Shi(a*d
/b+d*x)*sinh(-c+a*d/b)/a^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = &-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}\\ &-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}\\ &+\frac{d\sinh(c)\text{Chi}(dx)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}\end{aligned}$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (b*Cosh[c - (a
*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 + (d*CoshIntegral[d*x]*Sinh[c])/a +
 (d*Cosh[c]*SinhIntegral[d*x])/a - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (b*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a} - \frac{(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^2}$$
$$+ \frac{\left(b^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$
$$- \frac{(b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^2} + \frac{\left(b^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$
$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2}$$
$$+ \frac{b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a} + \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{a}$$
$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}$$
$$+ \frac{d\mathrm{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a}$$
$$- \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} + \frac{b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx$$
$$= \frac{-a\cosh(c+dx) + bx\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + \mathrm{Chi}(dx)(-bx\cosh(c)+adx\sinh(c)) + adx\cosh(c)\mathrm{S}}{a^2x}$$

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (-(a*Cosh[c + d*x]) + b*x*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + Cos
hIntegral[d*x]*(-(b*x*Cosh[c]) + a*d*x*Sinh[c]) + a*d*x*Cosh[c]*SinhIntegra
l[d*x] - b*x*Sinh[c]*SinhIntegral[d*x] + b*x*Sinh[c - (a*d)/b]*SinhIntegral
[d*(a/b + x)])/(a^2*x)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{e^{c}\,\mathrm{Ei}_1(-dx)adx - e^{-c}\,\mathrm{Ei}_1(dx)adx - e^{c}\,\mathrm{Ei}_1(-dx)bx + b\,e^{-\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)x - e^{-c}\,\mathrm{Ei}_1(dx)bx + b\,e^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-c}{b}\right.}{2a^2x}$

```
[In] int(cosh(d*x+c)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] -1/2*(exp(c)*Ei(1,-d*x)*a*d*x-exp(-c)*Ei(1,d*x)*a*d*x-exp(c)*Ei(1,-d*x)*b*x
+b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*x-exp(-c)*Ei(1,d*x)*b*x+b*exp
((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*x+exp(-d*x-c)*a+a*exp(d*x+c))/a^2/x

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = -\frac{2\,a\cosh\left(dx+c\right)-\left((ad-b)x\mathrm{Ei}(dx)-(ad+b)x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(bx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+bx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\mathrm{c}$$

[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")

[Out] -1/2*(2*a*cosh(d*x + c) - ((a*d - b)*x*Ei(d*x) - (a*d + b)*x*Ei(-d*x))*cosh
(c) - (b*x*Ei((b*d*x + a*d)/b) + b*x*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - ((a*d - b)*x*Ei(d*x) + (a*d + b)*x*Ei(-d*x))*sinh(c) + (b*x*Ei((b*d*
x + a*d)/b) - b*x*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*x)

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\cosh\left(c+dx\right)}{x^2\left(a+bx\right)}\,dx$$

[In] integrate(cosh(d*x+c)/x**2/(b*x+a),x)

[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.70

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = -\frac{1}{2}\,d\left(\frac{\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}}{a}+\frac{b^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d}+\frac{2\,b\cosh\left(dx+c\right)\log\left(\right.}{a^2d}\right.$$
$$+\left(\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log\left(x\right)}{a^2}-\frac{1}{ax}\right)\cosh\left(dx+c\right)$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*d*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)/a + b^2*(e^(-c + a*d/b)*exp_integra
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(a^2*d) + 2*b*cosh(d*x + c)*log(b*x + a)/(a^2*d) - 2*b*cosh(d*x + c)*log(
x)/(a^2*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/(a^2*d)) + (b*log(b*x + a)/a
^2 - b*log(x)/a^2 - 1/(a*x))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx =$$

$$-\frac{adx\text{Ei}(-dx)\,e^{(-c)} - adx\text{Ei}(dx)\,e^{c} + bx\text{Ei}(-dx)\,e^{(-c)} - bx\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + bx\text{Ei}(dx)\,e^{c} - bx\text{Ei}\left(-\frac{bd}{}\right.\cdots}{2\,a^2x}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c + b*x*Ei(-d*x)*e^(-c) - b*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b*x*Ei(d*x)*e^c - b*x*Ei(-(b*d*x + a*d
)/b)*e^(-c + a*d/b) + a*e^(d*x + c) + a*e^(-d*x - c))/(a^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,(a+b\,x)}\,dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)),x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)), x)
```

3.25 $\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 190

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = &-\frac{\cosh(c+dx)}{2ax^2}+\frac{b\cosh(c+dx)}{a^2x}+\frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3}\\&+\frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a}-\frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}\\&-\frac{bd\mathrm{Chi}(dx)\sinh(c)}{a^2}-\frac{d\sinh(c+dx)}{2ax}\\&-\frac{bd\cosh(c)\mathrm{Shi}(dx)}{a^2}+\frac{b^2\sinh(c)\mathrm{Shi}(dx)}{a^3}\\&+\frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a}-\frac{b^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}\end{aligned}$$

```
[Out] b^2*Chi(d*x)*cosh(c)/a^3+1/2*d^2*Chi(d*x)*cosh(c)/a-b^2*Chi(a*d/b+d*x)*cosh
(-c+a*d/b)/a^3-1/2*cosh(d*x+c)/a/x^2+b*cosh(d*x+c)/a^2/x-b*d*cosh(c)*Shi(d*
x)/a^2-b*d*Chi(d*x)*sinh(c)/a^2+b^2*Shi(d*x)*sinh(c)/a^3+1/2*d^2*Shi(d*x)*s
inh(c)/a+b^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3-1/2*d*sinh(d*x+c)/a/x
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used

= {6874, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{b^2\sinh(c)\mathrm{Shi}(dx)}{a^3} - \frac{b^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{bd\sinh(c)\mathrm{Chi}(dx)}{a^2} - \frac{bd\cosh(c)\mathrm{Shi}(dx)}{a^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} + \frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a} - \frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax}$$

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a*x^2) + (b*Cosh[c + d*x])/(a^2*x) + (b^2*Cosh[c]*CoshI
ntegral[d*x])/a^3 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) - (b^2*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^3 - (b*d*CoshIntegral[d*x]*Sinh[c])/
a^2 - (d*Sinh[c + d*x])/(2*a*x) - (b*d*Cosh[c]*SinhIntegral[d*x])/a^2 + (b^
2*Sinh[c]*SinhIntegral[d*x])/a^3 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a) -
(b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a^2x^2} + \frac{b^2\cosh(c+dx)}{a^3x} - \frac{b^3\cosh(c+dx)}{a^3(a+bx)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x^2} dx}{a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b^3\int \frac{\cosh(c+dx)}{a+bx} dx}{a^3}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{d\int \frac{\sinh(c+dx)}{x^2} dx}{2a} - \frac{(bd)\int \frac{\sinh(c+dx)}{x} dx}{a^2} + \frac{(b^2\cosh(c))\int \frac{\cosh(dx)}{x} dx}{a^3} - \frac{\left(b^3\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{a^3} + \frac{(b^2\sinh(c))\int \frac{\sinh(dx)}{x} dx}{a^3} - \frac{\left(b^3\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{a^3}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\text{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d\sinh(c+dx)}{2ax} + \frac{b^2\sinh(c)\text{Shi}(dx)}{a^3} - \frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d^2\int \frac{\cosh(c+dx)}{x} dx}{2a} - \frac{(bd\cosh(c))\int \frac{\sinh(dx)}{x} dx}{a^2} - \frac{(bd\sinh(c))\int \frac{\cosh(dx)}{x} dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\text{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{bd\text{Chi}(dx)\sinh(c)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{bd\cosh(c)\text{Shi}(dx)}{a^2} + \frac{b^2\sinh(c)\text{Shi}(dx)}{a^3} - \frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{(d^2\cosh(c))\int \frac{\cosh(dx)}{x} dx}{2a} + \frac{(d^2\sinh(c))\int \frac{\sinh(dx)}{x} dx}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\text{Chi}(dx)}{a^3} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{bd\text{Chi}(dx)\sinh(c)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{bd\cosh(c)\text{Shi}(dx)}{a^2} + \frac{b^2\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d^2\sinh(c)\text{Shi}(dx)}{2a} - \frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$
$$= \frac{-a^2\cosh(c+dx) + 2abx\cosh(c+dx) - 2b^2x^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + x^2\text{Chi}(dx)\left((2b^2+a^2d^2)\text{co}\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] (-(a^2*Cosh[c + d*x]) + 2*a*b*x*Cosh[c + d*x] - 2*b^2*x^2*Cosh[c - (a*d)/b]
*CoshIntegral[d*(a/b + x)] + x^2*CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[
c] - 2*a*b*d*Sinh[c]) - a^2*d*x*Sinh[c + d*x] - 2*a*b*d*x^2*Cosh[c]*SinhInt
egral[d*x] + 2*b^2*x^2*Sinh[c]*SinhIntegral[d*x] + a^2*d^2*x^2*Sinh[c]*Sinh
Integral[d*x] - 2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(2*a
^3*x^2)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.48

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}}{4ax}+\frac{\mathrm{e}^{-dx-c}b}{2a^2x}-\frac{\mathrm{e}^{-dx-c}}{4a\,x^2}-\frac{d^2\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)}{4a}-\frac{d\,\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)b}{2a^2}-\frac{\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)b^2}{2a^3}+\frac{b^2\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2a^3}+\frac{b}{}$

```
[In] int(cosh(d*x+c)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x+1/2*exp(-d*x-c)/a^2/x*b-1/4*exp(-d*x-c)/a/x^2-1/4*d^2
/a*exp(-c)*Ei(1,d*x)-1/2*d/a^2*exp(-c)*Ei(1,d*x)*b-1/2/a^3*exp(-c)*Ei(1,d*x
)*b^2+1/2*b^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*b/a^2/x*exp(
d*x+c)+1/2*d*b/a^2*exp(c)*Ei(1,-d*x)-1/2*b^2/a^3*exp(c)*Ei(1,-d*x)+1/2/a^3*
b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/4/a/x^2*exp(d*x+c)-1/4*d/a
/x*exp(d*x+c)-1/4*d^2/a*exp(c)*Ei(1,-d*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.45

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = -\frac{2\,a^2dx\sinh\left(dx+c\right)-2\left(2\,abx-a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^2-2\,abd+2\,b^2\right)x^2\mathrm{Ei}(dx)+\left(a^2d^2+2\,abd+2\right.\right.}{}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*sinh(d*x + c) - 2*(2*a*b*x - a^2)*cosh(d*x + c) - ((a^2*d^2
 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))
*cosh(c) + 2*(b^2*x^2*Ei((b*d*x + a*d)/b) + b^2*x^2*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 +
 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))*sinh(c) - 2*(b^2*x^2*Ei((b*d*x + a*d)/b) -
b^2*x^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*x^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\cosh\left(c+dx\right)}{x^3\left(a+bx\right)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.27

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \frac{1}{4}\,d\left(\frac{de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)}{a}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)b}{a^2}+\frac{2\,b^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\right.}}{}\right.}{a^3d}\right.$$
$$-\frac{1}{2}\left(\frac{2\,b^2\log\left(bx+a\right)}{a^3}-\frac{2\,b^2\log\left(x\right)}{a^3}-\frac{2\,bx-a}{a^2x^2}\right)\cosh\left(dx+c\right)$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*d*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))/a + 2*(Ei(-d*x)*e^
(-c) - Ei(d*x)*e^c)*b/a^2 + 2*b^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x +
a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a^3*d) + 4*
b^2*cosh(d*x + c)*log(b*x + a)/(a^3*d) - 4*b^2*cosh(d*x + c)*log(x)/(a^3*d)
 + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/(a^3*d)) - 1/2*(2*b^2*log(b*x + a)
/a^3 - 2*b^2*log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \frac{a^2d^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^2x^2\mathrm{Ei}(dx)\,e^{c} + 2\,abdx^2\mathrm{Ei}(-dx)\,e^{(-c)} - 2\,abdx^2\mathrm{Ei}(dx)\,e^{c} + 2\,b^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c + 2*a*b*d*x^2*Ei
(-d*x)*e^(-c) - 2*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) - 2*b^2
*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^2*x^2*Ei(d*x)*e^c - 2*b^2*x^2*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x
 - c) + 2*a*b*x*e^(d*x + c) + 2*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*
e^(-d*x - c))/(a^3*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,(a+b\,x)}\,dx$$

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x)),x)
```

```
[Out] int(cosh(c + d*x)/(x^3*(a + b*x)), x)
```

3.26 $\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 231

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} dx = \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{a^4d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^6} + \frac{2\sinh(c+dx)}{b^2d^3} + \frac{3a^2\sinh(c+dx)}{b^4d} - \frac{2ax\sinh(c+dx)}{b^3d} + \frac{x^2\sinh(c+dx)}{b^2d} + \frac{a^4d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^6} - \frac{4a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}$$

```
[Out] -4*a^3*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^5+2*a*cosh(d*x+c)/b^3/d^2-2*x*cosh(d
*x+c)/b^2/d^2-a^4*cosh(d*x+c)/b^5/(b*x+a)+a^4*d*cosh(-c+a*d/b)*Shi(a*d/b+d*
x)/b^6-a^4*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^6+4*a^3*Shi(a*d/b+d*x)*sinh(-c
+a*d/b)/b^5+2*sinh(d*x+c)/b^2/d^3+3*a^2*sinh(d*x+c)/b^4/d-2*a*x*sinh(d*x+c)
/b^3/d+x^2*sinh(d*x+c)/b^2/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6874, 2717, 3377, 2718, 3378, 3384, 3379, 3382}

$$\int \frac{x^4\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{a^4 d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^6}+\frac{a^4 d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)}-\frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{4a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{3a^2\sinh(c+dx)}{b^4d} + \frac{2a\cosh(c+dx)}{b^3d^2}-\frac{2ax\sinh(c+dx)}{b^3d} + \frac{2\sinh(c+dx)}{b^2d^3}-\frac{2x\cosh(c+dx)}{b^2d^2}+\frac{x^2\sinh(c+dx)}{b^2d}$$

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*a*Cosh[c + d*x])/(b^3*d^2) - (2*x*Cosh[c + d*x])/(b^2*d^2) - (a^4*Cosh[c
 + d*x])/(b^5*(a + b*x)) - (4*a^3*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b +
d*x])/b^5 + (a^4*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^6 + (2*
Sinh[c + d*x])/(b^2*d^3) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (2*a*x*Sinh[c +
d*x])/(b^3*d) + (x^2*Sinh[c + d*x])/(b^2*d) + (a^4*d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b^6 - (4*a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/
b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3a^2\cosh(c+dx)}{b^4} - \frac{2ax\cosh(c+dx)}{b^3} + \frac{x^2\cosh(c+dx)}{b^2} + \frac{a^4\cosh(c+dx)}{b^4(a+bx)^2} - \frac{4a^3\cosh(c+dx)}{b^4(a+bx)}\right) dx \\
&= \frac{(3a^2)\int \cosh(c+dx)\,dx}{b^4} - \frac{(4a^3)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^4\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^4} \\
&\quad - \frac{(2a)\int x\cosh(c+dx)\,dx}{b^3} + \frac{\int x^2\cosh(c+dx)\,dx}{b^2} \\
&= -\frac{a^4\cosh(c+dx)}{b^5(a+bx)} + \frac{3a^2\sinh(c+dx)}{b^4d} - \frac{2ax\sinh(c+dx)}{b^3d} + \frac{x^2\sinh(c+dx)}{b^2d} \\
&\quad + \frac{(2a)\int \sinh(c+dx)\,dx}{b^3d} - \frac{2\int x\sinh(c+dx)\,dx}{b^2d} + \frac{(a^4d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^5} \\
&\quad - \frac{\left(4a^3\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4} - \frac{\left(4a^3\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} \\
&\quad - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2\sinh(c+dx)}{b^4d} - \frac{2ax\sinh(c+dx)}{b^3d} \\
&\quad + \frac{x^2\sinh(c+dx)}{b^2d} - \frac{4a^3\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{2\int\cosh(c+dx)\,dx}{b^2d^2} \\
&\quad + \frac{\left(a^4d\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^5} + \frac{\left(a^4d\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^5} \\
&= \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} \\
&\quad + \frac{a^4d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^6} + \frac{2\sinh(c+dx)}{b^2d^3} + \frac{3a^2\sinh(c+dx)}{b^4d} - \frac{2ax\sinh(c+dx)}{b^3d} \\
&\quad + \frac{x^2\sinh(c+dx)}{b^2d} + \frac{a^4d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^6} - \frac{4a^3\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.75

$$
\int \frac{x^4\cosh(c+dx)}{(a+bx)^2}\,dx
= \frac{-\frac{b(-2a^2b^2+a^4d^2+2b^4x^2)\cosh(c+dx)}{d^2(a+bx)} + a^3\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(-4b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right) + \frac{b^2(3a^2d^2-2abd^2x\cdots}{}}{b^6}
$$

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (-((b*(-2*a^2*b^2 + a^4*d^2 + 2*b^4*x^2)*Cosh[c + d*x])/(d^2*(a + b*x))) +
a^3*CoshIntegral[d*(a/b + x)]*(-4*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/
b]) + (b^2*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
 + a^3*(a*d*Cosh[c - (a*d)/b] - 4*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b
+ x)])/b^6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(236) = 472.

Time = 0.29 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.90

method	result
risch	$-\frac{2\,e^{-dx-c}b^5x+2\,e^{-dx-c}a\,b^4-4\,e^{-\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^3b^2d^3x-e^{-dx-c}a\,b^4d^2x^2-2\,e^{dx+c}b^5x-2\,e^{dx+c}a\,b^4+e^{dx+c}a\,b^4d^2x\cdots}{}$

```
[In] int(x^4*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d^3*(2*exp(-d*x-c)*b^5*x+2*exp(-d*x-c)*a*b^4-4*exp(-(a*d-b*c)/b)*Ei(1,
-d*x-c-(a*d-b*c)/b)*a^3*b^2*d^3*x-exp(-d*x-c)*a*b^4*d^2*x^2-2*exp(d*x+c)*b^
5*x-2*exp(d*x+c)*a*b^4+exp(d*x+c)*a*b^4*d^2*x^2-4*exp(-(a*d-b*c)/b)*Ei(1,-d
*x-c-(a*d-b*c)/b)*a^4*b*d^3-exp(d*x+c)*a^2*b^3*d^2*x+exp(-d*x-c)*b^5*d^2*x^
3-exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^5*d^4+exp(-d*x-c)*a^4*b*d^3+2*
exp(-d*x-c)*b^5*d*x^2+3*exp(-d*x-c)*a^3*b^2*d^2-2*exp(-d*x-c)*a^2*b^3*d-4*e
xp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*b*d^3+exp(-d*x-c)*a^2*b^3*d^2*x
+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4*b*d^4*x-exp((a*d-b*c)/b)*Ei
(1,d*x+c+(a*d-b*c)/b)*a^4*b*d^4*x-4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b
)*a^3*b^2*d^3*x-exp(d*x+c)*b^5*d^2*x^3+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b
*c)/b)*a^5*d^4+exp(d*x+c)*a^4*b*d^3+2*exp(d*x+c)*b^5*d*x^2-3*exp(d*x+c)*a^3
*b^2*d^2-2*exp(d*x+c)*a^2*b^3*d)/b^6/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.61

$$\int \frac{x^4\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{2\left(a^4bd^3+2\,b^5dx^2-2\,a^2b^3d\right)\cosh\left(dx+c\right)-\left(\left(a^5d^4-4\,a^4bd^3+\left(a^4bd^4-4\,a^3b^2d^3\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^5d^4\right.\right.}{}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4*b*d^3 + 2*b^5*d*x^2 - 2*a^2*b^3*d)*cosh(d*x + c) - ((a^5*d^4 -
 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^
4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh
(-(b*c - a*d)/b) - 2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 + 2*a*b^4
 + (a^2*b^3*d^2 + 2*b^5)*x)*sinh(d*x + c) + ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*
b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^4 + 4*a^4*b*d^3 + (a
^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b
^7*d^3*x + a*b^6*d^3)
```

Sympy [F]

$$\int \frac{x^4\cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4\cosh\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

```
[In] integrate(x**4*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.76

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \frac{1}{6}\left(3\,a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^6} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^6}\right) + \frac{12\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)}{b}\right.}{b}\right.}{b^4d}\right. - \frac{1}{3}\left(\frac{3\,a^4}{b^6x+ab^5} + \frac{12\,a^3\log(bx+a)}{b^5} - \frac{b^2x^3-3\,abx^2+9\,a^2x}{b^4}\right)\cosh(dx+c)$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^6 - e^(c - a*
d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^6) + 12*a^3*(e^(-c + a*d/b)*exp_in
tegral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d
/b)/b)/(b^4*d) - 9*a^2*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c
)/d^2)/b^4 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2
+ 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3 - ((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*
e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d
^4)/b^2 + 24*a^3*cosh(d*x + c)*log(b*x + a)/(b^5*d))*d - 1/3*(3*a^4/(b^6*x
+ a*b^5) + 12*a^3*log(b*x + a)/b^5 - (b^2*x^3 - 3*a*b*x^2 + 9*a^2*x)/b^4)*c
osh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2979 vs. 2(236) = 472.

Time = 0.33 (sec) , antiderivative size = 2979, normalized size of antiderivative = 12.90

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^4*b
*c*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^
((b*c - a*d)/b) + a^5*d^5*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*
x + a) + d)*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
```

a*d)/b)*e^(-(b*c - a*d)/b) + a^4*b*c*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^5*d^5*Ei(-((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) -
4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 4*a^3*
b^2*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
*e^((b*c - a*d)/b) - 4*a^4*b*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a
) - a*d/(b*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 4*a^3*b^2*c*d^3*Ei(-((b*x + a)*(b*c
/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 4*a^4*
b*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^
(-(b*c - a*d)/b) - a^4*b*d^4*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) - a^4*b*d^4*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b
*x + a)^3*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^3*e^((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d)/b) - 3*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x
+ a) + d)^2*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 3*(b*x
+ a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c^2*e^((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) - b^5*c^3*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d)/b) - (b*x + a)^2*a*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*e^
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*a*b^3*(b*c/
(b*x + a) - a*d/(b*x + a) + d)*c*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b) - a*b^4*c^2*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b) + (b*x + a)*a^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*e^((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^2*b^3*c*d^2*e^((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b) + 3*a^3*b^2*d^3*e^((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) - (b*x + a)^3*b^2*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 3*(b*x + a)^2
b^3(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*c*e^(-(b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b) - 3*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d
)*c^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^5*c^3*e^(-(b
x + a)(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^2*a*b^2*(b*c/(b*
x + a) - a*d/(b*x + a) + d)^2*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b) - 2*(b*x + a)*a*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*e^(-(
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^4*c^2*d*e^(-(b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a^2*b^2*(b*c/(b*x + a)
- a*d/(b*x + a) + d)*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b) + a^2*b^3*c*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3
*a^3*b^2*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x
+ a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) + 4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*e^((
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*e^((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)^2*b^3*(b*c/(b*x + a)
- a*d/(b*x + a) + d)^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b

```
) + 4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*e^(-(b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*e^(-(b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b) + 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e
^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c*e^((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a*b^4*d*e^((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*b^5*c*e^(-(b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*b^4*d*e^(-(b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^8*(b*c/(b*x + a) -
a*d/(b*x + a) + d)*d^2 - b^9*c*d^2 + a*b^8*d^3)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{(a+b\,x)^2}\,dx$$

```
[In] int((x^4*cosh(c + d*x))/(a + b*x)^2,x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x)^2, x)
```

3.27 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 182

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{\cosh(c+dx)}{b^2d^2}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} -\frac{a^3d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^5}-\frac{2a\sinh(c+dx)}{b^3d}+\frac{x\sinh(c+dx)}{b^2d} -\frac{a^3d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{3a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}$$

```
[Out] 3*a^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4-cosh(d*x+c)/b^2/d^2+a^3*cosh(d*x+c)
/b^4/(b*x+a)-a^3*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5+a^3*d*Chi(a*d/b+d*x)*s
inh(-c+a*d/b)/b^5-3*a^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4-2*a*sinh(d*x+c)/b
^3/d+x*sinh(d*x+c)/b^2/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6874, 2717, 3377, 2718, 3378, 3384, 3379, 3382}

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{a^3d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} +\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} +\frac{3a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{2a\sinh(c+dx)}{b^3d} -\frac{\cosh(c+dx)}{b^2d^2}+\frac{x\sinh(c+dx)}{b^2d}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -(Cosh[c + d*x]/(b^2*d^2)) + (a^3*Cosh[c + d*x])/(b^4*(a + b*x)) + (3*a^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d*CoshIntegral[(a*
d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 - (2*a*Sinh[c + d*x])/(b^3*d) + (x*Sinh[
c + d*x])/(b^2*d) - (a^3*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b
^5 + (3*a^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{2a\cosh(c+dx)}{b^3} + \frac{x\cosh(c+dx)}{b^2} - \frac{a^3\cosh(c+dx)}{b^3(a+bx)^2} + \frac{3a^2\cosh(c+dx)}{b^3(a+bx)}\right) dx$$

$$= -\frac{(2a)\int \cosh(c+dx)\,dx}{b^3} + \frac{(3a^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^3} - \frac{a^3\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^3} + \frac{\int x\cosh(c+dx)\,dx}{b^2}$$

$$= \frac{a^3\cosh(c+dx)}{b^4(a+bx)} - \frac{2a\sinh(c+dx)}{b^3d} + \frac{x\sinh(c+dx)}{b^2d} - \frac{\int \sinh(c+dx)\,dx}{b^2d} - \frac{(a^3d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^4} + \frac{\left(3a^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} + \frac{\left(3a^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}$$

$$= -\frac{\cosh(c+dx)}{b^2d^2} + \frac{a^3\cosh(c+dx)}{b^4(a+bx)} + \frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2a\sinh(c+dx)}{b^3d} + \frac{x\sinh(c+dx)}{b^2d} + \frac{3a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{\left(a^3d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4} - \frac{\left(a^3d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}$$

$$= -\frac{\cosh(c+dx)}{b^2d^2} + \frac{a^3\cosh(c+dx)}{b^4(a+bx)} + \frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^5} - \frac{2a\sinh(c+dx)}{b^3d} + \frac{x\sinh(c+dx)}{b^2d} - \frac{a^3d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\, dx = \frac{a^2 \mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(3b\cosh\left(c-\frac{ad}{b}\right)-ad\sinh\left(c-\frac{ad}{b}\right)\right)+\frac{b\left(\left(-ab^2+a^3d^2-b^3x\right)\cosh(c+dx)+bd\left(-2a^2-abx+b^2x^2\right)\sinh(c+}{d^2(a+bx)}}{b^5}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a^2*CoshIntegral[d*(a/b + x)]*(3*b*Cosh[c - (a*d)/b] - a*d*Sinh[c - (a*d)/
b]) + (b*((-(a*b^2) + a^3*d^2 - b^3*x)*Cosh[c + d*x] + b*d*(-2*a^2 - a*b*x
+ b^2*x^2)*Sinh[c + d*x]))/(d^2*(a + b*x)) - a^2*(a*d*Cosh[c - (a*d)/b] - 3
*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(185) = 370$.

Time = 0.20 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.02

method	result
risch	$\frac{\mathrm{e}^{-\frac{da-cb}{b}} \mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^3b\,d^3x-\mathrm{e}^{\frac{da-cb}{b}} \mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^3b\,d^3x+\mathrm{e}^{-\frac{da-cb}{b}} \mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^4d^3-3\,\mathrm{e}^{-\frac{da-cb}{b}} \mathrm{Ei}_1\left(-\right.}{}$

```
[In] int(x^3*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d^2*(exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b*d^3*x-exp((a*d-b*
c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b*d^3*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*
d-b*c)/b)*a^4*d^3-3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2*d^2*
x-exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*d^3-3*exp((a*d-b*c)/b)*Ei(1,
d*x+c+(a*d-b*c)/b)*a^2*b^2*d^2*x-3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/
b)*a^3*b*d^2-exp(-d*x-c)*b^4*d*x^2-3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/
b)*a^3*b*d^2+exp(d*x+c)*b^4*d*x^2+exp(-d*x-c)*a^3*b*d^2+exp(-d*x-c)*a*b^3*d
*x+exp(d*x+c)*a^3*b*d^2-exp(d*x+c)*a*b^3*d*x+2*exp(-d*x-c)*a^2*b^2*d-exp(-d
*x-c)*b^4*x-2*exp(d*x+c)*a^2*b^2*d-exp(d*x+c)*b^4*x-exp(-d*x-c)*a*b^3-exp(d
*x+c)*a*b^3)/b^5/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.83

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = \frac{2\left(a^3bd^2 - b^4x - ab^3\right)\cosh(dx+c) - \left((a^4d^3 - 3a^3bd^2 + (a^3bd^3 - 3a^2b^2d^2)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - (a^4d^3 + 3a^3bd^2\right.}{}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b*d^2 - b^4*x - a*b^3)*cosh(d*x + c) - ((a^4*d^3 - 3*a^3*b*d^2
+ (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^3 + 3*a^3*b*d
^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)
/b) + 2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*sinh(d*x + c) + ((a^4*d^3 - 3
*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^3
+ 3*a^3*b*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-
(b*c - a*d)/b))/(b^6*d^2*x + a*b^5*d^2)
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.71

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{1}{4}\left(2a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^5} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^5}\right) + \frac{6a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)}{b}\right.}{b}\right.}{b^3d}\right.$$
$$+\frac{1}{2}\left(\frac{2a^3}{b^5x+ab^4} + \frac{6a^2\log(bx+a)}{b^4} + \frac{bx^2-4ax}{b^3}\right)\cosh(dx+c)$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^5 - e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^5) + 6*a^2*(e^(-c + a*d/b)*exp_in
tegral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d
/b)/b)/(b^3*d) - 4*a*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/
d^2)/b^3 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*
x + 2)*e^(-d*x - c)/d^3)/b^2 + 12*a^2*cosh(d*x + c)*log(b*x + a)/(b^4*d))*d
 + 1/2*(2*a^3/(b^5*x + a*b^4) + 6*a^2*log(b*x + a)/b^4 + (b*x^2 - 4*a*x)/b^
3)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(185) = 370$.

Time = 0.33 (sec) , antiderivative size = 1991, normalized size of antiderivative = 10.94

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^3*
b*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e
^((b*c - a*d)/b) + a^4*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*e^(-(b*c - a*d)/b) + a^3*b*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^4*d^4*Ei(-((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) -
 3*(b*x + a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 3*a^2
*b^2*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
)*e^((b*c - a*d)/b) - 3*a^3*b*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 3*(b*x + a)*a^2*b*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
 d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 3*a^2*b^2*c*d^2*Ei(-((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 3*a^3
*b*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e
^(-(b*c - a*d)/b) - a^3*b*d^3*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)/b) - a^3*b*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (
b*x + a)^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^((b*x + a)*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - b^4*c^2*e^
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*a*b^2*(b*c/(b
```

```
*x + a) - a*d/(b*x + a) + d)*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)/b) - a*b^3*c*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2
*a^2*b^2*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a
)^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^(-(b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) +
d)*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^4*c^2*e^(-(b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a*b^2*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
)/b) + a*b^3*c*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a
^2*b^2*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)
*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e^((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d)/b) - b^4*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b) + a*b^3*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a
)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b) - b^4*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) + a*b^3*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(
((b*x + a)*b^7*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d - b^8*c*d + a*b^7*d^2)
*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{{(a+b\,x)}^2}\,dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x)^2,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x)^2, x)
```

3.28 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 147

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} dx = -\frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2 d \mathrm{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sinh(c+dx)}{b^2 d} + \frac{a^2 d \cosh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2a \sinh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}$$

```
[Out] -2*a*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-a^2*cosh(d*x+c)/b^3/(b*x+a)+a^2*d*co
sh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4-a^2*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+2*a
*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3+sinh(d*x+c)/b^2/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 2717, 3378, 3384, 3379, 3382}

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} dx = \frac{a^2 d \sinh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{a^2 d \cosh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{2a \sinh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{\sinh(c+dx)}{b^2 d}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/(b^3*(a + b*x))) - (2*a*Cosh[c - (a*d)/b]*CoshIntegra
l[(a*d)/b + d*x])/b^3 + (a^2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b
])/b^4 + Sinh[c + d*x]/(b^2*d) + (a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d
)/b + d*x])/b^4 - (2*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\cosh(c+dx)}{b^2}+\frac{a^2\cosh(c+dx)}{b^2(a+bx)^2}-\frac{2a\cosh(c+dx)}{b^2(a+bx)}\right)dx\\ &= \frac{\int \cosh(c+dx)\,dx}{b^2}-\frac{(2a)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^2}+\frac{a^2\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^2}\end{aligned}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)} + \frac{\sinh(c+dx)}{b^2 d} + \frac{(a^2 d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^3}$$
$$- \frac{\left(2a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} - \frac{\left(2a\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$
$$= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)} - \frac{2a\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}$$
$$+ \frac{\sinh(c+dx)}{b^2 d} - \frac{2a\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}$$
$$+ \frac{\left(a^2 d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} + \frac{\left(a^2 d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3}$$
$$= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)} - \frac{2a\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2 d\,\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^4}$$
$$+ \frac{\sinh(c+dx)}{b^2 d} + \frac{a^2 d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{2a\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{a\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(-2b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)+b\left(-\frac{a^2\cosh(c+dx)}{a+bx}+\frac{b\sinh(c+dx)}{d}\right)+a\left(ad\cosh\left(c-\right.\right.}{b^4}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CoshIntegral[d*(a/b + x)]*(-2*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b
]) + b*(-((a^2*Cosh[c + d*x])/(a + b*x)) + (b*Sinh[c + d*x])/d) + a*(a*d*Co
sh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(152) = 304$.

Time = 0.24 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.97

method	result
risch	$-\frac{e^{-\frac{da-cb}{b}}\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^2 b d^2 x - e^{\frac{da-cb}{b}}\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^2 b d^2 x + e^{-\frac{da-cb}{b}}\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)a^3 d^2 - 2e^{-\frac{da-cb}{b}}\mathrm{Ei}_1}{}$

```
[In] int(x^2*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

[Out] -1/2/d*(exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b*d^2*x-exp((a*d-b*c
)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b*d^2*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d
-b*c)/b)*a^3*d^2-2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^2*d*x-exp
((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*d^2-2*exp((a*d-b*c)/b)*Ei(1,d*x+c
+(a*d-b*c)/b)*a*b^2*d*x-2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b*
d-2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b*d+exp(-d*x-c)*a^2*b*d+ex
p(-d*x-c)*b^3*x+exp(d*x+c)*a^2*b*d-exp(d*x+c)*b^3*x+exp(-d*x-c)*a*b^2-exp(d
*x+c)*a*b^2)/b^4/(b*x+a)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.86

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{2\,a^2bd\cosh\left(dx+c\right) - \left(\left(a^3d^2 - 2\,a^2bd + \left(a^2bd^2 - 2\,ab^2d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - \left(a^3d^2 + 2\,a^2bd + \left(a^2bd^2 + 2\,ab^2\right.\right.\right.}{}$$

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(2*a^2*b*d*cosh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^
2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d
)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*x + a*b^2)*sinh(d*
x + c) + ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d
)/b) + (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/
b))*sinh(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^2 \cosh\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**2,x)

[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.61

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{1}{2}\left(a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^4} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^4}\right) + \frac{2a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d}\right.$$
$$-\left(\frac{a^2}{b^4x+ab^3} - \frac{x}{b^2} + \frac{2a\log(bx+a)}{b^3}\right)\cosh(dx+c)$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^4 - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/b^4) + 2*a*(e^(-c + a*d/b)*exp_integra
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(b^2*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2
+ 4*a*cosh(d*x + c)*log(b*x + a)/(b^3*d))*d - (a^2/(b^4*x + a*b^3) - x/b^2
+ 2*a*log(b*x + a)/b^3)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. 2(152) = 304.

Time = 0.30 (sec) , antiderivative size = 1308, normalized size of antiderivative = 8.90

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^2*b
*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^
((b*c - a*d)/b) + a^3*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*
x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
a*d)/b)*e^(-(b*c - a*d)/b) + a^2*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^3*d^3*Ei(-((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) -
2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b
```

```
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 2*a*b^2*c*
d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c
 - a*d)/b) - 2*a^2*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
- b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a
*d)/b)*e^(-(b*c - a*d)/b) + 2*a*b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 2*a^2*b*d^2*Ei(-((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b)
 - a^2*b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^2*b*d^
2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*b^2*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)/b) - b^3*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^
2*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*b^2*(b*
c/(b*x + a) - a*d/(b*x + a) + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b) + b^3*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a
*b^2*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a
)*b^6*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^7*c + a*b^6*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} \, dx = \int \frac{x^2 \cosh(c+d\,x)}{(a+b\,x)^2} \, dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x)^2,x)
[Out] int((x^2*cosh(c + d*x))/(a + b*x)^2, x)
```

3.29 $\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{a\cosh(c+dx)}{b^2(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{ad\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} - \frac{ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^2+a*cosh(d*x+c)/b^2/(b*x+a)-a*d*cosh(-c+a*d
/b)*Shi(a*d/b+d*x)/b^3+a*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-Shi(a*d/b+d*x)
*sinh(-c+a*d/b)/b^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{ad\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{a\cosh(c+dx)}{b^2(a+bx)}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*Cosh[c + d*x])/(b^2*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b
 + d*x])/b^2 - (a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 - (a
*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (Sinh[c - (a*d)/b]*
SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{a\cosh(c+dx)}{b(a+bx)^2}+\frac{\cosh(c+dx)}{b(a+bx)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b} \\
&= \frac{a\cosh(c+dx)}{b^2(a+bx)}-\frac{(ad)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^2} \\
&\quad +\frac{\cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}
\end{aligned}
$$

$$= \frac{a\cosh(c+dx)}{b^2(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{\left(ad\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{b^2} - \frac{\left(ad\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}dx}{b^2}$$

$$= \frac{a\cosh(c+dx)}{b^2(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{ad\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} - \frac{ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{\frac{ab\cosh(c+dx)}{a+bx} + \mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right) - ad\sinh\left(c-\frac{ad}{b}\right)\right) + \left(-ad\cosh\left(c-\frac{ad}{b}\right) + b\sinh\left(c-\frac{ad}{b}\right)\right)}{b^3}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] ((a*b*Cosh[c + d*x])/(a + b*x) + CoshIntegral[d*(a/b + x)]*(b*Cosh[c - (a*d
)/b] - a*d*Sinh[c - (a*d)/b]) + (-(a*d*Cosh[c - (a*d)/b]) + b*Sinh[c - (a*d
)/b])*SinhIntegral[d*(a/b + x)])/b^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(129) = 258.

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.35

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}a}{2b^2(dxb+da)} - \frac{d\,\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a}{2b^3} - \frac{\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2b^2} + \mathrm{e}^{-\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)abdx+\mathrm{e}^{-\frac{da-cb}{b}}$

```
[In] int(x*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*exp(-d*x-c)/b^2/(b*d*x+a*d)*a-1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(
a*d-b*c)/b)*a-1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*(exp(-(a
*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b*d*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(
a*d-b*c)/b)*a^2*d-exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*b^2*x-exp(-(a*
d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b+exp(d*x+c)*a*b)/b^3/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.60

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{2\,ab\cosh\left(dx+c\right) - \left(\left(a^2d-ab+\left(abd-b^2\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - \left(a^2d+ab+\left(abd+b^2\right)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\right.}{2\left(b^4x+\right.}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cosh(d*x + c) - ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d
)/b) - (a^2*d + a*b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a
*d)/b) + ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d + a*
b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*x + a
*b^3)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\cosh\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{1}{2}\left(a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^3} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^3}\right) + \frac{\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}}{bd} + \frac{2\,c\right. + \left(\frac{a}{b^3x+ab^2} + \frac{\log\left(bx+a\right)}{b^2}\right)\cosh\left(dx+c\right)$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^3 - e^(c - a*d/b
)*exp_integral_e(1, -(b*x + a)*d/b)/b^3) + (e^(-c + a*d/b)*exp_integral_e(1
, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*
d) + 2*cosh(d*x + c)*log(b*x + a)/(b^2*d))*d + (a/(b^3*x + a*b^2) + log(b*x
 + a)/b^2)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(129) = 258$.

Time = 0.31 (sec) , antiderivative size = 994, normalized size of antiderivative = 7.95

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{\left((bx+a)a\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\text{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-abcd^2\text{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}{b}\right.\right.}{}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c*
d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b
*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a
) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/
b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - (b*x +
a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + b^2*c*d*Ei(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*
b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(
(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/
b) + b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d
)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*e^((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d)/b) - a*b*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b))*b/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)
- b^5*c + a*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \cosh(c + dx)}{(a + bx)^2} \, dx = \int \frac{x \cosh(c + d\,x)}{\left(a + b\,x\right)^2} \, dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x)^2,x)
[Out] int((x*cosh(c + d*x))/(a + b*x)^2, x)
```

3.30 $\int \frac{\cosh(c+dx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{\cosh(c+dx)}{(a+bx)^2} dx = -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^2} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] -cosh(d*x+c)/b/(b*x+a)+d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^2-d*Chi(a*d/b+d*x)
*sinh(-c+a*d/b)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{(a+bx)^2} dx = \frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{\cosh(c+dx)}{b(a+bx)}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Cosh[c + d*x]/(b*(a + b*x))) + (d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a
*d)/b])/b^2 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b}\\
&= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} + \frac{\left(d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}\\
&= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d\,\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^2} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$
\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{-\frac{b\cosh(c+dx)}{a+bx} + d\,\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\sinh\left(c-\frac{ad}{b}\right) + d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2}
$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (-((b*Cosh[c + d*x])/(a + b*x)) + d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d
)/b] + d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b^2
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

method	result	size
risch	$-\frac{d\,\mathrm{e}^{-dx-c}}{2b(dxb+da)}+\frac{d\,\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2b^2}-\frac{d\,\mathrm{e}^{dx+c}}{2b^2\left(\frac{da}{b}+dx\right)}-\frac{d\,\mathrm{e}^{-\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)}{2b^2}$	132

```
[In] int(cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*exp(-d*x-c)/b/(b*d*x+a*d)+1/2*d/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d
-b*c)/b)-1/2*d/b^2*exp(d*x+c)/(d/b*a+d*x)-1/2*d/b^2*exp(-(a*d-b*c)/b)*Ei(1,
-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(74) = 148.

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.10

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{2\,b\cosh\left(dx+c\right)-\left((bdx+ad)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-(bdx+ad)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+\left((bdx+ad)\mathrm{Ei}\left(\frac{b}{}\right.\right.}{2\left(b^3x+ab^2\right)}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cosh(d*x + c) - ((b*d*x + a*d)*Ei((b*d*x + a*d)/b) - (b*d*x + a*d
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b*d*x + a*d)*Ei((b*d*x + a
*d)/b) + (b*d*x + a*d)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^3*x +
 a*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{2\,b} - \frac{\cosh\left(dx+c\right)}{(bx+a)b}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - e^(c - a*d/b)*ex
p_integral_e(1, -(b*x + a)*d/b)/b)/b - cosh(d*x + c)/((b*x + a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(74) = 148.

Time = 0.31 (sec) , antiderivative size = 615, normalized size of antiderivative = 8.66

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{\left((bx+a)\left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right)d^2\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)} - bcd^2\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right.\right.}{2\left((bx+a)b^4\left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right) - \right.}$$
$$- \frac{\left((bx+a)\left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right)d^2\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)} - bcd^2\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right.}{b}\right.\right.}{2\left((bx+a)b^4\left(\frac{bc}{bx+a} - \frac{ad}{bx+a}\right.\right.}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - b*c*d^2*E
i(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c -
a*d)/b) + a*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a
*d)/b)*e^((b*c - a*d)/b) - b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^5*c
+ a*b^4*d)*d) - 1/2*((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(-
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a
*d)/b) - b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*e^(-(b*c - a*d)/b) + a*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*
x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + b*d^2*e^(-(b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b^5*c + a*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{\cosh(c+d\,x)}{\left(a+b\,x\right)^2}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*x)^2,x)
[Out] int(cosh(c + d*x)/(a + b*x)^2, x)
```

3.31 $\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx = \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{ab} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{ab} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2}$$

```
[Out] Chi(d*x)*cosh(c)/a^2-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^2+cosh(d*x+c)/a/(b*x+a
)-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a/b+Shi(d*x)*sinh(c)/a^2+d*Chi(a*d/b+d*x)
*sinh(-c+a*d/b)/a/b+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3384, 3379, 3382, 3378}

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx = -\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{ab} - \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{ab} + \frac{\cosh(c+dx)}{a(a+bx)}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] Cosh[c + d*x]/(a*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 - (d*CoshIntegral[(a*d)/b + d*x]*S
inh[c - (a*d)/b])/(a*b) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c - (a*
d)/b]*SinhIntegral[(a*d)/b + d*x])/(a*b) - (Sinh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a(a+bx)^2} - \frac{b\cosh(c+dx)}{a^2(a+bx)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a}$$

$$
\begin{aligned}
&= \frac{\cosh(c+dx)}{a(a+bx)} - \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^2} \\
&\quad - \frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&\quad + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a^2} - \frac{\left(b\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&= \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} \\
&\quad + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2} \\
&\quad - \frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} - \frac{\left(d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} \\
&= \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} \\
&\quad - \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{ab} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} \\
&\quad - \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{ab} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$
\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx
$$
$$
= \frac{\frac{a\cosh(c)\cosh(dx)}{a+bx} + \cosh(c)\text{Chi}(dx) - \frac{\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)}{b} + \frac{a\sinh(c)\sinh(dx)}{a+bx} + \sinh(c)\text{Shi}(dx)
$$
$$
a^2
$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] ((a*Cosh[c]*Cosh[d*x])/(a + b*x) + Cosh[c]*CoshIntegral[d*x] - (CoshIntegra
l[d*(a/b + x)]*(b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b]))/b + (a*Sinh[c
]*Sinh[d*x])/(a + b*x) + Sinh[c]*SinhIntegral[d*x] - (a*d*Cosh[c - (a*d)/b]
*SinhIntegral[d*(a/b + x)])/b - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]
)/a^2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.69

method	result
risch	$\frac{\mathrm{e}^{-dx-c} d}{2a((dx+c)b+da-cb)} - \frac{\mathrm{e}^{-c} \mathrm{Ei}_1(dx)}{2a^2} - \frac{\mathrm{e}^{\frac{da-cb}{b}} \mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right) d}{2ba} + \frac{\mathrm{e}^{\frac{da-cb}{b}} \mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2a^2} - \frac{\mathrm{e}^{c} \mathrm{Ei}_1(-dx)}{2a^2} + \frac{d\,\mathrm{e}^{dx}}{2ab\left(\frac{da}{b}\right.}$

[In] int(cosh(d*x+c)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*exp(-d*x-c)*d/a/((d*x+c)*b+d*a-c*b)-1/2/a^2*exp(-c)*Ei(1,d*x)-1/2/b/a*e
xp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a^2*exp((a*d-b*c)/b)*Ei(1,d*x
+c+(a*d-b*c)/b)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/2/a*d/b*exp(d*x+c)/(d/b*a+d*x)+
1/2/a*d/b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2/a^2*exp(-(a*d-b*c)
/b)*Ei(1,-d*x-c-(a*d-b*c)/b)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.80

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \frac{2\,ab\cosh\left(dx+c\right) + \left(\left(b^2x+ab\right)\mathrm{Ei}(dx) + \left(b^2x+ab\right)\mathrm{Ei}(-dx)\right)\cosh\left(c\right) - \left(\left(a^2d+ab+\left(abd+b^2\right)x\right)\mathrm{Ei}\left(\frac{bdx+}{b}\right.\right.}{}$$

[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(2*a*b*cosh(d*x + c) + ((b^2*x + a*b)*Ei(d*x) + (b^2*x + a*b)*Ei(-d*x))
*cosh(c) - ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) - (a^2*d -
a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b^2*x
 + a*b)*Ei(d*x) - (b^2*x + a*b)*Ei(-d*x))*sinh(c) + ((a^2*d + a*b + (a*b*d
+ b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x
+ a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{x\left(a+bx\right)^2}\,dx$$

[In] integrate(cosh(d*x+c)/x/(b*x+a)**2,x)

[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.51

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = -\frac{1}{2}\,d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{ab} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{ab} - \frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d} - \frac{2}{\ldots}\right.$$
$$+\left(\frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}\right)\cosh(dx+c)$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/(a*b) - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/(a*b) - b*(e^(-c + a*d/b)*exp_integral
_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)
/(a^2*d) - 2*cosh(d*x + c)*log(b*x + a)/(a^2*d) + 2*cosh(d*x + c)*log(x)/(a
^2*d) - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a^2*d)) + (1/(a*b*x + a^2) - log(b
*x + a)/a^2 + log(x)/a^2)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. 2(152) = 304.

Time = 0.31 (sec) , antiderivative size = 1329, normalized size of antiderivative = 8.86

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c*
d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b
*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a
) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/
b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - (b*x +
a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-(b*x + a)*(b*c/(b*x + a) - a
```

```
*d/(b*x + a) + d)/b + c)*e^(-c) + b^2*c*d*Ei(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b + c)*e^(-c) - a*b*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d)/b + c)*e^(-c) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c + b^2*c*
d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - a*b*d^2*Ei(
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c + (b*x + a)*b*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - b^2*c*d*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + a*b*d^2*Ei(
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*
d)/b) + (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - b^2*
c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-
(b*c - a*d)/b) + a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*e^((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d)/b) - a*b*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b))*b^3/(((b*x + a)*a^2*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - a
^2*b^5*c + a^3*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\cosh(c+d\,x)}{x\,{(a+b\,x)}^2}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x)^2),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x)^2), x)
```

3.32 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx = -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\text{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d\text{Chi}(dx)\sinh(c)}{a^2} + \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2} + \frac{d\cosh(c)\text{Shi}(dx)}{a^2} - \frac{2b\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{2b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}$$

```
[Out] -2*b*Chi(d*x)*cosh(c)/a^3+2*b*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-cosh(d*x+c)
/a^2/x-b*cosh(d*x+c)/a^2/(b*x+a)+d*cosh(c)*Shi(d*x)/a^2+d*cosh(-c+a*d/b)*Sh
i(a*d/b+d*x)/a^2+d*Chi(d*x)*sinh(c)/a^2-2*b*Shi(d*x)*sinh(c)/a^3-d*Chi(a*d/
b+d*x)*sinh(-c+a*d/b)/a^2-2*b*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used

= {6874, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = -\frac{2b\cosh(c)\mathrm{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{2b\sinh(c)\mathrm{Shi}(dx)}{a^3} + \frac{2b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^2} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^2} - \frac{b\cosh(c+dx)}{a^2(a+bx)} + \frac{d\sinh(c)\mathrm{Chi}(dx)}{a^2} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2} - \frac{\cosh(c+dx)}{a^2x}$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] -(Cosh[c + d*x]/(a^2*x)) - (b*Cosh[c + d*x])/(a^2*(a + b*x)) - (2*b*Cosh[c]
*CoshIntegral[d*x])/a^3 + (2*b*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x
])/a^3 + (d*CoshIntegral[d*x]*Sinh[c])/a^2 + (d*CoshIntegral[(a*d)/b + d*x]
*Sinh[c - (a*d)/b])/a^2 + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (2*b*Sinh[c]*
SinhIntegral[d*x])/a^3 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/
a^2 + (2*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{2b\cosh(c+dx)}{a^3x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^2} + \frac{2b^2\cosh(c+dx)}{a^3(a+bx)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} - \frac{(2b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} + \frac{(2b^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^2} + \frac{(bd)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^2} \\
&\quad - \frac{(2b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^3} + \frac{\left(2b^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&\quad - \frac{(2b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^3} + \frac{\left(2b^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\mathrm{Chi}(dx)}{a^3} \\
&\quad + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{2b\sinh(c)\mathrm{Shi}(dx)}{a^3} \\
&\quad + \frac{2b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^2} \\
&\quad + \frac{\left(bd\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&\quad + \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^2} + \frac{\left(bd\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\mathrm{Chi}(dx)}{a^3} \\
&\quad + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d\mathrm{Chi}(dx)\sinh(c)}{a^2} \\
&\quad + \frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2} - \frac{2b\sinh(c)\mathrm{Shi}(dx)}{a^3} \\
&\quad + \frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{2b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \frac{-\frac{a(a+2bx)\cosh(c)\cosh(dx)}{x(a+bx)} - 2b\cosh(c)\text{Chi}(dx) + 2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + ad\text{Chi}(dx)\sinh(c) + ad\text{C}}{}$$

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]

[Out] (-((a*(a + 2*b*x)*Cosh[c]*Cosh[d*x])/(x*(a + b*x))) - 2*b*Cosh[c]*CoshInteg
ral[d*x] + 2*b*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + a*d*CoshIntegr
al[d*x]*Sinh[c] + a*d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d)/b] - (a*(a +
 2*b*x)*Sinh[c]*Sinh[d*x])/(x*(a + b*x)) + a*d*Cosh[c]*SinhIntegral[d*x] -
2*b*Sinh[c]*SinhIntegral[d*x] + a*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b +
 x)] + 2*b*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a^3

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.68

method	result
risch	$-\frac{d\,\mathrm{e}^{-dx-c}b}{a^2(dxb+da)} - \frac{d\,\mathrm{e}^{-dx-c}}{2ax(dxb+da)} + \frac{d\,\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)}{2a^2} + \frac{\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)b}{a^3} + \frac{d\,\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2a^2} - \frac{\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-c}{b}\right.}{a^3}$

[In] int(cosh(d*x+c)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -d*exp(-d*x-c)/a^2/(b*d*x+a*d)*b-1/2*d*exp(-d*x-c)/a/x/(b*d*x+a*d)+1/2*d/a^
2*exp(-c)*Ei(1,d*x)+1/a^3*exp(-c)*Ei(1,d*x)*b+1/2*d/a^2*exp((a*d-b*c)/b)*Ei
(1,d*x+c+(a*d-b*c)/b)-1/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b-1/2/
a^2/x*exp(d*x+c)-1/2*d/a^2*exp(c)*Ei(1,-d*x)+1/a^3*b*exp(c)*Ei(1,-d*x)-1/2*
d/a^2*exp(d*x+c)/(d/b*a+d*x)-1/2*d/a^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b
*c)/b)-b/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.03

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = -\frac{2\,(2\,abx+a^2)\cosh\left(dx+c\right)-(((abd-2\,b^2)x^2+(a^2d-2\,ab)x)\mathrm{Ei}(dx)-((abd+2\,b^2)x^2+(a^2d+2\,ab)x}{}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*a*b*x + a^2)*cosh(d*x + c) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*
a*b)*x)*Ei(d*x) - ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*cosh(
c) - (((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) - ((a*b
*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(d*x) + ((a*b*d + 2*b^2
)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*sinh(c) + (((a*b*d + 2*b^2)*x^2 + (a^2
*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) + ((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)
*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b*x^2 + a^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx+a\right)^2x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3353 vs. 2(191) = 382.

Time = 0.33 (sec) , antiderivative size = 3353, normalized size of antiderivative = 18.03

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(-(b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)/b - 2*(b*x + a)*a*(b*c/(
b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d)/b + c)*e^(-c) + a*b*c^2*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d)/b + c)*e^(-c) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)*d^3*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)/b
- a^2*c*d^3*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)
 - (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c/b + 2*(b*x + a)*a*(b*c/(b*x + a
) - a*d/(b*x + a) + d)*c*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b - c)*e^c - a*b*c^2*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
)/b - c)*e^c - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c/b + a^2*c*d^3*Ei((b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - (b*x + a)^2*a*(b*c/(
b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)/b + 2*(b*x + a)*a*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c^2*d^2*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a
^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)/b + a^2*c*d^3*Ei(((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)
+ (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(-((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b)/b - 2*
(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei(-((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a*b*c^2*
d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-
(b*c - a*d)/b) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(-
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a
*d)/b)/b - a^2*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b
*c + a*d)/b)*e^(-(b*c - a*d)/b) + 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)^2*d*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c
) - 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei(-(b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) + 2*b^2*c^2*d*Ei(-(b*x + a)*
(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) + 2*(b*x + a)*a*(b*c/(b*x
```

```
 + a) - a*d/(b*x + a) + d)*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)/b + c)*e^(-c) - 2*a*b*c*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)/b + c)*e^(-c) + 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^
2*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - 4*(b*x +
a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei((b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b - c)*e^c + 2*b^2*c^2*d*Ei((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d)/b - c)*e^c + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - 2*a
*b*c*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - 2*(b
*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*Ei(((b*x + a)*(b*c/(b*x +
 a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 4*(b*x + a)*b*
(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*b^2*c^2*d*Ei(((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2
*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 2*a*b*c*d^2
*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c
- a*d)/b) - 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*Ei(-((b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b
) + 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei(-((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 2*b^
2*c^2*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*
e^(-(b*c - a*d)/b) - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*
Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c
 - a*d)/b) + 2*a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*
b*c*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a^2*d^3*e^((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*a*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) - 2*a*b*c*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) +
a^2*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x +
a)^2*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2 - 2*(b*x + a)*a^3*b^2*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*c + a^3*b^3*c^2 + (b*x + a)*a^4*b*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)*d - a^4*b^2*c*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,{(a+b\,x)}^2}\,dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)^2),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)^2), x)
```

3.33 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 264

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx = \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3d^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^6} + \frac{3a^2d\text{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^5} + \frac{\sinh(c+dx)}{b^3d} + \frac{a^3d \sinh(c+dx)}{2b^5(a+bx)} + \frac{3a^2d \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3d^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{2b^6}$$

```
[Out] -3*a*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4-1/2*a^3*d^2*Chi(a*d/b+d*x)*cosh(-c+a
*d/b)/b^6+1/2*a^3*cosh(d*x+c)/b^4/(b*x+a)^2-3*a^2*cosh(d*x+c)/b^4/(b*x+a)+3
*a^2*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5-3*a^2*d*Chi(a*d/b+d*x)*sinh(-c+a*d
/b)/b^5+3*a*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+1/2*a^3*d^2*Shi(a*d/b+d*x)*si
nh(-c+a*d/b)/b^6+sinh(d*x+c)/b^3/d+1/2*a^3*d*sinh(d*x+c)/b^5/(b*x+a)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used

$= \{6874, 2717, 3378, 3384, 3379, 3382\}$

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{a^3d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{2b^6} - \frac{a^3d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{2b^6} + \frac{a^3d\sinh(c+dx)}{2b^5(a+bx)} + \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} + \frac{3a^2d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{3a^2d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{3a\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{\sinh(c+dx)}{b^3d}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a^3*Cosh[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Cosh[c + d*x])/(b^4*(a + b
*x)) - (3*a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^6) + (3*a^2*d*CoshIntegr
al[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 + Sinh[c + d*x]/(b^3*d) + (a^3*d*S
inh[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(
a*d)/b + d*x])/b^5 - (3*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^
4 - (a^3*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^6)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{b^3} - \frac{a^3\cosh(c+dx)}{b^3(a+bx)^3} + \frac{3a^2\cosh(c+dx)}{b^3(a+bx)^2} - \frac{3a\cosh(c+dx)}{b^3(a+bx)}\right) dx \\
&= \frac{\int \cosh(c+dx)\,dx}{b^3} - \frac{(3a)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^3} + \frac{(3a^2)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^3} - \frac{a^3\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b^3} \\
&= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} + \frac{\sinh(c+dx)}{b^3 d} \\
&\quad + \frac{(3a^2d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^4} - \frac{(a^3d)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^4} \\
&\quad - \frac{\left(3a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} - \frac{\left(3a\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} \\
&= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\sinh(c+dx)}{b^3 d} \\
&\quad + \frac{a^3 d\sinh(c+dx)}{2b^5(a+bx)} - \frac{3a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{(a^3d^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^5} \\
&\quad + \frac{\left(3a^2d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4} + \frac{\left(3a^2d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^4}
\end{aligned}$$

$$= \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{3a^2 d\text{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^5} + \frac{\sinh(c+dx)}{b^3 d} + \frac{a^3 d \sinh(c+dx)}{2b^5(a+bx)} + \frac{3a^2 d \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{\left(a^3 d^2 \cosh\left(c-\frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{2b^5} - \frac{\left(a^3 d^2 \sinh\left(c-\frac{ad}{b}\right)\right) \int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{2b^5}$$

$$= \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3 d^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^6} + \frac{3a^2 d\text{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^5} + \frac{\sinh(c+dx)}{b^3 d} + \frac{a^3 d \sinh(c+dx)}{2b^5(a+bx)} + \frac{3a^2 d \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3 d^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{2b^6}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx = -\frac{b \cosh(dx) \left(a^2 bd(5a+6bx) \cosh(c) - (a+bx) \left(2ab^2 + a^3 d^2 + 2b^3 x\right) \sinh(c)\right) - b\left((a+bx) \left(2ab^2 + a^3 d^2 + \right.\right.}{}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*Cosh[d*x]*(a^2*b*d*(5*a + 6*b*x)*Cosh[c] - (a + b*x)*(2*a*b^2 + a^3
*d^2 + 2*b^3*x)*Sinh[c]) - b*((a + b*x)*(2*a*b^2 + a^3*d^2 + 2*b^3*x)*Cosh[
c] - a^2*b*d*(5*a + 6*b*x)*Sinh[c])*Sinh[d*x] + a*d*(a + b*x)^2*(CoshIntegr
al[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 6*a*b*d*Sinh[c - (a*
d)/b]) + (-6*a*b*d*Cosh[c - (a*d)/b] + (6*b^2 + a^2*d^2)*Sinh[c - (a*d)/b])
*SinhIntegral[d*(a/b + x)]))/(b^6*d*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. 2(262) = 524.

Time = 0.36 (sec) , antiderivative size = 1055, normalized size of antiderivative = 4.00

method	result	size
risch	Expression too large to display	1055

```
[In] int(x^3*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*exp(d*x+c)*b^5*x^2+2*exp(d*x+c)*a^2*b^3+exp(d*x+c)*a^3*b^2*d^2*x-6*e
xp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4*b*d^2-6*exp(d*x+c)*a^2*b^3*d*
x+6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b^2*d-2*exp(-d*x-c)*a^2*
b^3-2*exp(-d*x-c)*b^5*x^2+exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^5*d^3-
exp(-d*x-c)*a^4*b*d^2-5*exp(-d*x-c)*a^3*b^2*d-4*exp(-d*x-c)*a*b^4*x+6*exp((
a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a*b^4*d*x^2+12*exp((a*d-b*c)/b)*Ei(1,d*
x+c+(a*d-b*c)/b)*a^2*b^3*d*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3
*b^2*d^3*x^2+2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4*b*d^3*x-6*exp
(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^3*d^2*x^2-12*exp(-(a*d-b*c)/b
)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b^2*d^2*x+6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a
*d-b*c)/b)*a*b^4*d*x^2+12*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^
3*d*x-exp(-d*x-c)*a^3*b^2*d^2*x+6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*
a^4*b*d^2-6*exp(-d*x-c)*a^2*b^3*d*x+6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)
/b)*a^3*b^2*d+exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b^2*d^3*x^2+2*ex
p((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*b*d^3*x+6*exp((a*d-b*c)/b)*Ei(1,
d*x+c+(a*d-b*c)/b)*a^2*b^3*d^2*x^2+12*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)
/b)*a^3*b^2*d^2*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^5*d^3+exp(d*
x+c)*a^4*b*d^2-5*exp(d*x+c)*a^3*b^2*d+4*exp(d*x+c)*a*b^4*x)/d/b^6/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(262) = 524.

Time = 0.26 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.14

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx =$$
$$-\frac{2\left(6\,a^2b^3dx+5\,a^3b^2d\right)\cosh\left(dx+c\right)+\left(\left(a^5d^3-6\,a^4bd^2+6\,a^3b^2d+\left(a^3b^2d^3-6\,a^2b^3d^2+6\,ab^4d\right)x^2+2\,(\right.\right.}{}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*cosh(d*x + c) + ((a^5*d^3 - 6*a^4*b*d
^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b
*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^3 + 6*a
```

```
^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*
(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b
*c - a*d)/b) - 2*(a^4*b*d^2 + 2*b^5*x^2 + 2*a^2*b^3 + (a^3*b^2*d^2 + 4*a*b^
4)*x)*sinh(d*x + c) - ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3
- 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3
*d)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^
2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a
^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b
^7*d*x + a^2*b^6*d)
```

Sympy [F]

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx=\int \frac{x^3\cosh{(c+dx)}}{(a+bx)^3}\,dx$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx=\int \frac{x^3\cosh{(dx+c)}}{(bx+a)^3}\,dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*d*integrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^
3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3/2*a^2*d*integrate(x/(b^5*d
^2*x^4*e^(d*x + c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x
 + c) + 4*a^3*b^2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) - 3*a*b*in
tegrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 +
4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3*a*b*integrate(x/(b^5*d^2*x^4*e^(d*x +
c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x + c) + 4*a^3*b^
2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) + 1/2*((b*d*x^3*e^(2*c) -
3*a*x*e^(2*c))*e^(d*x) - (b*d*x^3 + 3*a*x)*e^(-d*x))/(b^4*d^2*x^3*e^c + 3*a
*b^3*d^2*x^2*e^c + 3*a^2*b^2*d^2*x*e^c + a^3*b*d^2*e^c) - 3/2*a^2*e^(-c + a
*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b^2*d^2) - 3/2*a^2*e^(c
 - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(262) = 524.

Time = 0.27 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.33

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} \, dx =$$
$$-\frac{a^3 b^2 d^3 x^2 \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^3 b^2 d^3 x^2 \text{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + 2\,a^4 b d^3 x \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} - 6\,a^2 b^3 d^2 x^2}{}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*b^2*d^3*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^3*b^2*d^3*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^4*b*d^3*x*Ei((b*d*x + a*d)/b)*e^(c
 - a*d/b) - 6*a^2*b^3*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^4*b*d
^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a^2*b^3*d^2*x^2*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) + a^5*d^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^3*
b^2*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a*b^4*d*x^2*Ei((b*d*x + a*d
)/b)*e^(c - a*d/b) + a^5*d^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^3*b
^2*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a*b^4*d*x^2*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) - a^3*b^2*d^2*x*e^(d*x + c) + a^3*b^2*d^2*x*e^(-d*x -
 c) - 6*a^4*b*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a^2*b^3*d*x*Ei((b*
d*x + a*d)/b)*e^(c - a*d/b) + 6*a^4*b*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/
b) + 12*a^2*b^3*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^4*b*d^2*e^(d*x
+ c) + 6*a^2*b^3*d*x*e^(d*x + c) - 2*b^5*x^2*e^(d*x + c) + a^4*b*d^2*e^(-d*
x - c) + 6*a^2*b^3*d*x*e^(-d*x - c) + 2*b^5*x^2*e^(-d*x - c) + 6*a^3*b^2*d*
Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a^3*b^2*d*Ei(-(b*d*x + a*d)/b)*e^(-c
+ a*d/b) + 5*a^3*b^2*d*e^(d*x + c) - 4*a*b^4*x*e^(d*x + c) + 5*a^3*b^2*d*e^
(-d*x - c) + 4*a*b^4*x*e^(-d*x - c) - 2*a^2*b^3*e^(d*x + c) + 2*a^2*b^3*e^(
-d*x - c))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} \, dx = \int \frac{x^3 \cosh(c + d\,x)}{(a + b\,x)^3} \, dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x^3*cosh(c + d*x))/(a + b*x)^3, x)
```

3.34 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 241

$$\begin{aligned}\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx = &-\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2}+\frac{2a\cosh(c+dx)}{b^3(a+bx)} \\ &+\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2b^5} \\ &-\frac{2ad\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^4}-\frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} \\ &-\frac{2ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} \\ &+\frac{a^2d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2b^5}\end{aligned}$$

```
[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3+1/2*a^2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)
/b^5-1/2*a^2*cosh(d*x+c)/b^3/(b*x+a)^2+2*a*cosh(d*x+c)/b^3/(b*x+a)-2*a*d*co
sh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4+2*a*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4-Shi
(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-1/2*a^2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^
5-1/2*a^2*d*sinh(d*x+c)/b^4/(b*x+a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx = {} & \frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^5} \\ & + \frac{a^2d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^5} - \frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} \\ & - \frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} - \frac{2ad\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} \\ & - \frac{2ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3} \\ & + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{2a\cosh(c+dx)}{b^3(a+bx)}\end{aligned}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(a^2*Cosh[c + d*x])/(b^3*(a + b*x)^2) + (2*a*Cosh[c + d*x])/(b^3*(a +
b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Cosh
[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^5) - (2*a*d*CoshIntegral[(a
*d)/b + d*x]*Sinh[c - (a*d)/b])/b^4 - (a^2*d*Sinh[c + d*x])/(2*b^4*(a + b*x
)) - (2*a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4 + (Sinh[c -
(a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Sinh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/(2*b^5)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{b^2(a+bx)^3} - \frac{2a\cosh(c+dx)}{b^2(a+bx)^2} + \frac{\cosh(c+dx)}{b^2(a+bx)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^2} - \frac{(2a)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^2} + \frac{a^2\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b^2} \\
&= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} - \frac{(2ad)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^3} + \frac{(a^2d)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^3} \\
&\quad + \frac{\cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} + \frac{\sinh\left(c-\frac{ad}{b}\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} \\
&= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} \\
&\quad - \frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{(a^2d^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^4} \\
&\quad - \frac{\left(2ad\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} - \frac{\left(2ad\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^3} \\
&= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} \\
&\quad - \frac{2ad\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^4} - \frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} \\
&\quad - \frac{2ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} \\
&\quad + \frac{\left(a^2d^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^4} + \frac{\left(a^2d^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^4}
\end{aligned}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^5} - \frac{2ad\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^4} - \frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} - \frac{2ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2b^5}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left((2b^2+a^2d^2)\cosh\left(c-\frac{ad}{b}\right)-4abd\sinh\left(c-\frac{ad}{b}\right)\right) - \frac{ab(-b(3a+4bx)\cosh(c+dx)+ad(a+bx)\sinh(c+dx))}{(a+bx)^2}}{2b^5}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (CoshIntegral[d*(a/b + x)]*((2*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 4*a*b*d*S
inh[c - (a*d)/b]) - (a*b*(-(b*(3*a + 4*b*x)*Cosh[c + d*x]) + a*d*(a + b*x)*
Sinh[c + d*x]))/(a + b*x)^2 + (-4*a*b*d*Cosh[c - (a*d)/b] + (2*b^2 + a^2*d^
2)*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/(2*b^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(240) = 480$.

Time = 0.28 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.80

method	result
risch	$-\frac{e^{\frac{da-cb}{b}}\,\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^2b^2d^2x^2+2\,e^{\frac{da-cb}{b}}\,\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a^3b\,d^2x+4\,e^{\frac{da-cb}{b}}\,\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)a\,b^3d\,x^2+8\,e^{\frac{da-cb}{b}}\,\text{Ei}_1}{}$

```
[In] int(x^2*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^2*d^2*x^2+2*exp((a*d-b
*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b*d^2*x+4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a
*d-b*c)/b)*a*b^3*d*x^2+8*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^2*d
*x-3*exp(d*x+c)*a^2*b^2-3*exp(-d*x-c)*a^2*b^2-exp(-d*x-c)*a^2*b^2*d*x+4*exp
((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b*d+4*exp((a*d-b*c)/b)*Ei(1,d*x+c
+(a*d-b*c)/b)*a*b^3*x-4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b*d+
4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^3*x+exp(d*x+c)*a^2*b^2*d*x
```

```
+exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*d^2+2*exp((a*d-b*c)/b)*Ei(1,d
*x+c+(a*d-b*c)/b)*b^4*x^2-exp(-d*x-c)*a^3*b*d-4*exp(-d*x-c)*a*b^3*x+2*exp((
a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^2+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(
a*d-b*c)/b)*a^4*d^2+2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*b^4*x^2+2*
exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2+exp(d*x+c)*a^3*b*d-4*exp
(d*x+c)*a*b^3*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2*d^2*x^2+
2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b*d^2*x-4*exp(-(a*d-b*c)/b
)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^3*d*x^2-8*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d
-b*c)/b)*a^2*b^2*d*x)/b^5/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.97

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{2\,(4\,ab^3x+3\,a^2b^2)\cosh\left(dx+c\right)+\left((a^4d^2-4\,a^3bd+2\,a^2b^2+(a^2b^2d^2-4\,ab^3d+2\,b^4)x^2+2\,(a^3bd^2-4\,a^2b^2\cdots\right.}{\cdots}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(4*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + ((a^4*d^2 - 4*a^3*b*d + 2*a^
2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d
+ 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2
*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a^2*b^2*d*x + a^3*b*d)*si
nh(d*x + c) - ((a^4*d^2 - 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d
+ 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b)
 - (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2
 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*
c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^2\cosh\left(c+dx\right)}{\left(a+bx\right)^3}\,dx$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^2 \cosh(dx+c)}{(bx+a)^3} \, dx$$

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")

[Out] -3/2*a*d*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 3/2*a*d*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + b*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + b*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b^3*d^2*x^3*e^c + 3*a*b^2*d^2*x^2*e^c + 3*a^2*b*d^2*x*e^c + a^3*d^2*e^c) + 1/2*a*e^(-c + a*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d^2) + 1/2*a*e^(c - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d^2)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(240) = 480$.

Time = 0.32 (sec) , antiderivative size = 741, normalized size of antiderivative = 3.07

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx = \frac{a^2b^2d^2x^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + a^2b^2d^2x^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2\,a^3bd^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 4\,ab^3dx^2\text{Ei}\left(\cdots\right.}{}$$

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")

[Out] 1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 4*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 8*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^4*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 8*a^2*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x*e^(d*x + c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a^3*b*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*x*Ei(-(b*d*x + a*d)/b)*

```
e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) + 4*a*b^3*x*e^(d*x + c) + a^3*b*d*e^(-
d*x - c) + 4*a*b^3*x*e^(-d*x - c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*
d/b) + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 3*a^2*b^2*e^(d*x + c
) + 3*a^2*b^2*e^(-d*x - c))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{{(a+b\,x)}^3}\,dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x)^3,x)
[Out] int((x^2*cosh(c + d*x))/(a + b*x)^3, x)
```

3.35 $\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 178

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^4}+\frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3}+\frac{ad\sinh(c+dx)}{2b^3(a+bx)}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2b^4}$$

```
[Out] -1/2*a*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4+1/2*a*cosh(d*x+c)/b^2/(b*x+a)^
2-cosh(d*x+c)/b^2/(b*x+a)+d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3-d*Chi(a*d/b+d
*x)*sinh(-c+a*d/b)/b^3+1/2*a*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+1/2*a*d*
sinh(d*x+c)/b^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^4}-\frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\sinh(c+dx)}{2b^3(a+bx)}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{a\cosh(c+dx)}{2b^2(a+bx)^2}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a*Cosh[c + d*x])/(2*b^2*(a + b*x)^2) - Cosh[c + d*x]/(b^2*(a + b*x)) - (a*
d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^4) + (d*CoshIntegra
l[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 + (a*d*Sinh[c + d*x])/(2*b^3*(a + b
*x)) + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 - (a*d^2*Sinh[
c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a\cosh(c+dx)}{b(a+bx)^3}+\frac{\cosh(c+dx)}{b(a+bx)^2}\right)\,dx \\
&= \frac{\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b} \\
&= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^2}-\frac{(ad)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^2}
\end{aligned}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} + \frac{ad\sinh(c+dx)}{2b^3(a+bx)} - \frac{(ad^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^3} + \frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2} + \frac{\left(d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} + \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} + \frac{ad\sinh(c+dx)}{2b^3(a+bx)} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{\left(ad^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^3} - \frac{\left(ad^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^3}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} - \frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^4} + \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} + \frac{ad\sinh(c+dx)}{2b^3(a+bx)} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2b^4}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{b\cosh(dx)(b(a+2bx)\cosh(c)-ad(a+bx)\sinh(c))-b(ad(a+bx)\cosh(c)-b(a+2bx)\sinh(c))\sinh(\ldots}{\ldots}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*Cosh[d*x]*(b*(a + 2*b*x)*Cosh[c] - a*d*(a + b*x)*Sinh[c]) - b*(a*d*
(a + b*x)*Cosh[c] - b*(a + 2*b*x)*Sinh[c])*Sinh[d*x] + d*(a + b*x)^2*(CoshI
ntegral[d*(a/b + x)]*(a*d*Cosh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b]) + (-2*
b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)]))/(b
^4*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(175) = 350.

Time = 0.26 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.44

method	result
risch	$-\frac{d^{3} \mathrm{e}^{-d x -c} a x}{4 b^{2} \left(x^{2} d^{2} b^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}-\frac{d^{3} \mathrm{e}^{-d x -c} a^{2}}{4 b^{3} \left(x^{2} d^{2} b^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}-\frac{d^{2} \mathrm{e}^{-d x -c} x}{2 b \left(x^{2} d^{2} b^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}-\frac{d^{2} \mathrm{e}^{-d x -c} a}{4 b^{2} \left(x^{2} d^{2} b^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}+$

```
[In] int(x*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x-1/4*d^3*exp(
-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/2*d^2*exp(-d*x-c)/b/(b^
2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x-1/4*d^2*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b
*d^2*x+a^2*d^2)*a+1/4*d^2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/
2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4*d^2/b^4*exp(d*x+c)/(d/
b*a+d*x)^2*a+1/4*d^2/b^4*exp(d*x+c)/(d/b*a+d*x)*a+1/4*d^2/b^4*exp(-(a*d-b*c
)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a-1/2*d/b^3*exp(d*x+c)/(d/b*a+d*x)-1/2*d/b^3*
exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(175) = 350.

Time = 0.25 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.10

$$\int \frac{x \cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{2\left(2\,b^{3}x+ab^{2}\right)\cosh\left(dx+c\right)+\left(\left(a^{3}d^{2}-2\,a^{2}bd+\left(ab^{2}d^{2}-2\,b^{3}d\right)x^{2}+2\left(a^{2}bd^{2}-2\,ab^{2}d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(\right.}{}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*b^3*x + a*b^2)*cosh(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^
2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) + (a^3*
d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*
Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*sinh(d
*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2
- 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2
*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c
 - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\cosh(dx+c)}{(bx+a)^3}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
[Out] b*integrate(x*e^(d*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4*
a^3*b*d*x + a^4*d), x) - b*integrate(x/(b^4*d*x^4*e^(d*x + c) + 4*a*b^3*d*x
^3*e^(d*x + c) + 6*a^2*b^2*d*x^2*e^(d*x + c) + 4*a^3*b*d*x*e^(d*x + c) + a^
4*d*e^(d*x + c)), x) + 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^3*e^c +
3*a*b^2*d*x^2*e^c + 3*a^2*b*d*x*e^c + a^3*d*e^c) - 1/2*a*e^(-c + a*d/b)*exp
_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d) + 1/2*a*e^(c - a*d/b)*exp_i
ntegral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(175) = 350.

Time = 0.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.97

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{ab^2d^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + ab^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2a^2bd^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 2b^3dx^2\mathrm{Ei}\Big(\frac{b}{}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
[Out] -1/4*(a*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*b^2*d^2*x^2*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^2*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a
*d/b) - 2*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d^2*x*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a
*d/b) + a^3*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 4*a*b^2*d*x*Ei((b*d*x +
 a*d)/b)*e^(c - a*d/b) + a^3*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*
```

```
b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b^2*d*x*e^(d*x + c) + a*b^2
*d*x*e^(-d*x - c) - 2*a^2*b*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b*d*e^(d*x + c) + 2*b^3*x*e^(d*x
 + c) + a^2*b*d*e^(-d*x - c) + 2*b^3*x*e^(-d*x - c) + a*b^2*e^(d*x + c) + a
*b^2*e^(-d*x - c))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\cosh(c+d\,x)}{(a+b\,x)^3}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x)^3, x)
```

3.36 $\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2b^3} - \frac{d\sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

```
[Out] 1/2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-1/2*cosh(d*x+c)/b/(b*x+a)^2-1/2*d
^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-1/2*d*sinh(d*x+c)/b^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{2b^3} + \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{2b^3} - \frac{d\sinh(c+dx)}{2b^2(a+bx)} - \frac{\cosh(c+dx)}{2b(a+bx)^2}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Cosh[c + d*x]/(b*(a + b*x)^2) + (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a
*d)/b + d*x])/(2*b^3) - (d*Sinh[c + d*x])/(2*b^2*(a + b*x)) + (d^2*Sinh[c -
 (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{d\int\frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b}\\
&= -\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{d^2\int\frac{\cosh(c+dx)}{a+bx}\,dx}{2b^2}\\
&= -\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{\left(d^2\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}\\
&\quad+\frac{\left(d^2\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}\\
&= -\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2b^3}\\
&\quad-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)-\frac{b(b\cosh(c+dx)+d(a+bx)\sinh(c+dx))}{(a+bx)^2}+d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^3}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^3,x]
```

```
[Out] (d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - (b*(b*Cosh[c + d*x] + d*
(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + d^2*Sinh[c - (a*d)/b]*SinhIntegral[
d*(a/b + x)])/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(98) = 196$.

Time = 0.22 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.65

method	result
risch	$\frac{d^3 e^{-dx-c} x}{4b(x^2d^2b^2+2ab\,d^2x+a^2d^2)}+\frac{d^3 e^{-dx-c} a}{4b^2(x^2d^2b^2+2ab\,d^2x+a^2d^2)}-\frac{d^2 e^{-dx-c}}{4b(x^2d^2b^2+2ab\,d^2x+a^2d^2)}-\frac{d^2 e^{\frac{da-cb}{b}}\,\text{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{4b^3}-$

```
[In] int(cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^3*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x+1/4*d^3*exp(-d*x-
c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a-1/4*d^2*exp(-d*x-c)/b/(b^2*d^2*x
^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b
)-1/4*d^2/b^3*exp(d*x+c)/(d/b*a+d*x)^2-1/4*d^2/b^3*exp(d*x+c)/(d/b*a+d*x)-1
/4*d^2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(98) = 196$.

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.43

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{2b^2\cosh(dx+c)-\left((b^2d^2x^2+2abd^2x+a^2d^2)\text{Ei}\left(\frac{bdx+ad}{b}\right)+(b^2d^2x^2+2abd^2x+a^2d^2)\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\text{co}}{}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*cosh(d*x + c) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*
x + a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) + 2*(b^2*d*x + a*b*d)*sinh(d*x + c) + ((b^2*d^2*x^2 + 2
*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^
2*d^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a
^2*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_2\left(\frac{(bx+a)d}{b}\right)}{(bx+a)b} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_2\left(-\frac{(bx+a)d}{b}\right)}{(bx+a)b}\right)}{4\,b} - \frac{\cosh\left(dx+c\right)}{2\left(bx+a\right)^2 b}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d*(e^(-c + a*d/b)*exp_integral_e(2, (b*x + a)*d/b)/((b*x + a)*b) - e^(c
 - a*d/b)*exp_integral_e(2, -(b*x + a)*d/b)/((b*x + a)*b))/b - 1/2*cosh(d*x
 + c)/((b*x + a)^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(98) = 196$.

Time = 0.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.87

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{b^2d^2x^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + b^2d^2x^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2\,abd^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + 2\,abd^2x\text{Ei}\left(-\frac{bdx+ad}{b}\right)}{}$$

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b^2*d^2*x^2*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) + 2*a*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) +
 2*a*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*d^2*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - b^2*d*x*e
^(d*x + c) + b^2*d*x*e^(-d*x - c) - a*b*d*e^(d*x + c) + a*b*d*e^(-d*x - c)
- b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{{(a+b\,x)}^3}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*x)^3,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x)^3, x)
```

3.37 $\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 262

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx &= \frac{\cosh(c+dx)}{2a(a+bx)^2}+\frac{\cosh(c+dx)}{a^2(a+bx)}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}\\ &\quad-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2ab^2}\\ &\quad-\frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2b}+\frac{d\sinh(c+dx)}{2ab(a+bx)}\\ &\quad+\frac{\sinh(c)\text{Shi}(dx)}{a^3}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2b}\\ &\quad-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2ab^2}\end{aligned}$$

```
[Out] Chi(d*x)*cosh(c)/a^3-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-1/2*d^2*Chi(a*d/b+d*
x)*cosh(-c+a*d/b)/a/b^2+1/2*cosh(d*x+c)/a/(b*x+a)^2+cosh(d*x+c)/a^2/(b*x+a)
-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a^2/b+Shi(d*x)*sinh(c)/a^3+d*Chi(a*d/b+d*x
)*sinh(-c+a*d/b)/a^2/b+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3+1/2*d^2*Shi(a*d/b+
d*x)*sinh(-c+a*d/b)/a/b^2+1/2*d*sinh(d*x+c)/a/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used

= {6874, 3384, 3379, 3382, 3378}

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = -\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} - \frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2b} - \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2b} + \frac{\cosh(c+dx)}{a^2(a+bx)} - \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2ab^2} - \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2ab^2} + \frac{d\sinh(c+dx)}{2ab(a+bx)} + \frac{\cosh(c+dx)}{2a(a+bx)^2}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x)^2) + Cosh[c + d*x]/(a^2*(a + b*x)) + (Cosh[c]*
CoshIntegral[d*x])/a^3 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^
3 - (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a*b^2) - (d*Cosh
Integral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a^2*b) + (d*Sinh[c + d*x])/(2*a
*b*(a + b*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 - (d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/(a^2*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b
+ d*x])/a^3 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{a(a+bx)^3} - \frac{b\cosh(c+dx)}{a^2(a+bx)^2} - \frac{b\cosh(c+dx)}{a^3(a+bx)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{a} \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} - \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2a} \\
&\quad + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^3} - \frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&\quad + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a^3} - \frac{\left(b\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} \\
&\quad + \frac{d\sinh(c+dx)}{2ab(a+bx)} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d^2\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2ab} \\
&\quad - \frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} - \frac{\left(d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} \\
&\quad - \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2b} + \frac{d\sinh(c+dx)}{2ab(a+bx)} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} \\
&\quad - \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2b} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} \\
&\quad - \frac{\left(d^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2ab} - \frac{\left(d^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2ab}
\end{aligned}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2ab^2} - \frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2b} + \frac{d\sinh(c+dx)}{2ab(a+bx)} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a^3} - \frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^2b} - \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2ab^2}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.72

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = -\frac{-3a^2b^2\cosh(c+dx) - 2ab^3x\cosh(c+dx) - 2b^2(a+bx)^2\cosh(c)\mathrm{Chi}(dx) + (a+bx)^2\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] -1/2*(-3*a^2*b^2*Cosh[c + d*x] - 2*a*b^3*x*Cosh[c + d*x] - 2*b^2*(a + b*x)^
2*Cosh[c]*CoshIntegral[d*x] + (a + b*x)^2*CoshIntegral[d*(a/b + x)]*((2*b^2
 + a^2*d^2)*Cosh[c - (a*d)/b] + 2*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*Sinh[c
 + d*x] - a^2*b^2*d*x*Sinh[c + d*x] - 2*a^2*b^2*Sinh[c]*SinhIntegral[d*x] -
 4*a*b^3*x*Sinh[c]*SinhIntegral[d*x] - 2*b^4*x^2*Sinh[c]*SinhIntegral[d*x]
+ 2*a^3*b*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a^2*b^2*d*x*Cos
h[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a*b^3*d*x^2*Cosh[c - (a*d)/b]*
SinhIntegral[d*(a/b + x)] + 2*a^2*b^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b
 + x)] + a^4*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a*b^3*x*Si
nh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x*Sinh[c - (a*d)/b]
*SinhIntegral[d*(a/b + x)] + 2*b^4*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/
b + x)] + a^2*b^2*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(a^3
*b^2*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{\mathrm{e}^{-dx-c} d\left(\left(dx+c\right)abd+a^2d^2-bcda-2\left(dx+c\right)b^2-3dab+2c\,b^2\right)}{4a^2b\left(\left(dx+c\right)^2b^2+2\left(dx+c\right)abd-2\left(dx+c\right)b^2c+a^2d^2-2bcda+b^2c^2\right)} - \frac{\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)}{2a^3} + \frac{\mathrm{e}^{\frac{da-cb}{b}}\,\mathrm{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)d^2}{4b^2a} - \frac{\mathrm{e}^{\frac{da-cb}{b}}}{}$

```
[In] int(cosh(d*x+c)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(-d*x-c)*d*((d*x+c)*a*b*d+a^2*d^2-b*c*d*a-2*(d*x+c)*b^2-3*d*a*b+2*c
*b^2)/a^2/b/((d*x+c)^2*b^2+2*(d*x+c)*a*b*d-2*(d*x+c)*b^2*c+a^2*d^2-2*b*c*d*
a+b^2*c^2)-1/2/a^3*exp(-c)*Ei(1,d*x)+1/4/b^2/a*exp((a*d-b*c)/b)*Ei(1,d*x+c+
(a*d-b*c)/b)*d^2-1/2/b/a^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a
^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/a^3*exp(c)*Ei(1,-d*x)+1/4/a
/b^2*d^2*exp(d*x+c)/(d/b*a+d*x)^2+1/4/a/b^2*d^2*exp(d*x+c)/(d/b*a+d*x)+1/4/
a/b^2*d^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2/a^2*d/b*exp(d*x+c)
/(d/b*a+d*x)+1/2/a^2*d/b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2/a^3
*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(258) = 516$.

Time = 0.29 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.29

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \frac{2\,(2\,ab^3x+3\,a^2b^2)\cosh\,(dx+c)+2\,((b^4x^2+2\,ab^3x+a^2b^2)\mathrm{Ei}(dx)+(b^4x^2+2\,ab^3x+a^2b^2)\mathrm{Ei}(-dx))\cosh\,(}{}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^
2*b^2)*Ei(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*cosh(c) - ((a^4*
d^2 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^
3*b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^3*
b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*
a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2
*b^2*d*x + a^3*b*d)*sinh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(d
*x) - (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*sinh(c) + ((a^4*d^2 + 2*a^3
*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 2
*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^2 - 2*a^3*b*d + 2*a^2
*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a^2*b^2*d +
 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a
^4*b^3*x + a^5*b^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh(c+dx)}{x\,(a+bx)^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)**3,x)
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx+a)^3x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(258) = 516$.

Time = 0.28 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.19

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = -\frac{a^2b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + a^2b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2\,a^3bd^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + 2\,ab^3dx^2\mathrm{E}}{}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
[Out] -1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c
 - a*d/b) + 2*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e
^(-c + a*d/b) - 2*b^4*x^2*Ei(-d*x)*e^(-c) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(
c - a*d/b) + 4*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei
((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*b^4*x^2*Ei(d*x)*e^c + a^4*d^2*Ei(-(b*d*
x + a*d)/b)*e^(-c + a*d/b) - 4*a^2*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d
/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x*e^(d*x +
c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a*b^3*x*Ei(-d*x)*e^(-c) + 2*a^3*b*d*Ei((b
*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b)
- 4*a*b^3*x*Ei(d*x)*e^c - 2*a^3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4
```

```
*a*b^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) - 2*a*b^
3*x*e^(d*x + c) + a^3*b*d*e^(-d*x - c) - 2*a*b^3*x*e^(-d*x - c) - 2*a^2*b^2
*Ei(-d*x)*e^(-c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*a^2*b^2*
Ei(d*x)*e^c + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 3*a^2*b^2*e^(
d*x + c) - 3*a^2*b^2*e^(-d*x - c))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x\,{(a+b\,x)}^3}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x)^3),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x)^3), x)
```

3.38 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 298

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = & -\frac{\cosh(c+dx)}{a^3x}-\frac{b\cosh(c+dx)}{2a^2(a+bx)^2}-\frac{2b\cosh(c+dx)}{a^3(a+bx)} \\ & -\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^4} \\ & +\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2a^2b}+\frac{d\text{Chi}(dx)\sinh(c)}{a^3} \\ & +\frac{2d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^3}-\frac{d\sinh(c+dx)}{2a^2(a+bx)}+\frac{d\cosh(c)\text{Shi}(dx)}{a^3} \\ & -\frac{3b\sinh(c)\text{Shi}(dx)}{a^4}+\frac{2d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} \\ & +\frac{3b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2a^2b}\end{aligned}$$

```
[Out] -3*b*Chi(d*x)*cosh(c)/a^4+3*b*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^4+1/2*d^2*Chi
(a*d/b+d*x)*cosh(-c+a*d/b)/a^2/b-cosh(d*x+c)/a^3/x-1/2*b*cosh(d*x+c)/a^2/(b
*x+a)^2-2*b*cosh(d*x+c)/a^3/(b*x+a)+d*cosh(c)*Shi(d*x)/a^3+2*d*cosh(-c+a*d/
b)*Shi(a*d/b+d*x)/a^3+d*Chi(d*x)*sinh(c)/a^3-3*b*Shi(d*x)*sinh(c)/a^4-2*d*C
hi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3-3*b*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^4-1/2*
d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2/b-1/2*d*sinh(d*x+c)/a^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^4} -\frac{3b\sinh(c)\text{Shi}(dx)}{a^4}+\frac{3b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^4} +\frac{2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{2d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3} -\frac{2b\cosh(c+dx)}{a^3(a+bx)}+\frac{d\sinh(c)\text{Chi}(dx)}{a^3}+\frac{d\cosh(c)\text{Shi}(dx)}{a^3} -\frac{\cosh(c+dx)}{a^3x}+\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^2b} +\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2a^2b}-\frac{d\sinh(c+dx)}{2a^2(a+bx)}-\frac{b\cosh(c+dx)}{2a^2(a+bx)^2}$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (b*Cosh[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Cos
h[c + d*x])/(a^3*(a + b*x)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (3*b*Co
sh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^4 + (d^2*Cosh[c - (a*d)/b]*C
oshIntegral[(a*d)/b + d*x])/(2*a^2*b) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 +
 (2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^3 - (d*Sinh[c + d*x]
)/(2*a^2*(a + b*x)) + (d*Cosh[c]*SinhIntegral[d*x])/a^3 - (3*b*Sinh[c]*Sinh
Integral[d*x])/a^4 + (2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^
3 + (3*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^4 + (d^2*Sinh[c -
 (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{a^3x^2} - \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^3} + \frac{2b^2\cosh(c+dx)}{a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^4} + \frac{(2b^2)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^3} + \frac{(2bd)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^3} + \frac{(bd)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2a^2} - \frac{(3b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4} - \frac{(3b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^4} - \frac{d\sinh(c+dx)}{2a^2(a+bx)} - \frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4} + \frac{3b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{d^2\int\frac{\cosh(c+dx)}{a+bx}\,dx}{2a^2} + \frac{(d\cosh(c))\int\frac{\sinh(dx)}{x}\,dx}{a^3} + \frac{\left(2bd\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3} + \frac{(d\sinh(c))\int\frac{\cosh(dx)}{x}\,dx}{a^3} + \frac{\left(2bd\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^3}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{d\mathrm{Chi}(dx)\sinh(c)}{a^3} + \frac{2d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^3} - \frac{d\sinh(c+dx)}{2a^2(a+bx)} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^3} - \frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4} + \frac{2d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{3b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{\left(d^2\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2a^2} + \frac{\left(d^2\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2a^2}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2a^2b} + \frac{d\mathrm{Chi}(dx)\sinh(c)}{a^3} + \frac{2d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^3} - \frac{d\sinh(c+dx)}{2a^2(a+bx)} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^3} - \frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4} + \frac{2d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{3b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2a^2b}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.82

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx$$
$$= \frac{-2a^3b\cosh(c+dx)-9a^2b^2x\cosh(c+dx)-6ab^3x^2\cosh(c+dx)+2bx(a+bx)^2\mathrm{Chi}(dx)(-3b\cosh(c)+a}{}$$

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]

[Out] (-2*a^3*b*Cosh[c + d*x] - 9*a^2*b^2*x*Cosh[c + d*x] - 6*a*b^3*x^2*Cosh[c + d*x] + 2*b*x*(a + b*x)^2*CoshIntegral[d*x]*(-3*b*Cosh[c] + a*d*Sinh[c]) + x*(a + b*x)^2*CoshIntegral[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 4*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*x*Sinh[c + d*x] - a^2*b^2*d*x^2*Sinh[c + d*x] + 2*a^3*b*d*x*Cosh[c]*SinhIntegral[d*x] + 4*a^2*b^2*d*x^2*Cosh[c]*SinhIntegral[d*x] + 2*a*b^3*d*x^3*Cosh[c]*SinhIntegral[d*x] - 6*a^2*b^2*x*Sinh[c]*SinhIntegral[d*x] - 12*a*b^3*x^2*Sinh[c]*SinhIntegral[d*x] - 6*b^4*x^3*Sinh[c]*SinhIntegral[d*x] + 4*a^3*b*d*x*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 8*a^2*b^2*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a*b^3*d*x^3*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a^2*b^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^4*d^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a*b^3*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*b^4*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(2*a^4*b*x*(a + b*x)^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(296) = 592.

Time = 0.36 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.16

method	result
risch	$\frac{e^{-dx-c} x\, d^3 b}{4a^2(x^2d^2b^2+2ab\,d^2x+a^2d^2)}+\frac{e^{-dx-c} d^3}{4a(x^2d^2b^2+2ab\,d^2x+a^2d^2)}-\frac{3\,e^{-dx-c} x\, d^2b^2}{2a^3(x^2d^2b^2+2ab\,d^2x+a^2d^2)}-\frac{9\,e^{-dx-c} d^2b}{4a^2(x^2d^2b^2+2ab\,d^2x+a^2d^2)}-$

[In] int(cosh(d*x+c)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*exp(-d*x-c)/a^2*x*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+1/4*exp(-d*x-c)/a*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-3/2*exp(-d*x-c)/a^3*x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2-9/4*exp(-d*x-c)/a^2*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+1/2*d/a^3*exp(-c)*Ei(1,d*x)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4/b/a^2*d^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+d/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-

```
b*c)/b)-3/2*b/a^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/a^3/x*exp(d*
x+c)-1/2*d/a^3*exp(c)*Ei(1,-d*x)+3/2/a^4*b*exp(c)*Ei(1,-d*x)-1/4/a^2*d^2/b*
exp(d*x+c)/(d/b*a+d*x)^2-1/4/a^2*d^2/b*exp(d*x+c)/(d/b*a+d*x)-1/4/a^2*d^2/b
*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-d/a^3*exp(d*x+c)/(d/b*a+d*x)-d/
a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-3/2*b/a^4*exp(-(a*d-b*c)/b)*
Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(296) = 592$.

Time = 0.26 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.56

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = -\frac{2\,(6\,ab^3x^2+9\,a^2b^2x+2\,a^3b)\cosh\left(dx+c\right)-2\,(((ab^3d-3\,b^4)x^3+2\,(a^2b^2d-3\,ab^3)x^2+(a^3bd-3\,a^2b^2)x}{}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*cosh(d*x + c) - 2*(((a*b^3*d
- 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3*b*d - 3*a^2*b^2)*x)*Ei(d*
x) - ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*b^3)*x^2 + (a^3*b*d + 3*a^
2*b^2)*x)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 + 4*a*b^3*d + 6*b^4)*x^3 + 2*(
a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 + 4*a^3*b*d + 6*a^2*b^2)*
x)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 4*a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*
d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a^3*b*d + 6*a^2*b^2)*x)*Ei(
-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2*b^2*d*x^2 + a^3*b*d*x)*sin
h(d*x + c) - 2*(((a*b^3*d - 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3
*b*d - 3*a^2*b^2)*x)*Ei(d*x) + ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*
b^3)*x^2 + (a^3*b*d + 3*a^2*b^2)*x)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 4*
a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2
 + 4*a^3*b*d + 6*a^2*b^2)*x)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 4*a*b^3*
d + 6*b^4)*x^3 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a
^3*b*d + 6*a^2*b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^4*b^3
*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx+a)^3x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. 2(296) = 592.

Time = 0.27 (sec) , antiderivative size = 1006, normalized size of antiderivative = 3.38

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
[Out] 1/4*(a^2*b^2*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^3*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^3*Ei(-d*x)*e^(-c) + 2*a^3*b
*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*d*x^3*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + 2*a*b^3*d*x^3*Ei(d*x)*e^c + 2*a^3*b*d^2*x^2*Ei(-(b*d*x
+ a*d)/b)*e^(-c + a*d/b) - 4*a*b^3*d*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
) - 4*a^2*b^2*d*x^2*Ei(-d*x)*e^(-c) - 6*b^4*x^3*Ei(-d*x)*e^(-c) + a^4*d^2*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 8*a^2*b^2*d*x^2*Ei((b*d*x + a*d)/b)*e^
(c - a*d/b) + 6*b^4*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^2*b^2*d*x^2
*Ei(d*x)*e^c - 6*b^4*x^3*Ei(d*x)*e^c + a^4*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b) - 8*a^2*b^2*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*b^4*x^3
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x^2*e^(d*x + c) + a^2*b^2*
d*x^2*e^(-d*x - c) - 2*a^3*b*d*x*Ei(-d*x)*e^(-c) - 12*a*b^3*x^2*Ei(-d*x)*e^
(-c) + 4*a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a*b^3*x^2*Ei((b*d
*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d*x*Ei(d*x)*e^c - 12*a*b^3*x^2*Ei(d*x)
*e^c - 4*a^3*b*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a*b^3*x^2*Ei(-(
```

```
b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x*e^(d*x + c) - 6*a*b^3*x^2*e^(d*x
 + c) + a^3*b*d*x*e^(-d*x - c) - 6*a*b^3*x^2*e^(-d*x - c) - 6*a^2*b^2*x*Ei(
-d*x)*e^(-c) + 6*a^2*b^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a^2*b^2*x*
Ei(d*x)*e^c + 6*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 9*a^2*b^2*x
*e^(d*x + c) - 9*a^2*b^2*x*e^(-d*x - c) - 2*a^3*b*e^(d*x + c) - 2*a^3*b*e^(
-d*x - c))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,{\left(a+b\,x\right)}^3}\,dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)^3), x)
```

3.39 $\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 377

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx &= -\frac{\cosh(c+dx)}{2a^3x^2}+\frac{3b\cosh(c+dx)}{a^4x}+\frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2}\\
&\quad+\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}+\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}\\
&\quad+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}-\frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^5}\\
&\quad-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2a^3}-\frac{3bd\text{Chi}(dx)\sinh(c)}{a^4}\\
&\quad-\frac{3bd\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^4}-\frac{d\sinh(c+dx)}{2a^3x}\\
&\quad+\frac{bd\sinh(c+dx)}{2a^3(a+bx)}-\frac{3bd\cosh(c)\text{Shi}(dx)}{a^4}+\frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5}\\
&\quad+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3}-\frac{3bd\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^4}\\
&\quad-\frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^5}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2a^3}\end{aligned}$$

```
[Out] 6*b^2*Chi(d*x)*cosh(c)/a^5+1/2*d^2*Chi(d*x)*cosh(c)/a^3-6*b^2*Chi(a*d/b+d*x
)*cosh(-c+a*d/b)/a^5-1/2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-1/2*cosh(d*x
+c)/a^3/x^2+3*b*cosh(d*x+c)/a^4/x+1/2*b^2*cosh(d*x+c)/a^3/(b*x+a)^2+3*b^2*c
osh(d*x+c)/a^4/(b*x+a)-3*b*d*cosh(c)*Shi(d*x)/a^4-3*b*d*cosh(-c+a*d/b)*Shi(
a*d/b+d*x)/a^4-3*b*d*Chi(d*x)*sinh(c)/a^4+6*b^2*Shi(d*x)*sinh(c)/a^5+1/2*d^
2*Shi(d*x)*sinh(c)/a^3+3*b*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/a^4+6*b^2*Shi(a*
d/b+d*x)*sinh(-c+a*d/b)/a^5+1/2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3-1/2*d
*sinh(d*x+c)/a^3/x+1/2*b*d*sinh(d*x+c)/a^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx =& \frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^5} \\ &+ \frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5} - \frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^5} \\ &+ \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} - \frac{3bd\sinh(c)\text{Chi}(dx)}{a^4} \\ &- \frac{3bd\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^4} - \frac{3bd\cosh(c)\text{Shi}(dx)}{a^4} \\ &- \frac{3bd\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^4} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} \\ &- \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^3} - \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2a^3} \\ &+ \frac{bd\sinh(c+dx)}{2a^3(a+bx)} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} \\ &+ \frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3} - \frac{\cosh(c+dx)}{2a^3x^2} - \frac{d\sinh(c+dx)}{2a^3x}\end{aligned}$$

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a^3*x^2) + (3*b*Cosh[c + d*x])/(a^4*x) + (b^2*Cosh[c + 
d*x])/(2*a^3*(a + b*x)^2) + (3*b^2*Cosh[c + d*x])/(a^4*(a + b*x)) + (6*b^2*
Cosh[c]*CoshIntegral[d*x])/a^5 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) -
(6*b^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^5 - (d^2*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a^3) - (3*b*d*CoshIntegral[d*x]*Sin
h[c])/a^4 - (3*b*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^4 - (d*
Sinh[c + d*x])/(2*a^3*x) + (b*d*Sinh[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*C
osh[c]*SinhIntegral[d*x])/a^4 + (6*b^2*Sinh[c]*SinhIntegral[d*x])/a^5 + (d^
2*Sinh[c]*SinhIntegral[d*x])/(2*a^3) - (3*b*d*Cosh[c - (a*d)/b]*SinhIntegra
l[(a*d)/b + d*x])/a^4 - (6*b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x
])/a^5 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{a^3x^3}-\frac{3b\cosh(c+dx)}{a^4x^2}+\frac{6b^2\cosh(c+dx)}{a^5x}-\frac{b^3\cosh(c+dx)}{a^3(a+bx)^3}-\frac{3b^3\cosh(c+dx)}{a^4(a+bx)^2}-\frac{6b^3\cosh(c+dx)}{a^5(a+bx)}\right)dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a^3}-\frac{(3b)\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^4}+\frac{(6b^2)\int \frac{\cosh(c+dx)}{x}\,dx}{a^5} \\
&\quad -\frac{(6b^3)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^5}-\frac{(3b^3)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^4}-\frac{b^3\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2}+\frac{3b\cosh(c+dx)}{a^4x}+\frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} \\
&\quad +\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}+\frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a^3}-\frac{(3bd)\int \frac{\sinh(c+dx)}{x}\,dx}{a^4} \\
&\quad -\frac{(3b^2d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^4}-\frac{(b^2d)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2a^3} \\
&\quad +\frac{(6b^2\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^5}-\frac{\left(6b^3\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^5} \\
&\quad +\frac{(6b^2\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^5}-\frac{\left(6b^3\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^5}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} \\
&\quad + \frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^5} \\
&\quad - \frac{d\sinh(c+dx)}{2a^3x} + \frac{bd\sinh(c+dx)}{2a^3(a+bx)} + \frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5} \\
&\quad - \frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^5} + \frac{d^2\int\frac{\cosh(c+dx)}{x}\,dx}{2a^3} - \frac{(bd^2)\int\frac{\cosh(c+dx)}{a+bx}\,dx}{2a^3} \\
&\quad - \frac{(3bd\cosh(c))\int\frac{\sinh(dx)}{x}\,dx}{a^4} - \frac{\left(3b^2d\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4} \\
&\quad - \frac{(3bd\sinh(c))\int\frac{\cosh(dx)}{x}\,dx}{a^4} - \frac{\left(3b^2d\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} \\
&\quad + \frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^5} \\
&\quad - \frac{3bd\text{Chi}(dx)\sinh(c)}{a^4} - \frac{3bd\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^4} - \frac{d\sinh(c+dx)}{2a^3x} \\
&\quad + \frac{bd\sinh(c+dx)}{2a^3(a+bx)} - \frac{3bd\cosh(c)\text{Shi}(dx)}{a^4} + \frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5} \\
&\quad - \frac{3bd\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^4} - \frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^5} \\
&\quad + \frac{(d^2\cosh(c))\int\frac{\cosh(dx)}{x}\,dx}{2a^3} - \frac{\left(bd^2\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2a^3} \\
&\quad + \frac{(d^2\sinh(c))\int\frac{\sinh(dx)}{x}\,dx}{2a^3} - \frac{\left(bd^2\sinh\left(c-\frac{ad}{b}\right)\right)\int\frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2a^3}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\mathrm{Chi}(dx)}{a^5} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a^3} - \frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^5} - \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2a^3} - \frac{3bd\mathrm{Chi}(dx)\sinh(c)}{a^4} - \frac{3bd\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^4} - \frac{d\sinh(c+dx)}{2a^3x} + \frac{bd\sinh(c+dx)}{2a^3(a+bx)} - \frac{3bd\cosh(c)\mathrm{Shi}(dx)}{a^4} + \frac{6b^2\sinh(c)\mathrm{Shi}(dx)}{a^5} + \frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a^3} - \frac{3bd\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^4} - \frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^5} - \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.66

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = -\frac{a^4\cosh(c+dx) - 4a^3bx\cosh(c+dx) - 18a^2b^2x^2\cosh(c+dx) - 12ab^3x^3\cosh(c+dx) - x^2(a+bx)^2\mathrm{C}}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*(a^4*Cosh[c + d*x] - 4*a^3*b*x*Cosh[c + d*x] - 18*a^2*b^2*x^2*Cosh[c +
 d*x] - 12*a*b^3*x^3*Cosh[c + d*x] - x^2*(a + b*x)^2*CoshIntegral[d*x]*((12
*b^2 + a^2*d^2)*Cosh[c] - 6*a*b*d*Sinh[c]) + x^2*(a + b*x)^2*CoshIntegral[d
*(a/b + x)]*((12*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 6*a*b*d*Sinh[c - (a*d)/
b]) + a^4*d*x*Sinh[c + d*x] + a^3*b*d*x^2*Sinh[c + d*x] + 6*a^3*b*d*x^2*Cos
h[c]*SinhIntegral[d*x] + 12*a^2*b^2*d*x^3*Cosh[c]*SinhIntegral[d*x] + 6*a*b
^3*d*x^4*Cosh[c]*SinhIntegral[d*x] - 12*a^2*b^2*x^2*Sinh[c]*SinhIntegral[d*
x] - a^4*d^2*x^2*Sinh[c]*SinhIntegral[d*x] - 24*a*b^3*x^3*Sinh[c]*SinhInteg
ral[d*x] - 2*a^3*b*d^2*x^3*Sinh[c]*SinhIntegral[d*x] - 12*b^4*x^4*Sinh[c]*S
inhIntegral[d*x] - a^2*b^2*d^2*x^4*Sinh[c]*SinhIntegral[d*x] + 6*a^3*b*d*x^
2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*d*x^3*Cosh[c - (
a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Cosh[c - (a*d)/b]*SinhInt
egral[d*(a/b + x)] + 12*a^2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b +
 x)] + a^4*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 24*a*b^3*x
^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^3*Sinh[c - (
a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*b^4*x^4*Sinh[c - (a*d)/b]*SinhIntegr
al[d*(a/b + x)] + a^2*b^2*d^2*x^4*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x
)])/(a^5*x^2*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. 2(367) = 734.

Time = 0.35 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.02

method	result
risch	$\frac{e^{-dx-c} d^3 b}{4a^2(x^2d^2b^2+2ab\,d^2x+a^2d^2)} + \frac{3d^2 e^{-dx-c} x\, b^3}{a^4(x^2d^2b^2+2ab\,d^2x+a^2d^2)} + \frac{d^3 e^{-dx-c}}{4ax(x^2d^2b^2+2ab\,d^2x+a^2d^2)} + \frac{9\, e^{-dx-c} d^2 b^2}{2a^3(x^2d^2b^2+2ab\,d^2x+a^2d^2)} +$

```
[In] int(cosh(d*x+c)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(-d*x-c)/a^2*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+3*d^2*exp(-d*x-
c)/a^4*x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^3+1/4*d^3*exp(-d*x-c)/a/x/(b^2
*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+9/2*exp(-d*x-c)/a^3*d^2/(b^2*d^2*x^2+2*a*b*d^
2*x+a^2*d^2)*b^2+d^2*exp(-d*x-c)/a^2/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-
1/4*exp(-d*x-c)/a/x^2*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/a^3*exp
(-c)*Ei(1,d*x)-3/2*d/a^4*exp(-c)*Ei(1,d*x)*b-3/a^5*exp(-c)*Ei(1,d*x)*b^2+1/
4*d^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-3/2*d/a^4*exp((a*d-b*c)/
b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+3/a^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)
*b^2+3/2/a^4*b/x*exp(d*x+c)+3/2*d/a^4*b*exp(c)*Ei(1,-d*x)-3/a^5*b^2*exp(c)*
Ei(1,-d*x)+1/4*d^2/a^3*exp(d*x+c)/(d/b*a+d*x)^2+1/4*d^2/a^3*exp(d*x+c)/(d/b
*a+d*x)+1/4*d^2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2*d/a^4*b*
exp(d*x+c)/(d/b*a+d*x)+3/2*d/a^4*b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/
b)+3*b^2/a^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/4/a^3/x^2*exp(d*x
+c)-1/4/a^3/x*d*exp(d*x+c)-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(367) = 734.

Time = 0.27 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.37

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \frac{2\,(12\,ab^3x^3 + 18\,a^2b^2x^2 + 4\,a^3bx - a^4)\cosh(dx+c) + (((a^2b^2d^2 - 6\,ab^3d + 12\,b^4)x^4 + 2\,(a^3bd^2 - 6\,a^2b^2d +}{}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*cosh(d*x + c) + ((
(a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*
b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) + ((a^2*b^2*d^2
+ 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a
^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 +
6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4
```

```
*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 6
*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*
d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/
b) - 2*(a^3*b*d*x^2 + a^4*d*x)*sinh(d*x + c) + (((a^2*b^2*d^2 - 6*a*b^3*d +
 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^
3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) - ((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4
+ 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^
2*b^2)*x^2)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 +
2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*
b^2)*x^2)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2
*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b
^2)*x^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b
*x^3 + a^7*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx+a)^3x^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(367) = 734$.

Time = 0.28 (sec) , antiderivative size = 1169, normalized size of antiderivative = 3.10

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^4*Ei(-d*x)*e^(-c) - a^2*b^2*d^2*x^4*Ei((b*d*x + a*d)/b)*
e^(c - a*d/b) + a^2*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*b^2*d^2*x^4*Ei(-(b*d*x +
a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x^3*Ei(-d*x)*e^(-c) + 6*a*b^3*d*x^4*Ei
(-d*x)*e^(-c) - 2*a^3*b*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a*b^3
*d*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x^3*Ei(d*x)*e^c - 6*
a*b^3*d*x^4*Ei(d*x)*e^c - 2*a^3*b*d^2*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/
b) + 6*a*b^3*d*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^4*d^2*x^2*Ei(-d*
x)*e^(-c) + 12*a^2*b^2*d*x^3*Ei(-d*x)*e^(-c) + 12*b^4*x^4*Ei(-d*x)*e^(-c) -
 a^4*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^2*b^2*d*x^3*Ei((b*d*x
 + a*d)/b)*e^(c - a*d/b) - 12*b^4*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a
^4*d^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*d*x^3*Ei(d*x)*e^c + 12*b^4*x^4*Ei(d*x)*
e^c - a^4*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^2*b^2*d*x^3*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 12*b^4*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c +
 a*d/b) + 6*a^3*b*d*x^2*Ei(-d*x)*e^(-c) + 24*a*b^3*x^3*Ei(-d*x)*e^(-c) - 6*
a^3*b*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 24*a*b^3*x^3*Ei((b*d*x + a*
d)/b)*e^(c - a*d/b) - 6*a^3*b*d*x^2*Ei(d*x)*e^c + 24*a*b^3*x^3*Ei(d*x)*e^c
+ 6*a^3*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 24*a*b^3*x^3*Ei(-(b*d
*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x^2*e^(d*x + c) + 12*a*b^3*x^3*e^(d*x
 + c) + a^3*b*d*x^2*e^(-d*x - c) + 12*a*b^3*x^3*e^(-d*x - c) + 12*a^2*b^2*x
^2*Ei(-d*x)*e^(-c) - 12*a^2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*
a^2*b^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
) - a^4*d*x*e^(d*x + c) + 18*a^2*b^2*x^2*e^(d*x + c) + a^4*d*x*e^(-d*x - c)
 + 18*a^2*b^2*x^2*e^(-d*x - c) + 4*a^3*b*x*e^(d*x + c) + 4*a^3*b*x*e^(-d*x
- c) - a^4*e^(d*x + c) - a^4*e^(-d*x - c))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7
*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx=\int \frac{\cosh(c+d\,x)}{x^3\,{(a+b\,x)}^3}\,dx$$

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x^3*(a + b*x)^3), x)
```

3.40 $\int x^3(a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 139

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = -\frac{120b\cosh(c+dx)}{d^6} - \frac{6a\cosh(c+dx)}{d^4} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{3ax^2\cosh(c+dx)}{d^2} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{120bx\sinh(c+dx)}{d^5} + \frac{6ax\sinh(c+dx)}{d^3} + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{ax^3\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+dx)}{d}$$

```
[Out] -120*b*cosh(d*x+c)/d^6-6*a*cosh(d*x+c)/d^4-60*b*x^2*cosh(d*x+c)/d^4-3*a*x^2
*cosh(d*x+c)/d^2-5*b*x^4*cosh(d*x+c)/d^2+120*b*x*sinh(d*x+c)/d^5+6*a*x*sinh
(d*x+c)/d^3+20*b*x^3*sinh(d*x+c)/d^3+a*x^3*sinh(d*x+c)/d+b*x^5*sinh(d*x+c)/
d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used

$= \{5395, 3377, 2718\}$

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3} - \frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d} - \frac{120b\cosh(c+dx)}{d^6}+\frac{120bx\sinh(c+dx)}{d^5} - \frac{60bx^2\cosh(c+dx)}{d^4}+\frac{20bx^3\sinh(c+dx)}{d^3} - \frac{5bx^4\cosh(c+dx)}{d^2}+\frac{bx^5\sinh(c+dx)}{d}$$

```
[In] Int[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c + d
*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (120
*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c + d*
x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(ax^3\cosh(c+dx)+bx^5\cosh(c+dx)\right)\,dx$$

$$= a\int x^3\cosh(c+dx)\,dx+b\int x^5\cosh(c+dx)\,dx$$

$$= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^5\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(5b)\int x^4\sinh(c+dx)\,dx}{d}$$

$$
\begin{aligned}
&= -\frac{3ax^2\cosh(c+dx)}{d^2} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{ax^3\sinh(c+dx)}{d} \\
&\quad + \frac{bx^5\sinh(c+dx)}{d} + \frac{(6a)\int x\cosh(c+dx)\,dx}{d^2} + \frac{(20b)\int x^3\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{3ax^2\cosh(c+dx)}{d^2} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} \\
&\quad + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{ax^3\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+dx)}{d} \\
&\quad - \frac{(6a)\int\sinh(c+dx)\,dx}{d^3} - \frac{(60b)\int x^2\sinh(c+dx)\,dx}{d^3} \\
&= -\frac{6a\cosh(c+dx)}{d^4} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{3ax^2\cosh(c+dx)}{d^2} \\
&\quad - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} + \frac{20bx^3\sinh(c+dx)}{d^3} \\
&\quad + \frac{ax^3\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+dx)}{d} + \frac{(120b)\int x\cosh(c+dx)\,dx}{d^4} \\
&= -\frac{6a\cosh(c+dx)}{d^4} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{3ax^2\cosh(c+dx)}{d^2} - \frac{5bx^4\cosh(c+dx)}{d^2} \\
&\quad + \frac{120bx\sinh(c+dx)}{d^5} + \frac{6ax\sinh(c+dx)}{d^3} + \frac{20bx^3\sinh(c+dx)}{d^3} \\
&\quad + \frac{ax^3\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+dx)}{d} - \frac{(120b)\int\sinh(c+dx)\,dx}{d^5} \\
&= -\frac{120b\cosh(c+dx)}{d^6} - \frac{6a\cosh(c+dx)}{d^4} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{3ax^2\cosh(c+dx)}{d^2} \\
&\quad - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{120bx\sinh(c+dx)}{d^5} + \frac{6ax\sinh(c+dx)}{d^3} \\
&\quad + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{ax^3\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

$$
\int x^3(a+bx^2)\cosh(c+dx)\,dx = \frac{-((3ad^2(2+d^2x^2)+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx))+dx(ad^2(6+d^2x^2)+b(120+20d^2x^2+d^4x}{d^6}
$$

```
[In] Integrate[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-((3*a*d^2*(2 + d^2*x^2) + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])
 + d*x*(a*d^2*(6 + d^2*x^2) + b*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x]
)/d^6
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{3\left(\left(\frac{5b x^2}{3}+a\right)d^2+20b\right)d^2x^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2dx\left(x^2\left(b\,x^2+a\right)d^4+2\left(10b\,x^2+3a\right)d^2+120b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(5b\,x^4+3a\,x^2\right)d}{d^6\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,x^5d^5+a\,d^5x^3-5b\,x^4d^4-3a\,d^4x^2+20b\,d^3x^3+6a\,d^3x-60b\,d^2x^2-6a\,d^2+120dxb-120b\right)e^{dx+c}}{2d^6}-\frac{\left(b\,x^5d^5+a\,d^5x^3+5b\,x^4d\right.}{}$
meijerg	$-\frac{32b\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib\sinh(c)\sqrt{\pi}\left(-\frac{ixd}{}\right.}{}$
parts	$\frac{b\,x^5\sinh(dx+c)}{d}+\frac{a\,x^3\sinh(dx+c)}{d}-\frac{\frac{5b\,c^4\cosh(dx+c)}{d^4}-\frac{20b\,c^3((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^4}+\frac{30b\,c^2\left((dx+c)^2\cosh(dx+c\right.}{}}{}$
derivativedivides	$\frac{-\frac{b\,c^5\sinh(dx+c)}{d^2}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+\frac{10b\,c^2}{}}{}$
default	$\frac{-\frac{b\,c^5\sinh(dx+c)}{d^2}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+\frac{10b\,c^2}{}}{}$

```
[In] int(x^3*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (3*((5/3*b*x^2+a)*d^2+20*b)*d^2*x^2*tanh(1/2*d*x+1/2*c)^2-2*d*x*(x^2*(b*x^2
+a)*d^4+2*(10*b*x^2+3*a)*d^2+120*b)*tanh(1/2*d*x+1/2*c)+(5*b*x^4+3*a*x^2)*d
^4+12*(5*b*x^2+a)*d^2+240*b)/d^6/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx =$$
$$-\frac{(5\,bd^4x^4+6\,ad^2+3\,(ad^4+20\,bd^2)x^2+120\,b)\cosh(dx+c)-(bd^5x^5+(ad^5+20\,bd^3)x^3+6\,(ad^3+20\,bd}{d^6}$$

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b*d^4*x^4 + 6*a*d^2 + 3*(a*d^4 + 20*b*d^2)*x^2 + 120*b)*cosh(d*x + c)
- (b*d^5*x^5 + (a*d^5 + 20*b*d^3)*x^3 + 6*(a*d^3 + 20*b*d)*x)*sinh(d*x + c)
)/d^6
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.21

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx$$
$$=\begin{cases}\frac{ax^3\sinh{(c+dx)}}{d}-\frac{3ax^2\cosh{(c+dx)}}{d^2}+\frac{6ax\sinh{(c+dx)}}{d^3}-\frac{6a\cosh{(c+dx)}}{d^4}+\frac{bx^5\sinh{(c+dx)}}{d}-\frac{5bx^4\cosh{(c+dx)}}{d^2}+\frac{20bx^3\sinh{(c+dx)}}{d^3}\\ \left(\frac{ax^4}{4}+\frac{bx^6}{6}\right)\cosh{(c)}\end{cases}$$

```
[In] integrate(x**3*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**5*sinh(c + d*x)/d - 5*b*x**
4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*
x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)),
 ((a*x**4/4 + b*x**6/6)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.80

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx =$$
$$-\frac{1}{24}\,d\left(\frac{3\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ae^{(dx)}}{d^5}+\frac{3\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+\right.$$
$$+\frac{1}{12}\left(2\,bx^6+3\,ax^4\right)\cosh\left(dx+c\right)$$

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3
*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b*e^(d*x)/d^7 + 2*(d^6*
x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b
*e^(-d*x - c)/d^7) + 1/12*(2*b*x^6 + 3*a*x^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.25

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx$$
$$=\frac{(bd^5x^5+ad^5x^3-5\,bd^4x^4-3\,ad^4x^2+20\,bd^3x^3+6\,ad^3x-60\,bd^2x^2-6\,ad^2+120\,bdx-120\,b)e^{(dx+c)}}{2\,d^6}$$
$$-\frac{(bd^5x^5+ad^5x^3+5\,bd^4x^4+3\,ad^4x^2+20\,bd^3x^3+6\,ad^3x+60\,bd^2x^2+6\,ad^2+120\,bdx+120\,b)e^{(-dx-c)}}{2\,d^6}$$

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^5*x^5 + a*d^5*x^3 - 5*b*d^4*x^4 - 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a
*d^3*x - 60*b*d^2*x^2 - 6*a*d^2 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*
(b*d^5*x^5 + a*d^5*x^3 + 5*b*d^4*x^4 + 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3
*x + 60*b*d^2*x^2 + 6*a*d^2 + 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx=\frac{x^3\sinh(c+d\,x)\,\left(a\,d^2+20\,b\right)}{d^3}-\frac{3\,x^2\cosh(c+d\,x)\,\left(a\,d^2+20\,b\right)}{d^4}-\frac{6\cosh(c+d\,x)\,\left(a\,d^2+20\,b\right)}{d^6}+\frac{6\,x\sinh(c+d\,x)\,\left(a\,d^2+20\,b\right)}{d^5}-\frac{5\,b\,x^4\cosh(c+d\,x)}{d^2}+\frac{b\,x^5\sinh(c+d\,x)}{d}$$

```
[In] int(x^3*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x^3*sinh(c + d*x)*(20*b + a*d^2))/d^3 - (3*x^2*cosh(c + d*x)*(20*b + a*d^2
))/d^4 - (6*cosh(c + d*x)*(20*b + a*d^2))/d^6 + (6*x*sinh(c + d*x)*(20*b +
a*d^2))/d^5 - (5*b*x^4*cosh(c + d*x))/d^2 + (b*x^5*sinh(c + d*x))/d
```

3.41 $\int x^2(a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int x^2\left(a+bx^2\right)\cosh(c+dx)\,dx = -\frac{24bx\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{2a\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}$$

```
[Out] -24*b*x*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-4*b*x^3*cosh(d*x+c)/d^2+24*b*
sinh(d*x+c)/d^5+2*a*sinh(d*x+c)/d^3+12*b*x^2*sinh(d*x+c)/d^3+a*x^2*sinh(d*x
+c)/d+b*x^4*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5395, 3377, 2717}

$$\int x^2\left(a+bx^2\right)\cosh(c+dx)\,dx = \frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c +
 d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x^2
*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^2\cosh(c+dx)+bx^4\cosh(c+dx)\right)\,dx\\
&= a\int x^2\cosh(c+dx)\,dx+b\int x^4\cosh(c+dx)\,dx\\
&= \frac{ax^2\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(2a)\int x\sinh(c+dx)\,dx}{d}-\frac{(4b)\int x^3\sinh(c+dx)\,dx}{d}\\
&= -\frac{2ax\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}\\
&\quad+\frac{bx^4\sinh(c+dx)}{d}+\frac{(2a)\int\cosh(c+dx)\,dx}{d^2}+\frac{(12b)\int x^2\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{2ax\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3}\\
&\quad+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(24b)\int x\sinh(c+dx)\,dx}{d^3}\\
&= -\frac{24bx\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}\\
&\quad+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}+\frac{(24b)\int\cosh(c+dx)\,dx}{d^4}
\end{aligned}
$$

$$= -\frac{24bx\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c+dx)}{d^5} + \frac{2a\sinh(c+dx)}{d^3} + \frac{12bx^2\sinh(c+dx)}{d^3} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \frac{-2dx(ad^2+2b(6+d^2x^2))\cosh(c+dx)+(ad^2(2+d^2x^2)+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

```
[In] Integrate[x^2*(a + b*x^2)*Cosh[c + d*x],x]
[Out] (-2*d*x*(a*d^2 + 2*b*(6 + d^2*x^2))*Cosh[c + d*x] + (a*d^2*(2 + d^2*x^2) +
b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{2d((2bx^2+a)d^2+12b)x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2((-bx^4-ax^2)d^4+2(-6bx^2-a)d^2-24b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2d((2bx^2+a)d^2+1}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(bx^4d^4+ad^4x^2-4bd^3x^3-2ad^3x+12bd^2x^2+2ad^2-24dxb+24b)e^{dx+c}}{2d^5}-\frac{(bx^4d^4+ad^4x^2+4bd^3x^3+2ad^3x+12bd^2x^2}{2d^5}$
parts	$\frac{bx^4\sinh(dx+c)}{d}+\frac{ax^2\sinh(dx+c)}{d}-\frac{2\left(-\frac{2bc^3\cosh(dx+c)}{d^3}+\frac{6bc^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^3}-\frac{6bc\left((dx+c)^2\cosh(dx\right.}{}\right.}{}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\frac{9}{2}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{bc^4\sinh(dx+c)}{d^2}-\frac{4bc^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{6bc^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4bc\left((d\right.}{}}{}$
default	$\frac{\frac{bc^4\sinh(dx+c)}{d^2}-\frac{4bc^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{6bc^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4bc\left((d\right.}{}}{}$

```
[In] int(x^2*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
[Out] 2*(d*((2*b*x^2+a)*d^2+12*b)*x*tanh(1/2*d*x+1/2*c)^2+((-b*x^4-a*x^2)*d^4+2*(
-6*b*x^2-a)*d^2-24*b)*tanh(1/2*d*x+1/2*c)+d*((2*b*x^2+a)*d^2+12*b)*x)/d^5/(
tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = -\frac{2\,(2\,bd^3x^3+(ad^3+12\,bd)x)\cosh{(dx+c)}-(bd^4x^4+2\,ad^2+(ad^4+12\,bd^2)x^2+24\,b)\sinh{(dx+c)}}{d^5}$$

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(2*b*d^3*x^3 + (a*d^3 + 12*b*d)*x)*cosh(d*x + c) - (b*d^4*x^4 + 2*a*d^2
 + (a*d^4 + 12*b*d^2)*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^2\sinh{(c+dx)}}{d} - \frac{2ax\cosh{(c+dx)}}{d^2} + \frac{2a\sinh{(c+dx)}}{d^3} + \frac{bx^4\sinh{(c+dx)}}{d} - \frac{4bx^3\cosh{(c+dx)}}{d^2} + \frac{12bx^2\sinh{(c+dx)}}{d^3} - \frac{24bx\cosh{(c+dx)}}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^5}{5}\right)\cosh{(c)} \end{cases}$$

```
[In] integrate(x**2*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x*
*2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5
, Ne(d, 0)), ((a*x**3/3 + b*x**5/5)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.96

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = -\frac{1}{30}\,d\left(\frac{5\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)ae^{(dx)}}{d^4}+\frac{5\,(d^3x^3+3\,d^2x^2+6\,dx+6)ae^{(-dx-c)}}{d^4}+\frac{3\,(d^5x^5e^c-\right. + \frac{1}{15}\,(3\,bx^5+5\,ax^3)\cosh{(dx+c)}$$

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/30*d*(5*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
+ 5*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^5*x^5*e^c -
 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b
*e^(d*x)/d^6 + 3*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x +
 120)*b*e^(-d*x - c)/d^6) + 1/15*(3*b*x^5 + 5*a*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^4x^4+ad^4x^2-4\,bd^3x^3-2\,ad^3x+12\,bd^2x^2+2\,ad^2-24\,bdx+24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4+ad^4x^2+4\,bd^3x^3+2\,ad^3x+12\,bd^2x^2+2\,ad^2+24\,bdx+24\,b)e^{(-dx-c)}}{2\,d^5}$$

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^2 - 4*b*d^3*x^3 - 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d
^2 - 24*b*d*x + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*x^2 + 4*b*d^
3*x^3 + 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d^2 + 24*b*d*x + 24*b)*e^(-d*x - c)/
d^5
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \frac{2\sinh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^5} + \frac{x^2\sinh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^3} - \frac{2\,x\cosh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^4} - \frac{4\,b\,x^3\cosh(c+d\,x)}{d^2} + \frac{b\,x^4\sinh(c+d\,x)}{d}$$

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (2*sinh(c + d*x)*(12*b + a*d^2))/d^5 + (x^2*sinh(c + d*x)*(12*b + a*d^2))/d
^3 - (2*x*cosh(c + d*x)*(12*b + a*d^2))/d^4 - (4*b*x^3*cosh(c + d*x))/d^2 +
 (b*x^4*sinh(c + d*x))/d
```

3.42 $\int x(a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int x(a+bx^2)\cosh(c+dx)\,dx = -\frac{6b\cosh(c+dx)}{d^4} - \frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d}$$

```
[Out] -6*b*cosh(d*x+c)/d^4-a*cosh(d*x+c)/d^2-3*b*x^2*cosh(d*x+c)/d^2+6*b*x*sinh(d
*x+c)/d^3+a*x*sinh(d*x+c)/d+b*x^3*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5395, 3377, 2718}

$$\int x(a+bx^2)\cosh(c+dx)\,dx = -\frac{a\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} - \frac{6b\cosh(c+dx)}{d^4} + \frac{6bx\sinh(c+dx)}{d^3} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{bx^3\sinh(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/
d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d
*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax\cosh(c+dx)+bx^3\cosh(c+dx)\right)\,dx\\
&= a\int x\cosh(c+dx)\,dx+b\int x^3\cosh(c+dx)\,dx\\
&= \frac{ax\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}-\frac{a\int\sinh(c+dx)\,dx}{d}-\frac{(3b)\int x^2\sinh(c+dx)\,dx}{d}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}\\
&\quad+\frac{bx^3\sinh(c+dx)}{d}+\frac{(6b)\int x\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}\\
&\quad+\frac{ax\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}-\frac{(6b)\int\sinh(c+dx)\,dx}{d^3}\\
&= -\frac{6b\cosh(c+dx)}{d^4}-\frac{a\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}\\
&\quad+\frac{6bx\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int x\left(a+bx^2\right)\cosh(c+dx)\,dx\\
&=\frac{-\left(\left(ad^2+3b(2+d^2x^2)\right)\cosh(c+dx)\right)+dx\left(ad^2+b(6+d^2x^2)\right)\sinh(c+dx)}{d^4}
\end{aligned}$$

```
[In] Integrate[x*(a + b*x^2)*Cosh[c + d*x],x]
[Out] (-((a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*x*(a*d^2 + b*(6 + d^2*x^2
))*Sinh[c + d*x])/d^4
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{3b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 x^2 d^2-2dx\left(\left(b\,x^2+a\right)d^2+6b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3b\,x^2+2a\right)d^2+12b}{d^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,d^3x^3+a\,d^3x-3b\,d^2x^2-a\,d^2+6dxb-6b\right)\mathrm{e}^{dx+c}}{2d^4}-\frac{\left(b\,d^3x^3+a\,d^3x+3b\,d^2x^2+a\,d^2+6dxb+6b\right)\mathrm{e}^{-dx-c}}{2d^4}$
parts	$\frac{b\,x^3\sinh(dx+c)}{d}+\frac{ax\sinh(dx+c)}{d}-\frac{\frac{3b\,c^2\cosh(dx+c)}{d^2}-\frac{6bc((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^2}+\frac{3b\left((dx+c)^2\cosh(dx+c)-2(dx+\right.}{d^2}}{d^2}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i\left(15x\right.}{}\right.}{d^4}$
derivativedivides	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx\right.}{}}{d^2}$
default	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx\right.}{}}{d^2}$

```
[In] int(x*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (3*b*tanh(1/2*d*x+1/2*c)^2*x^2*d^2-2*d*x*((b*x^2+a)*d^2+6*b)*tanh(1/2*d*x+1
/2*c)+(3*b*x^2+2*a)*d^2+12*b)/d^4/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int x\left(a+bx^2\right)\cosh(c+dx)\,dx = -\frac{\left(3\,bd^2x^2+ad^2+6\,b\right)\cosh\left(dx+c\right)-\left(bd^3x^3+\left(ad^3+6\,bd\right)x\right)\sinh\left(dx+c\right)}{d^4}$$

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b*d^2*x^2 + a*d^2 + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + (a*d^3 + 6*b*d)*
x)*sinh(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int x(a+bx^2)\cosh(c+dx)\,dx$$
$$= \begin{cases} \frac{ax\sinh(c+dx)}{d} - \frac{a\cosh(c+dx)}{d^2} + \frac{bx^3\sinh(c+dx)}{d} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} - \frac{6b\cosh(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^4}{4}\right)\cosh(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(b*x**2+a)*cosh(d*x+c),x)
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**3*sinh(c + d*x
)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c +
 d*x)/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(79) = 158.

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.70

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{(bx^2+a)^2\cosh(dx+c)}{4\,b} - \frac{\left(\frac{a^2e^{(dx+c)}}{d} + \frac{a^2e^{(-dx-c)}}{d} + \frac{2\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)abe^{(dx)}}{d^3} + \frac{2\,(d^2x^2 + 2\,dx + 2)abe^{(-dx-c)}}{d^3} + \frac{(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe}{d^5}\right.}{8\,b}$$

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
[Out] 1/4*(b*x^2 + a)^2*cosh(d*x + c)/b - 1/8*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
c)/d + 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^(d*x)/d^3 + 2*(d^2*x^2 + 2
*d*x + 2)*a*b*e^(-d*x - c)/d^3 + (d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*
e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*
x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^3x^3 + ad^3x - 3\,bd^2x^2 - ad^2 + 6\,bdx - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + ad^3x + 3\,bd^2x^2 + ad^2 + 6\,bdx + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 + a*d^3*x - 3*b*d^2*x^2 - a*d^2 + 6*b*d*x - 6*b)*e^(d*x + c)
/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x + 3*b*d^2*x^2 + a*d^2 + 6*b*d*x + 6*b)*e^(-
d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x(a+bx^2)\cosh(c+dx)\,dx=\frac{x\sinh(c+d\,x)\,\left(a\,d^2+6\,b\right)}{d^3}-\frac{\cosh(c+d\,x)\,\left(a\,d^2+6\,b\right)}{d^4}-\frac{3\,b\,x^2\cosh(c+d\,x)}{d^2}+\frac{b\,x^3\sinh(c+d\,x)}{d}$$

```
[In] int(x*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x*sinh(c + d*x)*(6*b + a*d^2))/d^3 - (cosh(c + d*x)*(6*b + a*d^2))/d^4 - (
3*b*x^2*cosh(c + d*x))/d^2 + (b*x^3*sinh(c + d*x))/d
```

3.43 $\int (a+bx^2) \cosh(c+dx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int (a+bx^2) \cosh(c+dx) \, dx = -\frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} + \frac{a\sinh(c+dx)}{d} + \frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] -2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*sinh(d*x+c)/d+b*x^2*sinh(d*x+c
)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5385, 2717, 3377}

$$\int (a+bx^2) \cosh(c+dx) \, dx = \frac{a\sinh(c+dx)}{d} + \frac{2b\sinh(c+dx)}{d^3} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{bx^2\sinh(c+dx)}{d}$$

```
[In] Int[(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d
+ (b*x^2*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a\cosh(c+dx)+bx^2\cosh(c+dx)\right)\,dx \\
&= a\int \cosh(c+dx)\,dx+b\int x^2\cosh(c+dx)\,dx \\
&= \frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d} \\
&= -\frac{2bx\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}+\frac{(2b)\int \cosh(c+dx)\,dx}{d^2} \\
&= -\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \left(a+bx^2\right)\cosh(c+dx)\,dx=\frac{-2bdx\cosh(c+dx)+\left(ad^2+b\left(2+d^2x^2\right)\right)\sinh(c+dx)}{d^3}$$

```
[In] Integrate[(a + b*x^2)*Cosh[c + d*x],x]
[Out] (-2*b*d*x*Cosh[c + d*x] + (a*d^2 + b*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result
parts	$\frac{b\,x^{2}\sinh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}-\frac{2b((dx+c)\cosh(dx+c)-\sinh(dx+c)-c\cosh(dx+c))}{d^{3}}$
parallelrisch	$\frac{2x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}bd+2\left(\left(-b\,x^{2}-a\right)d^{2}-2b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2dxb}{d^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,d^{2}x^{2}+a\,d^{2}-2dxb+2b\right){\mathrm e}^{dx+c}}{2d^{3}}-\frac{\left(b\,d^{2}x^{2}+a\,d^{2}+2dxb+2b\right){\mathrm e}^{-dx-c}}{2d^{3}}$
derivativedivides	$\frac{\frac{b\,c^{2}\sinh(dx+c)}{d^{2}}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}+\frac{b\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}+a\sinh(dx+c}{d}$
default	$\frac{\frac{b\,c^{2}\sinh(dx+c)}{d^{2}}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}+\frac{b\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}+a\sinh(dx+c}{d}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^{2}d^{2}}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^{3}}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^{2}d^{2}}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{3}}+$

```
[In] int((b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*x^2*sinh(d*x+c)/d+a*sinh(d*x+c)/d-2/d^3*b*((d*x+c)*cosh(d*x+c)-sinh(d*x+c
)-c*cosh(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int (a+bx^2)\cosh(c+dx)\,dx=-\frac{2\,bdx\cosh\left(dx+c\right)-\left(bd^2x^2+ad^2+2\,b\right)\sinh\left(dx+c\right)}{d^3}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*b*d*x*cosh(d*x + c) - (b*d^2*x^2 + a*d^2 + 2*b)*sinh(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int (a+bx^2)\cosh(c+dx)\,dx$$
$$=\begin{cases}\frac{a\sinh\left(c+dx\right)}{d}+\frac{bx^2\sinh\left(c+dx\right)}{d}-\frac{2bx\cosh\left(c+dx\right)}{d^2}+\frac{2b\sinh\left(c+dx\right)}{d^3} & \text{for}\ d\neq 0\\ \left(ax+\frac{bx^3}{3}\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**2*sinh(c + d*x)/d - 2*b*x*cosh(c + d*x)
/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*cosh(c), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{ae^{(dx+c)}}{2\,d} - \frac{ae^{(-dx-c)}}{2\,d} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)be^{(dx)}}{2\,d^3} - \frac{(d^2x^2 + 2\,dx + 2)be^{(-dx-c)}}{2\,d^3}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c +
 2*e^c)*b*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^2x^2 + ad^2 - 2\,bdx + 2\,b)e^{(dx+c)}}{2\,d^3} - \frac{(bd^2x^2 + ad^2 + 2\,bdx + 2\,b)e^{(-dx-c)}}{2\,d^3}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + a*d^2 - 2*b*d*x + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^2*x^2 +
a*d^2 + 2*b*d*x + 2*b)*e^(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a\,d^2+2\,b\right)}{d^3} - \frac{2\,b\,x\cosh(c+d\,x)}{d^2} + \frac{b\,x^2\sinh(c+d\,x)}{d}$$

```
[In] int(cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (sinh(c + d*x)*(2*b + a*d^2))/d^3 - (2*b*x*cosh(c + d*x))/d^2 + (b*x^2*sinh
(c + d*x))/d
```

3.44 $\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = -\frac{b\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{bx\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)$$

```
[Out] a*Chi(d*x)*cosh(c)-b*cosh(d*x+c)/d^2+a*Shi(d*x)*sinh(c)+b*x*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5395, 3384, 3379, 3382, 3377, 2718}

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx) + a\sinh(c)\text{Shi}(dx) - \frac{b\cosh(c+dx)}{d^2} + \frac{bx\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) + a*Cosh[c]*CoshIntegral[d*x] + (b*x*Sinh[c + d*x]
)/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x} + bx\cosh(c+dx)\right) dx \\
&= a\int \frac{\cosh(c+dx)}{x}\,dx + b\int x\cosh(c+dx)\,dx \\
&= \frac{bx\sinh(c+dx)}{d} - \frac{b\int \sinh(c+dx)\,dx}{d} \\
&\quad + (a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + (a\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{b\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{bx\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx)+\frac{b\cosh(dx)(-\cosh(c)+dx\sinh(c))}{d^2}+\frac{b(dx\cosh(c)-\sinh(c))\sinh(dx)}{d^2}+a\sinh(c)\text{Shi}(dx)$$

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*(-Cosh[c] + d*x*Sinh[c]))/d^2 +
(b*(d*x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2 + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{a\,\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)}{2}-\frac{a\,\mathrm{e}^{c}\,\mathrm{Ei}_1(-dx)}{2}-\frac{\mathrm{e}^{-dx-c}bx}{2d}+\frac{\mathrm{e}^{dx+c}bx}{2d}-\frac{\mathrm{e}^{-dx-c}b}{2d^2}-\frac{\mathrm{e}^{dx+c}b}{2d^2}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{a\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}+\frac{2\ \mathrm{Chi}(dx)-}{}\right.}{2}$

```
[In] int((b*x^2+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*exp(-c)*Ei(1,d*x)-1/2*a*exp(c)*Ei(1,-d*x)-1/2/d*exp(-d*x-c)*b*x+1/2/
d*exp(d*x+c)*b*x-1/2/d^2*exp(-d*x-c)*b-1/2/d^2*exp(d*x+c)*b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = \frac{2\,bdx\sinh(dx+c)-2\,b\cosh(dx+c)+(ad^2\text{Ei}(dx)+ad^2\text{Ei}(-dx))\cosh(c)+(ad^2\text{Ei}(dx)-ad^2\text{Ei}(-dx))\,\text{s}}{2\,d^2}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*sinh(d*x + c) - 2*b*cosh(d*x + c) + (a*d^2*Ei(d*x) + a*d^2*Ei(
-d*x))*cosh(c) + (a*d^2*Ei(d*x) - a*d^2*Ei(-d*x))*sinh(c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\sinh{(c)}\operatorname{Shi}{(dx)} + a\cosh{(c)}\operatorname{Chi}{(dx)} + b\left(\begin{cases} \frac{x\sinh{(c+dx)}}{d} - \frac{\cosh{(c+dx)}}{d^2} & \text{for}\ d \neq 0 \\ \frac{x^2\cosh{(c)}}{2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x,x)
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x*sinh(c + d*x)/d -
cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(41) = 82.

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.98

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = -\frac{1}{4}\left(b\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)e^{(-dx-c)}}{d^3}\right) + \frac{2\,a\cosh{(dx+c)}\log{(x^2)}}{d} - \frac{2\,(\operatorname{Ei}(\right. + \frac{1}{2}\left(bx^2 + a\log{\left(x^2\right)}\right)\cosh{(dx+c)}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="maxima")
[Out] -1/4*(b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x +
 2)*e^(-d*x - c)/d^3) + 2*a*cosh(d*x + c)*log(x^2)/d - 2*(Ei(-d*x)*e^(-c) +
 Ei(d*x)*e^c)*a/d)*d + 1/2*(b*x^2 + a*log(x^2))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = \frac{ad^2\operatorname{Ei}(-dx)\,e^{(-c)} + ad^2\operatorname{Ei}(dx)\,e^c + bdxe^{(dx+c)} - bdxe^{(-dx-c)} - be^{(dx+c)} - be^{(-dx-c)}}{2\,d^2}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="giac")
[Out] 1/2*(a*d^2*Ei(-d*x)*e^(-c) + a*d^2*Ei(d*x)*e^c + b*d*x*e^(d*x + c) - b*d*x*
e^(-d*x - c) - b*e^(d*x + c) - b*e^(-d*x - c))/d^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\,\mathrm{coshint}(d\,x)\,\cosh(c) + a\,\mathrm{sinhint}(d\,x)\,\sinh(c) - \frac{b\,(\cosh(c+d\,x) - d\,x\sinh(c+d\,x))}{d^2}$$

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) - (b*(cosh(c + d*x) - d*x*s
inh(c + d*x)))/d^2
```

3.45 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c+dx)}{x} + ad\text{Chi}(dx)\sinh(c) + \frac{b\sinh(c+dx)}{d} + ad\cosh(c)\text{Shi}(dx)$$

```
[Out] -a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c)+b*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5395, 2717, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = ad\sinh(c)\text{Chi}(dx) + ad\cosh(c)\text{Shi}(dx) - \frac{a\cosh(c+dx)}{x} + \frac{b\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x^2}\right) dx\\
&= a\int \frac{\cosh(c+dx)}{x^2}\,dx+b\int \cosh(c+dx)\,dx\\
&= -\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}+(ad)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}+(ad\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(ad\sinh(c))\int \frac{\cosh(dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{b\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c+dx)}{x} + ad\text{Chi}(dx)\sinh(c) + \frac{b\sinh(c+dx)}{d} + ad\cosh(c)\text{Shi}(dx)$$

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)a\,d^2x - e^{-c}\,\text{Ei}_1(dx)a\,d^2x + d\,e^{-dx-c}a + e^{-dx-c}bx + ad\,e^{dx+c} - e^{dx+c}bx}{2dx}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d} - \frac{b\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d} + \frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}} - \frac{4i\,\text{Shi}(dx)}{\sqrt{\pi}}\right)}{4} + \frac{a\sinh(c)\sqrt{\pi}\,d\left(\frac{4\gamma-4+4\ln(x)+4\ln}{\sqrt{\pi}}\right.}{}$

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(exp(c)*Ei(1,-d*x)*a*d^2*x-exp(-c)*Ei(1,d*x)*a*d^2*x+d*exp(-d*x-c)*a
+exp(-d*x-c)*b*x+a*d*exp(d*x+c)-exp(d*x+c)*b*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{2\,ad\cosh(dx+c) - 2\,bx\sinh(dx+c) - (ad^2x\text{Ei}(dx) - ad^2x\text{Ei}(-dx))\cosh(c) - (ad^2x\text{Ei}(dx) + ad^2x\text{E}}{2\,dx}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cosh(d*x + c) - 2*b*x*sinh(d*x + c) - (a*d^2*x*Ei(d*x) - a*d^2*
x*Ei(-d*x))*cosh(c) - (a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx^2)\cosh{(c+dx)}}{x^2}\,dx$$

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**2,x)
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx \\ = -\frac{1}{2}\left(a\text{Ei}(-dx)\,e^{(-c)} - a\text{Ei}(dx)\,e^{c} + \frac{(dxe^{c}-e^{c})be^{(dx)}}{d^2} + \frac{(dx+1)be^{(-dx-c)}}{d^2}\right)d \\ + \left(bx - \frac{a}{x}\right)\cosh{(dx+c)}\end{aligned}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
[Out] -1/2*(a*Ei(-d*x)*e^(-c) - a*Ei(d*x)*e^c + (d*x*e^c - e^c)*b*e^(d*x)/d^2 + (
d*x + 1)*b*e^(-d*x - c)/d^2)*d + (b*x - a/x)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx \\ = -\frac{ad^2x\text{Ei}(-dx)\,e^{(-c)} - ad^2x\text{Ei}(dx)\,e^{c} + ade^{(dx+c)} - bxe^{(dx+c)} + ade^{(-dx-c)} + bxe^{(-dx-c)}}{2\,dx}\end{aligned}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
[Out] -1/2*(a*d^2*x*Ei(-d*x)*e^(-c) - a*d^2*x*Ei(d*x)*e^c + a*d*e^(d*x + c) - b*x
*e^(d*x + c) + a*d*e^(-d*x - c) + b*x*e^(-d*x - c))/(d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^2}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^2,x)
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^2, x)
```

3.46 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = -\frac{a\cosh(c+dx)}{2x^2} + b\cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{2x} + b\sinh(c)\text{Shi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

```
[Out] b*Chi(d*x)*cosh(c)+1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2+b*Shi(d
*x)*sinh(c)+1/2*a*d^2*Shi(d*x)*sinh(c)-1/2*a*d*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx) - \frac{a\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{2x} + b\cosh(c)\text{Chi}(dx) + b\sinh(c)\text{Shi}(dx)$$

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x])/x^2 + b*Cosh[c]*CoshIntegral[d*x] + (a*d^2*Cosh[c]*C
oshIntegral[d*x])/2 - (a*d*Sinh[c + d*x])/(2*x) + b*Sinh[c]*SinhIntegral[d*
x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^3}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&\quad +(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\text{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x} \\
&\quad +b\sinh(c)\text{Shi}(dx)+\frac{1}{2}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x}\,dx
\end{aligned}$$

$$= -\frac{a\cosh(c+dx)}{2x^2} + b\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{2x} + b\sinh(c)\text{Shi}(dx) + \frac{1}{2}\left(ad^2\cosh(c)\right)\int\frac{\cosh(dx)}{x}\,dx + \frac{1}{2}\left(ad^2\sinh(c)\right)\int\frac{\sinh(dx)}{x}\,dx$$

$$= -\frac{a\cosh(c+dx)}{2x^2} + b\cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{2x} + b\sinh(c)\text{Shi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int\frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = b\cosh(c)\text{Chi}(dx) - \frac{a\cosh(dx)(\cosh(c)+dx\sinh(c))}{2x^2} - \frac{a(dx\cosh(c)+\sinh(c))\sinh(dx)}{2x^2} + b\sinh(c)\text{Shi}(dx) + \frac{1}{2}ad^2(\cosh(c)\text{Chi}(dx)+\sinh(c)\text{Shi}(dx))$$

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

```
[Out] b*Cosh[c]*CoshIntegral[d*x] - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2)
 - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + b*Sinh[c]*SinhIntegral[d
*x] + (a*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)a\,d^2x^2+e^{-c}\,\text{Ei}_1(dx)a\,d^2x^2+2\,e^{c}\,\text{Ei}_1(-dx)b\,x^2+2\,e^{-c}\,\text{Ei}_1(dx)b\,x^2-e^{-dx-c}adx+e^{dx+c}adx+e^{-dx-c}a+a\,e^{dx+c}}{4x^2}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}+\frac{2\ \text{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)}{2} + b\ \text{Shi}\,(dx)\sinh\,(c) - \frac{a\cosh(c)\sqrt{\pi}\,d^2\Bigg(\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2(2\gamma-3+2\ln(x)+}{\sqrt{\pi}}}{}$

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(exp(c)*Ei(1,-d*x)*a*d^2*x^2+exp(-c)*Ei(1,d*x)*a*d^2*x^2+2*exp(c)*Ei(1
,-d*x)*b*x^2+2*exp(-c)*Ei(1,d*x)*b*x^2-exp(-d*x-c)*a*d*x+exp(d*x+c)*a*d*x+e
xp(-d*x-c)*a+a*exp(d*x+c))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = -\frac{2\,adx\sinh(dx+c)+2\,a\cosh(dx+c)-((ad^2+2\,b)x^2\mathrm{Ei}(dx)+(ad^2+2\,b)x^2\mathrm{Ei}(-dx))\cosh(c)-((ad}{4\,x^2}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((a*d^2 + 2*b)*x^2*Ei(d*x
) + (a*d^2 + 2*b)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) - (a*d
^2 + 2*b)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$$

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a-\frac{2\,b\cosh(dx+c)\log(x^2)}{d}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b}{d}\right)d + \frac{1}{2}\left(b\log(x^2)-\frac{a}{x^2}\right)\cosh(dx+c)$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a - 2*b*cosh(d*x + c
)*log(x^2)/d + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/2*(b*log(x^2) -
 a/x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{ad^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + ad^2x^2\mathrm{Ei}(dx)\,e^{c} + 2\,bx^2\mathrm{Ei}(-dx)\,e^{(-c)} + 2\,bx^2\mathrm{Ei}(dx)\,e^{c} - adxe^{(dx+c)} + adxe^{(-dx-c)} - a}{4\,x^2}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c + 2*b*x^2*Ei(-d*x)*e
^(-c) + 2*b*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) - a*e^
(d*x + c) - a*e^(-d*x - c))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\ \left(b\,x^2+a\right)}{x^3}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^3, x)
```

3.47 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 105

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\mathrm{Chi}(dx)\sinh(c) + \frac{1}{6}ad^3\mathrm{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{6x^2} + bd\cosh(c)\mathrm{Shi}(dx) + \frac{1}{6}ad^3\cosh(c)\mathrm{Shi}(dx)$$

```
[Out] -1/3*a*cosh(d*x+c)/x^3-b*cosh(d*x+c)/x-1/6*a*d^2*cosh(d*x+c)/x+b*d*cosh(c)*
Shi(d*x)+1/6*a*d^3*cosh(c)*Shi(d*x)+b*d*Chi(d*x)*sinh(c)+1/6*a*d^3*Chi(d*x)
*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}ad^3\sinh(c)\mathrm{Chi}(dx) + \frac{1}{6}ad^3\cosh(c)\mathrm{Shi}(dx) - \frac{ad^2\cosh(c+dx)}{6x} - \frac{a\cosh(c+dx)}{3x^3} - \frac{ad\sinh(c+dx)}{6x^2} + bd\sinh(c)\mathrm{Chi}(dx) + bd\cosh(c)\mathrm{Shi}(dx) - \frac{b\cosh(c+dx)}{x}$$

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a*Cosh[c + d*x])/x^3 - (b*Cosh[c + d*x])/x - (a*d^2*Cosh[c + d*x])/(6
*x) + b*d*CoshIntegral[d*x]*Sinh[c] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 -
```

```
(a*d*Sinh[c + d*x])/(6*x^2) + b*d*Cosh[c]*SinhIntegral[d*x] + (a*d^3*Cosh[
c]*SinhIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\cosh(c+dx)}{x^4}\,dx+b\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx+(bd)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}-\frac{ad\sinh(c+dx)}{6x^2}+\frac{1}{6}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&\quad+(bd\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(bd\sinh(c))\int \frac{\cosh(dx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\text{Chi}(dx)\sinh(c) \\
&\quad - \frac{ad\sinh(c+dx)}{6x^2} + bd\cosh(c)\text{Shi}(dx) + \frac{1}{6}\left(ad^3\right)\int \frac{\sinh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{6x^2} \\
&\quad + bd\cosh(c)\text{Shi}(dx) + \frac{1}{6}\left(ad^3\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx + \frac{1}{6}\left(ad^3\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\text{Chi}(dx)\sinh(c) \\
&\quad + \frac{1}{6}ad^3\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{6x^2} + bd\cosh(c)\text{Shi}(dx) \\
&\quad + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$
\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = -\frac{2a\cosh(c+dx) + 6bx^2\cosh(c+dx) + ad^2x^2\cosh(c+dx) - d(6b+ad^2)\,x^3\text{Chi}(dx)\sinh(c) + adx\sinh(}{6x^3}
$$

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a*Cosh[c + d*x] + 6*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] -
 d*(6*b + a*d^2)*x^3*CoshIntegral[d*x]*Sinh[c] + a*d*x*Sinh[c + d*x] - d*(6
*b + a*d^2)*x^3*Cosh[c]*SinhIntegral[d*x])/x^3
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)a\,d^3x^3 - e^{-c}\,\text{Ei}_1(dx)a\,d^3x^3 + 6\,e^{c}\,\text{Ei}_1(-dx)bd\,x^3 - 6\,e^{-c}\,\text{Ei}_1(dx)bd\,x^3 + e^{-dx-c}a\,d^2x^2 + e^{dx+c}a\,d^2x^2 - e^{-dx-c}adx + 6\,e^{-}}{12x^3}$
meijerg	$\frac{idb\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}} - \frac{4i\,\text{Shi}(dx)}{\sqrt{\pi}}\right)}{4} + \frac{db\sinh(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)+4\ln(id)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4\sinh(dx)}{\sqrt{\pi}\,xd} + \frac{4\,\text{Chi}(dx)-4\ln(dx)-4\gamma}{\sqrt{\pi}}\right)}{4} - \frac{ia\cosh}{}$

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(exp(c)*Ei(1,-d*x)*a*d^3*x^3-exp(-c)*Ei(1,d*x)*a*d^3*x^3+6*exp(c)*Ei(
1,-d*x)*b*d*x^3-6*exp(-c)*Ei(1,d*x)*b*d*x^3+exp(-d*x-c)*a*d^2*x^2+exp(d*x+c
```

```
)*a*d^2*x^2-exp(-d*x-c)*a*d*x+6*exp(-d*x-c)*b*x^2+exp(d*x+c)*a*d*x+6*exp(d*
x+c)*b*x^2+2*exp(-d*x-c)*a+2*a*exp(d*x+c))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = -\frac{2\,adx\sinh\left(dx+c\right)+2\left((ad^2+6\,b)x^2+2\,a\right)\cosh\left(dx+c\right)-\left((ad^3+6\,bd)x^3\mathrm{Ei}(dx)-(ad^3+6\,bd)x^3\mathrm{Ei}(-\right.}{12\,x^3}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*((a*d^2 + 6*b)*x^2 + 2*a)*cosh(d*x + c) -
((a*d^3 + 6*b*d)*x^3*Ei(d*x) - (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*cosh(c) - ((a*
d^3 + 6*b*d)*x^3*Ei(d*x) + (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)\cosh\left(c+dx\right)}{x^4}\,dx$$

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(ad^2e^{(-c)}\Gamma(-2,dx)-ad^2e^c\Gamma(-2,-dx)-3\,b\mathrm{Ei}(-dx)\,e^{(-c)}+3\,b\mathrm{Ei}(dx)\,e^c\right)d - \frac{(3\,bx^2+a)\cosh\left(dx+c\right)}{3\,x^3}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a*d^2*e^(-c)*gamma(-2, d*x) - a*d^2*e^c*gamma(-2, -d*x) - 3*b*Ei(-d*x)
*e^(-c) + 3*b*Ei(d*x)*e^c)*d - 1/3*(3*b*x^2 + a)*cosh(d*x + c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = -\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^{c} + 6\,bdx^3\mathrm{Ei}(-dx)\,e^{(-c)} - 6\,bdx^3\mathrm{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2x^2e^{\cdots}}{12\,x^3}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + 6*b*d*x^3*Ei(-d*
x)*e^(-c) - 6*b*d*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(-d
*x - c) + a*d*x*e^(d*x + c) + 6*b*x^2*e^(d*x + c) - a*d*x*e^(-d*x - c) + 6*
b*x^2*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^4}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^4, x)
```

3.48 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{24x^2}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{2x}-\frac{ad^3\sinh(c+dx)}{24x}+\frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)$$

```
[Out] 1/2*b*d^2*Chi(d*x)*cosh(c)+1/24*a*d^4*Chi(d*x)*cosh(c)-1/4*a*cosh(d*x+c)/x^
4-1/2*b*cosh(d*x+c)/x^2-1/24*a*d^2*cosh(d*x+c)/x^2+1/2*b*d^2*Shi(d*x)*sinh(
c)+1/24*a*d^4*Shi(d*x)*sinh(c)-1/12*a*d*sinh(d*x+c)/x^3-1/2*b*d*sinh(d*x+c)
/x-1/24*a*d^3*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{a\cosh(c+dx)}{4x^4}-\frac{ad\sinh(c+dx)}{12x^3}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{2x^2}-\frac{bd\sinh(c+dx)}{2x}$$

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a*Cosh[c + d*x])/x^4 - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(24*x^2) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^4*Cosh[c]*CoshInt
egral[d*x])/24 - (a*d*Sinh[c + d*x])/(12*x^3) - (b*d*Sinh[c + d*x])/(2*x) -
 (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a*d^
4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^3}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^5}\,dx+b\int \frac{\cosh(c+dx)}{x^3}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} + \frac{1}{4}(ad)\int \frac{\sinh(c+dx)}{x^4}\,dx + \frac{1}{2}(bd)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} \\
&\quad + \frac{1}{12}(ad^2)\int \frac{\cosh(c+dx)}{x^3}\,dx + \frac{1}{2}(bd^2)\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} \\
&\quad - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} + \frac{1}{24}(ad^3)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&\quad + \frac{1}{2}(bd^2\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + \frac{1}{2}(bd^2\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} \\
&\quad + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} \\
&\quad - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx) + \frac{1}{24}(ad^4)\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) \\
&\quad - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx) \\
&\quad + \frac{1}{24}(ad^4\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + \frac{1}{24}(ad^4\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) \\
&\quad + \frac{1}{24}ad^4\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} \\
&\quad - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx) + \frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$
\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \\
-\frac{6a\cosh(c+dx) + 12bx^2\cosh(c+dx) + ad^2x^2\cosh(c+dx) - d^2(12b+ad^2)\,x^4\cosh(c)\text{Chi}(dx) + 2adx\,\text{si}}{24x^4}
$$

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/24*(6*a*Cosh[c + d*x] + 12*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x]
 - d^2*(12*b + a*d^2)*x^4*Cosh[c]*CoshIntegral[d*x] + 2*a*d*x*Sinh[c + d*x]
 + 12*b*d*x^3*Sinh[c + d*x] + a*d^3*x^3*Sinh[c + d*x] - d^2*(12*b + a*d^2)*
x^4*Sinh[c]*SinhIntegral[d*x])/x^4
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{e^{c} \operatorname{Ei}_{1}(-dx) a\, d^{4} x^{4}+e^{-c} \operatorname{Ei}_{1}(dx) a\, d^{4} x^{4}+12\, e^{c} \operatorname{Ei}_{1}(-dx) b\, d^{2} x^{4}+12\, e^{-c} \operatorname{Ei}_{1}(dx) b\, d^{2} x^{4}-e^{-dx-c} a\, d^{3} x^{3}+e^{dx+c} a\, d^{3} x^{3}+e^{-dx-c} a\, d^{2} x}{48}$
meijerg	$-\frac{d^{2} b \cosh(c) \sqrt{\pi}\left(\frac{4}{\sqrt{\pi}\, x^{2} d^{2}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}-\frac{4\left(\frac{9x^{2}d^{2}}{2}+3\right)}{3\sqrt{\pi}\, x^{2} d^{2}}+\frac{4\cosh(dx)}{\sqrt{\pi}\, x^{2} d^{2}}+\frac{4\sinh(dx)}{\sqrt{\pi}\, x d}-\frac{4(\operatorname{Chi}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}\right)}{8}+\frac{i d^{2} b \sinh(c)}{}$

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(exp(c)*Ei(1,-d*x)*a*d^4*x^4+exp(-c)*Ei(1,d*x)*a*d^4*x^4+12*exp(c)*Ei
(1,-d*x)*b*d^2*x^4+12*exp(-c)*Ei(1,d*x)*b*d^2*x^4-exp(-d*x-c)*a*d^3*x^3+exp
(d*x+c)*a*d^3*x^3+exp(-d*x-c)*a*d^2*x^2-12*exp(-d*x-c)*b*d*x^3+exp(d*x+c)*a
*d^2*x^2+12*exp(d*x+c)*b*d*x^3-2*exp(-d*x-c)*a*d*x+12*exp(-d*x-c)*b*x^2+2*e
xp(d*x+c)*a*d*x+12*exp(d*x+c)*b*x^2+6*exp(-d*x-c)*a+6*a*exp(d*x+c))/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = -\frac{2\left((ad^2+12\,b)x^2+6\,a\right)\cosh\left(dx+c\right)-\left((ad^4+12\,bd^2)x^4\mathrm{Ei}(dx)+(ad^4+12\,bd^2)x^4\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+}{4}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a*d^2 + 12*b)*x^2 + 6*a)*cosh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4
*Ei(d*x) + (a*d^4 + 12*b*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*((a*d^3 + 12*b*d)*x
^3 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*Ei(d*x) - (a*d^4 + 12
*b*d^2)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)\cosh{(c+dx)}}{x^5}\,dx$$

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \frac{1}{8}\left(ad^3e^{(-c)}\Gamma(-3,dx)+ad^3e^c\Gamma(-3,-dx)+2\,bde^{(-c)}\Gamma(-1,dx)+2\,bde^c\Gamma(-1,-dx)\right)d - \frac{(2\,bx^2+a)\cosh{(dx+c)}}{4\,x^4}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(a*d^3*e^(-c)*gamma(-3, d*x) + a*d^3*e^c*gamma(-3, -d*x) + 2*b*d*e^(-c)
*gamma(-1, d*x) + 2*b*d*e^c*gamma(-1, -d*x))*d - 1/4*(2*b*x^2 + a)*cosh(d*x
 + c)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \frac{ad^4x^4\mathrm{Ei}(-dx)\,e^{(-c)}+ad^4x^4\mathrm{Ei}(dx)\,e^c+12\,bd^2x^4\mathrm{Ei}(-dx)\,e^{(-c)}+12\,bd^2x^4\mathrm{Ei}(dx)\,e^c-ad^3x^3e^{(dx+c)}+ad^3x^3e}{}$$

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c + 12*b*d^2*x^4*Ei(-
d*x)*e^(-c) + 12*b*d^2*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*
e^(-d*x - c) - a*d^2*x^2*e^(d*x + c) - 12*b*d*x^3*e^(d*x + c) - a*d^2*x^2*e
^(-d*x - c) + 12*b*d*x^3*e^(-d*x - c) - 2*a*d*x*e^(d*x + c) - 12*b*x^2*e^(d
*x + c) + 2*a*d*x*e^(-d*x - c) - 12*b*x^2*e^(-d*x - c) - 6*a*e^(d*x + c) -
6*a*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^5}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^5, x)
```

3.49 $\int x^2(a+bx^2)^2\cosh(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = -\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{720b^2\sinh(c+dx)}{d^7}+\frac{48ab\sinh(c+dx)}{d^5}+\frac{2a^2\sinh(c+dx)}{d^3}+\frac{360b^2x^2\sinh(c+dx)}{d^5}+\frac{24abx^2\sinh(c+dx)}{d^3}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh(c+dx)}{d^3}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d}$$

```
[Out] -720*b^2*x*cosh(d*x+c)/d^6-48*a*b*x*cosh(d*x+c)/d^4-2*a^2*x*cosh(d*x+c)/d^2
-120*b^2*x^3*cosh(d*x+c)/d^4-8*a*b*x^3*cosh(d*x+c)/d^2-6*b^2*x^5*cosh(d*x+c
)/d^2+720*b^2*sinh(d*x+c)/d^7+48*a*b*sinh(d*x+c)/d^5+2*a^2*sinh(d*x+c)/d^3+
360*b^2*x^2*sinh(d*x+c)/d^5+24*a*b*x^2*sinh(d*x+c)/d^3+a^2*x^2*sinh(d*x+c)/
d+30*b^2*x^4*sinh(d*x+c)/d^3+2*a*b*x^4*sinh(d*x+c)/d+b^2*x^6*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5395, 3377, 2717}

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = \frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh(c+dx)}{d^4}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{720b^2\sinh(c+dx)}{d^7}-\frac{720b^2x\cosh(c+dx)}{d^6}+\frac{360b^2x^2\sinh(c+dx)}{d^5}-\frac{120b^2x^3\cosh(c+dx)}{d^4}+\frac{30b^2x^4\sinh(c+dx)}{d^3}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{b^2x^6\sinh(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*
x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48
*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c +
 d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (
30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh
[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2x^2\cosh(c+dx)+2abx^4\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)\,dx \\
&= a^2\int x^2\cosh(c+dx)\,dx+(2ab)\int x^4\cosh(c+dx)\,dx+b^2\int x^6\cosh(c+dx)\,dx \\
&= \frac{a^2x^2\sinh(c+dx)}{d}+\frac{2abx^4\sinh(c+dx)}{d} \\
&\quad+\frac{b^2x^6\sinh(c+dx)}{d}-\frac{(2a^2)\int x\sinh(c+dx)\,dx}{d} \\
&\quad-\frac{(8ab)\int x^3\sinh(c+dx)\,dx}{d}-\frac{(6b^2)\int x^5\sinh(c+dx)\,dx}{d} \\
&= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d} \\
&\quad+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d}+\frac{(2a^2)\int\cosh(c+dx)\,dx}{d^2} \\
&\quad+\frac{(24ab)\int x^2\cosh(c+dx)\,dx}{d^2}+\frac{(30b^2)\int x^4\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{2a^2\sinh(c+dx)}{d^3} \\
&\quad+\frac{24abx^2\sinh(c+dx)}{d^3}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh(c+dx)}{d^3}+\frac{2abx^4\sinh(c+dx)}{d} \\
&\quad+\frac{b^2x^6\sinh(c+dx)}{d}-\frac{(48ab)\int x\sinh(c+dx)\,dx}{d^3}-\frac{(120b^2)\int x^3\sinh(c+dx)\,dx}{d^3} \\
&= -\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4} \\
&\quad-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2} \\
&\quad+\frac{2a^2\sinh(c+dx)}{d^3}+\frac{24abx^2\sinh(c+dx)}{d^3}+\frac{a^2x^2\sinh(c+dx)}{d} \\
&\quad+\frac{30b^2x^4\sinh(c+dx)}{d^3}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d} \\
&\quad+\frac{(48ab)\int\cosh(c+dx)\,dx}{d^4}+\frac{(360b^2)\int x^2\cosh(c+dx)\,dx}{d^4}
\end{aligned}
$$

$$= -\frac{48abx\cosh(c+dx)}{d^4} - \frac{2a^2x\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{360b^2x^2\sinh(c+dx)}{d^5} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{a^2x^2\sinh(c+dx)}{d} + \frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{b^2x^6\sinh(c+dx)}{d} - \frac{(720b^2)\int x\sinh(c+dx)\,dx}{d^5}$$

$$= -\frac{720b^2x\cosh(c+dx)}{d^6} - \frac{48abx\cosh(c+dx)}{d^4} - \frac{2a^2x\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{360b^2x^2\sinh(c+dx)}{d^5} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{a^2x^2\sinh(c+dx)}{d} + \frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{b^2x^6\sinh(c+dx)}{d} + \frac{(720b^2)\int\cosh(c+dx)\,dx}{d^6}$$

$$= -\frac{720b^2x\cosh(c+dx)}{d^6} - \frac{48abx\cosh(c+dx)}{d^4} - \frac{2a^2x\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{720b^2\sinh(c+dx)}{d^7} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{360b^2x^2\sinh(c+dx)}{d^5} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{a^2x^2\sinh(c+dx)}{d} + \frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{b^2x^6\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.59

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = \frac{-2dx(a^2d^4+4abd^2(6+d^2x^2)+3b^2(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(a^2d^4(2+d^2x^2)+2abd^2(24+$$
$$d^7$$

```
[In] Integrate[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*x*(a^2*d^4 + 4*a*b*d^2*(6 + d^2*x^2) + 3*b^2*(120 + 20*d^2*x^2 + d^4*
x^4))*Cosh[c + d*x] + (a^2*d^4*(2 + d^2*x^2) + 2*a*b*d^2*(24 + 12*d^2*x^2 +
 d^4*x^4) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/
d^7
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{2\left(\left(3b\,x^{2}+a\right)\left(b\,x^{2}+a\right)d^{4}+12\left(5x^{2}b^{2}+2ab\right)d^{2}+360b^{2}\right)dx\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\left(-x^{2}\left(b\,x^{2}+a\right)^{2}d^{6}+2\left(-15b^{2}x^{4}-12ab\,x^{2}-a^{2}\right)d^{4}\right.}{d^{7}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\right.}$
risch	$\frac{\left(b^{2}x^{6}d^{6}+2ab\,d^{6}x^{4}-6b^{2}x^{5}d^{5}+a^{2}d^{6}x^{2}-8ab\,d^{5}x^{3}+30b^{2}x^{4}d^{4}-2a^{2}d^{5}x+24ab\,d^{4}x^{2}-120b^{2}d^{3}x^{3}+2a^{2}d^{4}-48ab\,d^{3}x+360x^{2}d^{2}\right.}{2d^{7}}$
meijerg	$\frac{64ib^{2}\cosh\left(c\right)\sqrt{\pi}\left(\frac{ixd\left(\frac{21}{8}d^{4}x^{4}+\frac{105}{2}x^{2}d^{2}+315\right)\cosh\left(dx\right)}{28\sqrt{\pi}}-\frac{i\left(\frac{7}{16}x^{6}d^{6}+\frac{105}{8}d^{4}x^{4}+\frac{315}{2}x^{2}d^{2}+315\right)\sinh\left(dx\right)}{28\sqrt{\pi}}\right)}{d^{7}}+\frac{64b^{2}\sinh\left(c\right)\sqrt{\pi}}{}$
parts	$\frac{b^{2}x^{6}\sinh\left(dx+c\right)}{d}+\frac{2ab\,x^{4}\sinh\left(dx+c\right)}{d}+\frac{a^{2}x^{2}\sinh\left(dx+c\right)}{d}-\frac{2\left(\frac{15b^{2}c^{4}\left(\left(dx+c\right)\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)}{d^{5}}-\frac{30b^{2}c^{3}\left(\left(dx+c\right)\right.}{}\right.}{}$
derivativedivides	$\frac{a^{2}c^{2}\sinh\left(dx+c\right)+a^{2}\left(\left(dx+c\right)^{2}\sinh\left(dx+c\right)-2\left(dx+c\right)\cosh\left(dx+c\right)+2\sinh\left(dx+c\right)\right)+\frac{b^{2}c^{6}\sinh\left(dx+c\right)}{d^{4}}+\frac{b^{2}\left(\left(dx+c\right)^{6}\sinh\left(dx+c\right)\right.}{}}{}$
default	$\frac{a^{2}c^{2}\sinh\left(dx+c\right)+a^{2}\left(\left(dx+c\right)^{2}\sinh\left(dx+c\right)-2\left(dx+c\right)\cosh\left(dx+c\right)+2\sinh\left(dx+c\right)\right)+\frac{b^{2}c^{6}\sinh\left(dx+c\right)}{d^{4}}+\frac{b^{2}\left(\left(dx+c\right)^{6}\sinh\left(dx+c\right)\right.}{}}{}$

```
[In] int(x^2*(b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(((3*b*x^2+a)*(b*x^2+a)*d^4+12*(5*b^2*x^2+2*a*b)*d^2+360*b^2)*d*x*tanh(1/
2*d*x+1/2*c)^2+(-x^2*(b*x^2+a)^2*d^6+2*(-15*b^2*x^4-12*a*b*x^2-a^2)*d^4+24*
(-15*b^2*x^2-2*a*b)*d^2-720*b^2)*tanh(1/2*d*x+1/2*c)+((3*b*x^2+a)*(b*x^2+a)
*d^4+12*(5*b^2*x^2+2*a*b)*d^2+360*b^2)*d*x)/d^7/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.66

$$\int x^2\left(a+bx^2\right)^2\cosh(c+dx)\,dx = -\frac{2\,(3\,b^2d^5x^5+4\,(abd^5+15\,b^2d^3)x^3+(a^2d^5+24\,abd^3+360\,b^2d)x)\cosh\left(dx+c\right)-(b^2d^6x^6+2\,a^2d^4+2\,(a}{d^7}$$

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(3*b^2*d^5*x^5 + 4*(a*b*d^5 + 15*b^2*d^3)*x^3 + (a^2*d^5 + 24*a*b*d^3 +
 360*b^2*d)*x)*cosh(d*x + c) - (b^2*d^6*x^6 + 2*a^2*d^4 + 2*(a*b*d^6 + 15*b
^2*d^4)*x^4 + 48*a*b*d^2 + (a^2*d^6 + 24*a*b*d^4 + 360*b^2*d^2)*x^2 + 720*b
^2)*sinh(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.22

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx$$
$$=\begin{cases}\frac{a^2x^2\sinh(c+dx)}{d}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{2a^2\sinh(c+dx)}{d^3}+\frac{2abx^4\sinh(c+dx)}{d}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{48abx\cosh\ldots}{}\\ \left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right)\cosh(c)\end{cases}$$

```
[In] integrate(x**2*(b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2
*sinh(c + d*x)/d**3 + 2*a*b*x**4*sinh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)
/d**2 + 24*a*b*x**2*sinh(c + d*x)/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a
*b*sinh(c + d*x)/d**5 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*
x)/d**2 + 30*b**2*x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**
4 + 360*b**2*x**2*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*
b**2*sinh(c + d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**
7/7)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.64

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx =$$
$$-\frac{1}{210}d\left(\frac{35\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)a^2e^{(dx)}}{d^4}+\frac{35\,(d^3x^3+3\,d^2x^2+6\,dx+6)a^2e^{(-dx-c)}}{d^4}+\frac{42\,(d^5\ldots}{}\right.$$
$$+\frac{1}{105}\left(15\,b^2x^7+42\,abx^5+35\,a^2x^3\right)\cosh(dx+c)$$

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/210*d*(35*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/
d^4 + 35*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 42*(d^5*x
^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 12
0*e^c)*a*b*e^(d*x)/d^6 + 42*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2
+ 120*d*x + 120)*a*b*e^(-d*x - c)/d^6 + 15*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 4
2*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040
*d*x*e^c - 5040*e^c)*b^2*e^(d*x)/d^8 + 15*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5
 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b^2*e^(-d*x
- c)/d^8) + 1/105*(15*b^2*x^7 + 42*a*b*x^5 + 35*a^2*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.30

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx$$
$$=\frac{(b^2d^6x^6+2\,abd^6x^4-6\,b^2d^5x^5+a^2d^6x^2-8\,abd^5x^3+30\,b^2d^4x^4-2\,a^2d^5x+24\,abd^4x^2-120\,b^2d^3x^3+2\,a^2d}{2\,d^7}$$
$$-\frac{(b^2d^6x^6+2\,abd^6x^4+6\,b^2d^5x^5+a^2d^6x^2+8\,abd^5x^3+30\,b^2d^4x^4+2\,a^2d^5x+24\,abd^4x^2+120\,b^2d^3x^3+2\,a}{2\,d^7}$$

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 - 6*b^2*d^5*x^5 + a^2*d^6*x^2 - 8*a*b*d^5*
x^3 + 30*b^2*d^4*x^4 - 2*a^2*d^5*x + 24*a*b*d^4*x^2 - 120*b^2*d^3*x^3 + 2*a
^2*d^4 - 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2*d*x + 720*b^
2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + 6*b^2*d^5*x^5 + a^2
*d^6*x^2 + 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 + 2*a^2*d^5*x + 24*a*b*d^4*x^2 +
120*b^2*d^3*x^3 + 2*a^2*d^4 + 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 +
 720*b^2*d*x + 720*b^2)*e^(-d*x - c)/d^7
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.78

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx=\frac{2\sinh(c+d\,x)\,\left(a^2\,d^4+24\,a\,b\,d^2+360\,b^2\right)}{d^7}-\frac{6\,b^2\,x^5\cosh(c+d\,x)}{d^2}+\frac{b^2\,x^6\sinh(c+d\,x)}{d}-\frac{2\,x\cosh(c+d\,x)\,\left(a^2\,d^4+24\,a\,b\,d^2+360\,b^2\right)}{d^6}+\frac{x^2\sinh(c+d\,x)\,\left(a^2\,d^4+24\,a\,b\,d^2+360\,b^2\right)}{d^5}-\frac{8\,x^3\cosh(c+d\,x)\,\left(15\,b^2+a\,b\,d^2\right)}{d^4}+\frac{2\,x^4\sinh(c+d\,x)\,\left(15\,b^2+a\,b\,d^2\right)}{d^3}$$

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (2*sinh(c + d*x)*(360*b^2 + a^2*d^4 + 24*a*b*d^2))/d^7 - (6*b^2*x^5*cosh(c
+ d*x))/d^2 + (b^2*x^6*sinh(c + d*x))/d - (2*x*cosh(c + d*x)*(360*b^2 + a^2
*d^4 + 24*a*b*d^2))/d^6 + (x^2*sinh(c + d*x)*(360*b^2 + a^2*d^4 + 24*a*b*d^
2))/d^5 - (8*x^3*cosh(c + d*x)*(15*b^2 + a*b*d^2))/d^4 + (2*x^4*sinh(c + d*
x)*(15*b^2 + a*b*d^2))/d^3
```

3.50 $\int x(a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$\begin{aligned}\int x(a+bx^2)^2 \cosh(c+dx)\,dx = &-\frac{120b^2\cosh(c+dx)}{d^6}-\frac{12ab\cosh(c+dx)}{d^4}\\&-\frac{a^2\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}\\&-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}\\&+\frac{120b^2x\sinh(c+dx)}{d^5}+\frac{12abx\sinh(c+dx)}{d^3}\\&+\frac{a^2x\sinh(c+dx)}{d}+\frac{20b^2x^3\sinh(c+dx)}{d^3}\\&+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^5\sinh(c+dx)}{d}\end{aligned}$$

```
[Out] -120*b^2*cosh(d*x+c)/d^6-12*a*b*cosh(d*x+c)/d^4-a^2*cosh(d*x+c)/d^2-60*b^2*
x^2*cosh(d*x+c)/d^4-6*a*b*x^2*cosh(d*x+c)/d^2-5*b^2*x^4*cosh(d*x+c)/d^2+120
*b^2*x*sinh(d*x+c)/d^5+12*a*b*x*sinh(d*x+c)/d^3+a^2*x*sinh(d*x+c)/d+20*b^2*
x^3*sinh(d*x+c)/d^3+2*a*b*x^3*sinh(d*x+c)/d+b^2*x^5*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5395, 3377, 2718}

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = -\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d} -\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3} -\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d} -\frac{120b^2\cosh(c+dx)}{d^6}+\frac{120b^2x\sinh(c+dx)}{d^5} -\frac{60b^2x^2\cosh(c+dx)}{d^4}+\frac{20b^2x^3\sinh(c+dx)}{d^3} -\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{b^2x^5\sinh(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d
*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 -
 (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b*x*
Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d
^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2x\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^5\cosh(c+dx)\right)\,dx\\
&= a^2\int x\cosh(c+dx)\,dx+(2ab)\int x^3\cosh(c+dx)\,dx+b^2\int x^5\cosh(c+dx)\,dx\\
&= \frac{a^2x\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^5\sinh(c+dx)}{d}\\
&\quad-\frac{a^2\int\sinh(c+dx)\,dx}{d}-\frac{(6ab)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(5b^2)\int x^4\sinh(c+dx)\,dx}{d}\\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}\\
&\quad+\frac{a^2x\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^5\sinh(c+dx)}{d}\\
&\quad+\frac{(12ab)\int x\cosh(c+dx)\,dx}{d^2}+\frac{(20b^2)\int x^3\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}\\
&\quad+\frac{a^2x\sinh(c+dx)}{d}+\frac{20b^2x^3\sinh(c+dx)}{d^3}+\frac{2abx^3\sinh(c+dx)}{d}\\
&\quad+\frac{b^2x^5\sinh(c+dx)}{d}-\frac{(12ab)\int\sinh(c+dx)\,dx}{d^3}-\frac{(60b^2)\int x^2\sinh(c+dx)\,dx}{d^3}\\
&= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}\\
&\quad-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}\\
&\quad+\frac{a^2x\sinh(c+dx)}{d}+\frac{20b^2x^3\sinh(c+dx)}{d^3}+\frac{2abx^3\sinh(c+dx)}{d}\\
&\quad+\frac{b^2x^5\sinh(c+dx)}{d}+\frac{(120b^2)\int x\cosh(c+dx)\,dx}{d^4}\\
&= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}\\
&\quad-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{120b^2x\sinh(c+dx)}{d^5}\\
&\quad+\frac{12abx\sinh(c+dx)}{d^3}+\frac{a^2x\sinh(c+dx)}{d}+\frac{20b^2x^3\sinh(c+dx)}{d^3}\\
&\quad+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^5\sinh(c+dx)}{d}-\frac{(120b^2)\int\sinh(c+dx)\,dx}{d^5}
\end{aligned}$$

$$= -\frac{120b^2\cosh(c+dx)}{d^6} - \frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{120b^2x\sinh(c+dx)}{d^5} + \frac{12abx\sinh(c+dx)}{d^3} + \frac{a^2x\sinh(c+dx)}{d} + \frac{20b^2x^3\sinh(c+dx)}{d^3} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{b^2x^5\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx$$
$$= \frac{-((a^2d^4+6abd^2(2+d^2x^2)+5b^2(24+12d^2x^2+d^4x^4))\cosh(c+dx))+dx(a^2d^4+2abd^2(6+d^2x^2)+b^2(12}{d^6}$$

```
[In] Integrate[x*(a + b*x^2)^2*Cosh[c + d*x],x]
[Out] (-((a^2*d^4 + 6*a*b*d^2*(2 + d^2*x^2) + 5*b^2*(24 + 12*d^2*x^2 + d^4*x^4))*
Cosh[c + d*x]) + d*x*(a^2*d^4 + 2*a*b*d^2*(6 + d^2*x^2) + b^2*(120 + 20*d^2
*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{6d^2x^2\left(\left(\left(\frac{5bx^2}{6}+a\right)d^2+10b\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2dx\left((bx^2+a)^2d^4+4(5x^2b^2+3ab)d^2+120b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+(5b^2x^4+6abx\right.}{d^6\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b^2x^5d^5+2abd^5x^3-5b^2x^4d^4+a^2d^5x-6abd^4x^2+20b^2d^3x^3-a^2d^4+12abd^3x-60x^2d^2b^2-12ad^2b+120b^2dx-120b^2)e^{dx+c}}{2d^6}$
meijerg	$-\frac{32b^2\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib^2\sinh(c)\sqrt{\pi}\left(-\right.}{}$
parts	$\frac{b^2x^5\sinh(dx+c)}{d}+\frac{2abx^3\sinh(dx+c)}{d}+\frac{a^2x\sinh(dx+c)}{d}-\frac{\frac{5b^2c^4\cosh(dx+c)}{d^4}-\frac{20b^2c^3((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^4}+}{}$
derivativedivides	$\frac{\frac{5b^2c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}-\frac{10b^2c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{10b^2c^2\left((dx+c)^3\sinh(dx\right.}{}}{}$
default	$\frac{\frac{5b^2c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}-\frac{10b^2c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{10b^2c^2\left((dx+c)^3\sinh(dx\right.}{}}{}$

```
[In] int(x*(b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
[Out] (6*d^2*x^2*((5/6*b*x^2+a)*d^2+10*b)*b*tanh(1/2*d*x+1/2*c)^2-2*d*x*((b*x^2+a
)^2*d^4+4*(5*b^2*x^2+3*a*b)*d^2+120*b^2)*tanh(1/2*d*x+1/2*c)+(5*b^2*x^4+6*a
```

```
*b*x^2+2*a^2)*d^4+12*(5*b^2*x^2+2*a*b)*d^2+240*b^2)/d^6/(tanh(1/2*d*x+1/2*c
)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = -\frac{(5\,b^2d^4x^4+a^2d^4+12\,abd^2+6\,(abd^4+10\,b^2d^2)x^2+120\,b^2)\cosh{(dx+c)}-(b^2d^5x^5+2\,(abd^5+10\,b^2d^3}{d^6}$$

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b^2*d^4*x^4 + a^2*d^4 + 12*a*b*d^2 + 6*(a*b*d^4 + 10*b^2*d^2)*x^2 + 12
0*b^2)*cosh(d*x + c) - (b^2*d^5*x^5 + 2*(a*b*d^5 + 10*b^2*d^3)*x^3 + (a^2*d
^5 + 12*a*b*d^3 + 120*b^2*d)*x)*sinh(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2x\sinh{(c+dx)}}{d}-\frac{a^2\cosh{(c+dx)}}{d^2}+\frac{2abx^3\sinh{(c+dx)}}{d}-\frac{6abx^2\cosh{(c+dx)}}{d^2}+\frac{12abx\sinh{(c+dx)}}{d^3}-\frac{12ab\cosh{(c+dx)}}{d^4}+\frac{b^2x^5\sinh{(c}}{d} \\ \left(\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}\right)\cosh{(c)} \end{cases}$$

```
[In] integrate(x*(b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**3*si
nh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3
 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**5*sinh(c + d*x)/d - 5*b**2*x**4*cosh
(c + d*x)/d**2 + 20*b**2*x**3*sinh(c + d*x)/d**3 - 60*b**2*x**2*cosh(c + d*
x)/d**4 + 120*b**2*x*sinh(c + d*x)/d**5 - 120*b**2*cosh(c + d*x)/d**6, Ne(d
, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.92

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx=\frac{(bx^2+a)^3\cosh(dx+c)}{6b}$$
$$-\frac{\left(\frac{a^3e^{(dx+c)}}{d}+\frac{a^3e^{(-dx-c)}}{d}+\frac{3\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2be^{(dx)}}{d^3}+\frac{3\,(d^2x^2+2\,dx+2)a^2be^{(-dx-c)}}{d^3}+\frac{3\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dx}{d^5}\right.}{}$$

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3*cosh(d*x + c)/b - 1/12*(a^3*e^(d*x + c)/d + a^3*e^(-d*x -
 c)/d + 3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*b*e^(d*x)/d^3 + 3*(d^2*x^2
+ 2*d*x + 2)*a^2*b*e^(-d*x - c)/d^3 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d
^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^
3 + 12*d^2*x^2 + 24*d*x + 24)*a*b^2*e^(-d*x - c)/d^5 + (d^6*x^6*e^c - 6*d^5
*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c
 + 720*e^c)*b^3*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x
^3 + 360*d^2*x^2 + 720*d*x + 720)*b^3*e^(-d*x - c)/d^7)*d/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.30

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx$$
$$=\frac{(b^2d^5x^5+2\,abd^5x^3-5\,b^2d^4x^4+a^2d^5x-6\,abd^4x^2+20\,b^2d^3x^3-a^2d^4+12\,abd^3x-60\,b^2d^2x^2-12\,abd^2+1}{2\,d^6}$$
$$-\frac{(b^2d^5x^5+2\,abd^5x^3+5\,b^2d^4x^4+a^2d^5x+6\,abd^4x^2+20\,b^2d^3x^3+a^2d^4+12\,abd^3x+60\,b^2d^2x^2+12\,abd^2+}{2\,d^6}$$

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3 - 5*b^2*d^4*x^4 + a^2*d^5*x - 6*a*b*d^4*x^
2 + 20*b^2*d^3*x^3 - a^2*d^4 + 12*a*b*d^3*x - 60*b^2*d^2*x^2 - 12*a*b*d^2 +
 120*b^2*d*x - 120*b^2)*e^(d*x + c)/d^6 - 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3
+ 5*b^2*d^4*x^4 + a^2*d^5*x + 6*a*b*d^4*x^2 + 20*b^2*d^3*x^3 + a^2*d^4 + 12
*a*b*d^3*x + 60*b^2*d^2*x^2 + 12*a*b*d^2 + 120*b^2*d*x + 120*b^2)*e^(-d*x -
 c)/d^6
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx=\frac{b^2\,x^5\sinh(c+d\,x)}{d}-\frac{5\,b^2\,x^4\cosh(c+d\,x)}{d^2}-\frac{\cosh(c+d\,x)\,\left(a^2\,d^4+12\,a\,b\,d^2+120\,b^2\right)}{d^6}+\frac{x\sinh(c+d\,x)\,\left(a^2\,d^4+12\,a\,b\,d^2+120\,b^2\right)}{d^5}-\frac{6\,x^2\cosh(c+d\,x)\,\left(10\,b^2+a\,b\,d^2\right)}{d^4}+\frac{2\,x^3\sinh(c+d\,x)\,\left(10\,b^2+a\,b\,d^2\right)}{d^3}$$

```
[In] int(x*cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (b^2*x^5*sinh(c + d*x))/d - (5*b^2*x^4*cosh(c + d*x))/d^2 - (cosh(c + d*x)*
(120*b^2 + a^2*d^4 + 12*a*b*d^2))/d^6 + (x*sinh(c + d*x)*(120*b^2 + a^2*d^4
 + 12*a*b*d^2))/d^5 - (6*x^2*cosh(c + d*x)*(10*b^2 + a*b*d^2))/d^4 + (2*x^3
*sinh(c + d*x)*(10*b^2 + a*b*d^2))/d^3
```

3.51 $\int (a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{a^2\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d}$$

```
[Out] -24*b^2*x*cosh(d*x+c)/d^4-4*a*b*x*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^2
+24*b^2*sinh(d*x+c)/d^5+4*a*b*sinh(d*x+c)/d^3+a^2*sinh(d*x+c)/d+12*b^2*x^2*
sinh(d*x+c)/d^3+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5385, 2717, 3377}

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = \frac{a^2\sinh(c+dx)}{d} + \frac{4ab\sinh(c+dx)}{d^3} - \frac{4abx\cosh(c+dx)}{d^2} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{24b^2\sinh(c+dx)}{d^5} - \frac{24b^2x\cosh(c+dx)}{d^4} + \frac{12b^2x^2\sinh(c+dx)}{d^3} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{b^2x^4\sinh(c+dx)}{d}$$

```
[In] Int[(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Co
sh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3 +
 (a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sinh[c
 + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)\,dx\\
&= a^2\int \cosh(c+dx)\,dx+(2ab)\int x^2\cosh(c+dx)\,dx+b^2\int x^4\cosh(c+dx)\,dx\\
&= \frac{a^2\sinh(c+dx)}{d}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{b^2x^4\sinh(c+dx)}{d}\\
&\quad -\frac{(4ab)\int x\sinh(c+dx)\,dx}{d}-\frac{(4b^2)\int x^3\sinh(c+dx)\,dx}{d}\\
&= -\frac{4abx\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d}+\frac{2abx^2\sinh(c+dx)}{d}\\
&\quad +\frac{b^2x^4\sinh(c+dx)}{d}+\frac{(4ab)\int\cosh(c+dx)\,dx}{d^2}+\frac{(12b^2)\int x^2\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{4abx\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}\\
&\quad +\frac{a^2\sinh(c+dx)}{d}+\frac{12b^2x^2\sinh(c+dx)}{d^3}+\frac{2abx^2\sinh(c+dx)}{d}\\
&\quad +\frac{b^2x^4\sinh(c+dx)}{d}-\frac{(24b^2)\int x\sinh(c+dx)\,dx}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&\quad + \frac{4ab\sinh(c+dx)}{d^3} + \frac{a^2\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} \\
&\quad + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d} + \frac{(24b^2)\int\cosh(c+dx)\,dx}{d^4} \\
&= -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&\quad + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{a^2\sinh(c+dx)}{d} \\
&\quad + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$
\int \left(a+bx^2\right)^2\cosh(c+dx)\,dx
$$

$$
= \frac{-4bdx(ad^2+b(6+d^2x^2))\cosh(c+dx)+(a^2d^4+2abd^2(2+d^2x^2)+b^2(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}
$$

```
[In] Integrate[(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-4*b*d*x*(a*d^2 + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^4 + 2*a*b*d^2*(2
 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{4dxb\left((b\,x^2+a)\,d^2+6b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(-(b\,x^2+a)^2 d^4-4b(3b\,x^2+a)\,d^2-24b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4dxb\left((b\,x^2+a)\,d^2+6b\right)}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b^2x^4d^4+2ab\,d^4x^2-4b^2d^3x^3+a^2d^4-4ab\,d^3x+12x^2d^2b^2+4a\,d^2b-24b^2dx+24b^2)\,\mathrm{e}^{dx+c}}{2d^5}-\frac{(b^2x^4d^4+2ab\,d^4x^2+4b^2d^3x^3}{}$
parts	$\frac{b^2x^4\sinh(dx+c)}{d}+\frac{2ab\,x^2\sinh(dx+c)}{d}+\frac{a^2\sinh(dx+c)}{d}-\frac{4b\left(\frac{3b\,c^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^2}-\frac{b\,c^3\cosh(dx+c)}{d^2}\right.}{}$
meijerg	$-\frac{16ib^2\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b^2\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{b^2c^4\sinh(dx+c)}{d^4}-\frac{4b^2c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}+\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{2b\,c^2}{}}{}$
default	$\frac{\frac{b^2c^4\sinh(dx+c)}{d^4}-\frac{4b^2c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}+\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{2b\,c^2}{}}{}$

```
[In] int((b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*d*x*b*((b*x^2+a)*d^2+6*b)*tanh(1/2*d*x+1/2*c)^2+(-(b*x^2+a)^2*d^4-4*b*
(3*b*x^2+a)*d^2-24*b^2)*tanh(1/2*d*x+1/2*c)+2*d*x*b*((b*x^2+a)*d^2+6*b))/d^
5/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \left(a+bx^2\right)^2\cosh(c+dx)\,dx =$$
$$-\frac{4\,(b^2d^3x^3+(abd^3+6\,b^2d)x)\cosh\left(dx+c\right)-(b^2d^4x^4+a^2d^4+4\,abd^2+2\,(abd^4+6\,b^2d^2)x^2+24\,b^2)\sinh}{d^5}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(4*(b^2*d^3*x^3 + (a*b*d^3 + 6*b^2*d)*x)*cosh(d*x + c) - (b^2*d^4*x^4 + a^
2*d^4 + 4*a*b*d^2 + 2*(a*b*d^4 + 6*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^
5
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx$$
$$= \begin{cases} \frac{a^2\sinh{(c+dx)}}{d} + \frac{2abx^2\sinh{(c+dx)}}{d} - \frac{4abx\cosh{(c+dx)}}{d^2} + \frac{4ab\sinh{(c+dx)}}{d^3} + \frac{b^2x^4\sinh{(c+dx)}}{d} - \frac{4b^2x^3\cosh{(c+dx)}}{d^2} + \frac{12b^2x^2\sinh{(c+}}{d^3} \\ \left(a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}\right)\cosh{(c)} \end{cases}$$

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**2*sinh(c + d*x)/d - 4*a*b*x*cosh
(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**2*x**4*sinh(c + d*x)/d - 4*b
**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d*x)/d**3 - 24*b**2*x*c
osh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b
*x**3/3 + b**2*x**5/5)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx = {} & \frac{a^2e^{(dx+c)}}{2\,d} - \frac{a^2e^{(-dx-c)}}{2\,d} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)abe^{(dx)}}{d^3} \\ & - \frac{(d^2x^2 + 2\,dx + 2)abe^{(-dx-c)}}{d^3} \\ & + \frac{(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c)b^2e^{(dx)}}{2\,d^5} \\ & - \frac{(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24)b^2e^{(-dx-c)}}{2\,d^5}\end{aligned}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^2*x^2*e^c - 2*d*x*e^c +
 2*e^c)*a*b*e^(d*x)/d^3 - (d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3 + 1/2*
(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^
(d*x)/d^5 - 1/2*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*
x - c)/d^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.32

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = \frac{(b^2d^4x^4 + 2\,abd^4x^2 - 4\,b^2d^3x^3 + a^2d^4 - 4\,abd^3x + 12\,b^2d^2x^2 + 4\,abd^2 - 24\,b^2dx + 24\,b^2)e^{(dx+c)}}{2\,d^5} - \frac{(b^2d^4x^4 + 2\,abd^4x^2 + 4\,b^2d^3x^3 + a^2d^4 + 4\,abd^3x + 12\,b^2d^2x^2 + 4\,abd^2 + 24\,b^2dx + 24\,b^2)e^{(-dx-c)}}{2\,d^5}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^2 - 4*b^2*d^3*x^3 + a^2*d^4 - 4*a*b*d^3*x +
12*b^2*d^2*x^2 + 4*a*b*d^2 - 24*b^2*d*x + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^
2*d^4*x^4 + 2*a*b*d^4*x^2 + 4*b^2*d^3*x^3 + a^2*d^4 + 4*a*b*d^3*x + 12*b^2*
d^2*x^2 + 4*a*b*d^2 + 24*b^2*d*x + 24*b^2)*e^(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a^2\,d^4+4\,a\,b\,d^2+24\,b^2\right)}{d^5} - \frac{4\,b^2\,x^3\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^4\sinh(c+d\,x)}{d} - \frac{4\,x\cosh(c+d\,x)\,\left(6\,b^2+a\,b\,d^2\right)}{d^4} + \frac{2\,x^2\sinh(c+d\,x)\,\left(6\,b^2+a\,b\,d^2\right)}{d^3}$$

```
[In] int(cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (sinh(c + d*x)*(24*b^2 + a^2*d^4 + 4*a*b*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*
x))/d^2 + (b^2*x^4*sinh(c + d*x))/d - (4*x*cosh(c + d*x)*(6*b^2 + a*b*d^2))
/d^4 + (2*x^2*sinh(c + d*x)*(6*b^2 + a*b*d^2))/d^3
```

3.52 $\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = -\frac{6b^2\cosh(c+dx)}{d^4} - \frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\text{Chi}(dx) + \frac{6b^2x\sinh(c+dx)}{d^3} + \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d} + a^2\sinh(c)\text{Shi}(dx)$$

```
[Out] a^2*Chi(d*x)*cosh(c)-6*b^2*cosh(d*x+c)/d^4-2*a*b*cosh(d*x+c)/d^2-3*b^2*x^2*
cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+6*b^2*x*sinh(d*x+c)/d^3+2*a*b*x*sinh(d
*x+c)/d+b^2*x^3*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5395, 3384, 3379, 3382, 3377, 2718}

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = a^2\cosh(c)\text{Chi}(dx) + a^2\sinh(c)\text{Shi}(dx) - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} - \frac{6b^2\cosh(c+dx)}{d^4} + \frac{6b^2x\sinh(c+dx)}{d^3} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + \frac{b^2x^3\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3 +
 (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIn
tegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2\cosh(c+dx)}{x} + 2abx\cosh(c+dx) + b^2x^3\cosh(c+dx)\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x}\,dx + (2ab)\int x\cosh(c+dx)\,dx + b^2\int x^3\cosh(c+dx)\,dx$$

$$
\begin{aligned}
&= \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d} \\
&\quad - \frac{(2ab)\int \sinh(c+dx)\,dx}{d} - \frac{(3b^2)\int x^2\sinh(c+dx)\,dx}{d} \\
&\quad + \left(a^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx + \left(a^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{2abx\sinh(c+dx)}{d} \\
&\quad + \frac{b^2x^3\sinh(c+dx)}{d} + a^2\sinh(c)\mathrm{Shi}(dx) + \frac{(6b^2)\int x\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{6b^2x\sinh(c+dx)}{d^3} \\
&\quad + \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d} + a^2\sinh(c)\mathrm{Shi}(dx) - \frac{(6b^2)\int \sinh(c+dx)\,dx}{d^3} \\
&= -\frac{6b^2\cosh(c+dx)}{d^4} - \frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) \\
&\quad + \frac{6b^2x\sinh(c+dx)}{d^3} + \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d} + a^2\sinh(c)\mathrm{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = &-\frac{b(2ad^2+3b(2+d^2x^2))\cosh(c+dx)}{d^4} + a^2\cosh(c)\mathrm{Chi}(dx) \\
&+ \frac{bx(2ad^2+b(6+d^2x^2))\sinh(c+dx)}{d^3} + a^2\sinh(c)\mathrm{Shi}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*(2*a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x])/d^4) + a^2*Cosh[c]*CoshIn
tegral[d*x] + (b*x*(2*a*d^2 + b*(6 + d^2*x^2))*Sinh[c + d*x])/d^3 + a^2*Sin
h[c]*SinhIntegral[d*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(110) = 220.

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{e^{-dx-c}b^2x^3}{2d}+\frac{e^{dx+c}b^2x^3}{2d}-\frac{a^2e^c\operatorname{Ei}_1(-dx)}{2}-\frac{a^2e^{-c}\operatorname{Ei}_1(dx)}{2}-\frac{e^{-dx-c}abx}{d}-\frac{3\,e^{-dx-c}b^2x^2}{2d^2}+\frac{e^{dx+c}abx}{d}-\frac{3\,e^{dx+c}b^2x}{2d^2}$
meijerg	$\frac{8b^2\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib^2\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i\left(\frac{15x^2d^2}{2}+15\right)}{20\sqrt{\pi}}\right.}{d^4}$

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)*b^2*x^3+1/2/d*exp(d*x+c)*b^2*x^3-1/2*a^2*exp(c)*Ei(1,-d*
x)-1/2*a^2*exp(-c)*Ei(1,d*x)-1/d*exp(-d*x-c)*a*b*x-3/2/d^2*exp(-d*x-c)*b^2*
x^2+1/d*exp(d*x+c)*a*b*x-3/2/d^2*exp(d*x+c)*b^2*x^2-1/d^2*exp(-d*x-c)*a*b-3
/d^3*exp(-d*x-c)*b^2*x-1/d^2*exp(d*x+c)*a*b+3/d^3*exp(d*x+c)*b^2*x-3/d^4*ex
p(-d*x-c)*b^2-3/d^4*exp(d*x+c)*b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = -\frac{2\,(3\,b^2d^2x^2+2\,abd^2+6\,b^2)\cosh\,(dx+c)-(a^2d^4\operatorname{Ei}(dx)+a^2d^4\operatorname{Ei}(-dx))\cosh\,(c)-2\,(b^2d^3x^3+2\,(abd^3}{2\,d^4}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b^2*d^2*x^2 + 2*a*b*d^2 + 6*b^2)*cosh(d*x + c) - (a^2*d^4*Ei(d*x
) + a^2*d^4*Ei(-d*x))*cosh(c) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 + 3*b^2*d)*x)*s
inh(d*x + c) - (a^2*d^4*Ei(d*x) - a^2*d^4*Ei(-d*x))*sinh(c))/d^4
```

Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx$$

$$= a^2\sinh(c)\operatorname{Shi}(dx)+a^2\cosh(c)\operatorname{Chi}(dx)+2ab\left(\begin{cases}\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cosh(c)}{2} & \text{otherwise}\end{cases}\right)$$

$$+b^2\left(\begin{cases}\frac{x^3\sinh(c+dx)}{d}-\frac{3x^2\cosh(c+dx)}{d^2}+\frac{6x\sinh(c+dx)}{d^3}-\frac{6\cosh(c+dx)}{d^4} & \text{for } d\neq 0\\ \frac{x^4\cosh(c)}{4} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*sinh(c +
 d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True)) + b**2*Pie
cewise((x**3*sinh(c + d*x)/d - 3*x**2*cosh(c + d*x)/d**2 + 6*x*sinh(c + d*x
)/d**3 - 6*cosh(c + d*x)/d**4, Ne(d, 0)), (x**4*cosh(c)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(110) = 220$.

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x}\,dx =$$
$$-\frac{1}{8}\left(4\,ab\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)e^{(-dx-c)}}{d^3}\right) + b^2\left(\frac{(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e}{d^5}\right.\right.$$
$$+\frac{1}{4}\left(b^2x^4 + 4\,abx^2 + 2\,a^2\log\left(x^2\right)\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d
*x + 2)*e^(-d*x - c)/d^3) + b^2*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*
e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2
+ 24*d*x + 24)*e^(-d*x - c)/d^5) + 4*a^2*cosh(d*x + c)*log(x^2)/d - 4*(Ei(-
d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/4*(b^2*x^4 + 4*a*b*x^2 + 2*a^2*log(
x^2))*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x}\,dx$$
$$= \frac{b^2d^3x^3e^{(dx+c)} - b^2d^3x^3e^{(-dx-c)} + a^2d^4\text{Ei}(-dx)\,e^{(-c)} + a^2d^4\text{Ei}(dx)\,e^c + 2\,abd^3xe^{(dx+c)} - 3\,b^2d^2x^2e^{(dx+c)} - 2\,a}{}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^3*x^3*e^(d*x + c) - b^2*d^3*x^3*e^(-d*x - c) + a^2*d^4*Ei(-d*x)*
e^(-c) + a^2*d^4*Ei(d*x)*e^c + 2*a*b*d^3*x*e^(d*x + c) - 3*b^2*d^2*x^2*e^(d
*x + c) - 2*a*b*d^3*x*e^(-d*x - c) - 3*b^2*d^2*x^2*e^(-d*x - c) - 2*a*b*d^2
*e^(d*x + c) + 6*b^2*d*x*e^(d*x + c) - 2*a*b*d^2*e^(-d*x - c) - 6*b^2*d*x*e
^(-d*x - c) - 6*b^2*e^(d*x + c) - 6*b^2*e^(-d*x - c))/d^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x,x)
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x, x)
```

3.53 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} dx = -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + a^2d\mathrm{Chi}(dx)\sinh(c) + \frac{2b^2\sinh(c+dx)}{d^3} + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} + a^2d\cosh(c)\mathrm{Shi}(dx)$$

```
[Out] -a^2*cosh(d*x+c)/x-2*b^2*x*cosh(d*x+c)/d^2+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi
(d*x)*sinh(c)+2*b^2*sinh(d*x+c)/d^3+2*a*b*sinh(d*x+c)/d+b^2*x^2*sinh(d*x+c)
/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5395, 2717, 3378, 3384, 3379, 3382, 3377}

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} dx = a^2d\sinh(c)\mathrm{Chi}(dx) + a^2d\cosh(c)\mathrm{Shi}(dx) - \frac{a^2\cosh(c+dx)}{x} + \frac{2ab\sinh(c+dx)}{d} + \frac{2b^2\sinh(c+dx)}{d^3} - \frac{2b^2x\cosh(c+dx)}{d^2} + \frac{b^2x^2\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2ab\cosh(c+dx) + \frac{a^2\cosh(c+dx)}{x^2} + b^2x^2\cosh(c+dx)\right) dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx + (2ab)\int \cosh(c+dx)\,dx + b^2\int x^2\cosh(c+dx)\,dx \\
&= -\frac{a^2\cosh(c+dx)}{x} + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} \\
&\quad - \frac{(2b^2)\int x\sinh(c+dx)\,dx}{d} + (a^2d)\int \frac{\sinh(c+dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} \\
&\quad + \frac{(2b^2)\int \cosh(c+dx)\,dx}{d^2} + \left(a^2d\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx + \left(a^2d\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) + \frac{2b^2\sinh(c+dx)}{d^3} \\
&\quad + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx &= -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) \\
&\quad + \frac{2b^2\sinh(c+dx)}{d^3} + \frac{2ab\sinh(c+dx)}{d} \\
&\quad + \frac{b^2x^2\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)
\end{aligned}$$

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(95) = 190.

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.11

method	result
risch	$-\frac{e^{c} \mathrm{Ei}_1(-dx)a^2d^4x - e^{-c}\mathrm{Ei}_1(dx)a^2d^4x + d^2 e^{-dx-c}b^2x^3 - d^2 e^{dx+c}b^2x^3 + e^{-dx-c}a^2d^3 + 2\, e^{-dx-c}ab\,d^2x + 2d\, e^{-dx-c}b^2x^2 + e^{dx+c}a^2d^3}{2d^3x}$
meijerg	$\frac{4ib^2\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}} - \frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3} + \frac{4b^2\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}} - \frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3} + \frac{2ab\cosh(}{}$

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d^3*(exp(c)*Ei(1,-d*x)*a^2*d^4*x-exp(-c)*Ei(1,d*x)*a^2*d^4*x+d^2*exp(-
d*x-c)*b^2*x^3-d^2*exp(d*x+c)*b^2*x^3+exp(-d*x-c)*a^2*d^3+2*exp(-d*x-c)*a*b
*d^2*x+2*d*exp(-d*x-c)*b^2*x^2+exp(d*x+c)*a^2*d^3-2*exp(d*x+c)*a*b*d^2*x+2*
d*exp(d*x+c)*b^2*x^2+2*exp(-d*x-c)*b^2*x-2*exp(d*x+c)*b^2*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx = -\frac{2\,(a^2d^3+2\,b^2dx^2)\cosh(dx+c) - (a^2d^4x\mathrm{Ei}(dx) - a^2d^4x\mathrm{Ei}(-dx))\cosh(c) - 2\,(b^2d^2x^3+2\,(abd^2+b^2)x}{2\,d^3x}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2*d^3 + 2*b^2*d*x^2)*cosh(d*x + c) - (a^2*d^4*x*Ei(d*x) - a^2*d^
4*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^2*x^3 + 2*(a*b*d^2 + b^2)*x)*sinh(d*x + c)
 - (a^2*d^4*x*Ei(d*x) + a^2*d^4*x*Ei(-d*x))*sinh(c))/(d^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx$$

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx =$$
$$-\frac{1}{6}\left(3\,a^2\mathrm{Ei}(-dx)\,e^{(-c)}-3\,a^2\mathrm{Ei}(dx)\,e^{c}+\frac{6\,(dxe^{c}-e^{c})abe^{(dx)}}{d^2}+\frac{6\,(dx+1)abe^{(-dx-c)}}{d^2}+\frac{(d^3x^3e^{c}-3\,d^2x^2e^{c}\right.$$
$$+\frac{1}{3}\left(b^2x^3+6\,abx-\frac{3\,a^2}{x}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*Ei(-d*x)*e^(-c) - 3*a^2*Ei(d*x)*e^c + 6*(d*x*e^c - e^c)*a*b*e^(
d*x)/d^2 + 6*(d*x + 1)*a*b*e^(-d*x - c)/d^2 + (d^3*x^3*e^c - 3*d^2*x^2*e^c
+ 6*d*x*e^c - 6*e^c)*b^2*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b^
2*e^(-d*x - c)/d^4)*d + 1/3*(b^2*x^3 + 6*a*b*x - 3*a^2/x)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx =$$
$$-\frac{a^2d^4x\mathrm{Ei}(-dx)\,e^{(-c)}-a^2d^4x\mathrm{Ei}(dx)\,e^{c}-b^2d^2x^3e^{(dx+c)}+b^2d^2x^3e^{(-dx-c)}+a^2d^3e^{(dx+c)}-2\,abd^2xe^{(dx+c)}+2}{2\,d^3x}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^4*x*Ei(-d*x)*e^(-c) - a^2*d^4*x*Ei(d*x)*e^c - b^2*d^2*x^3*e^(d*
x + c) + b^2*d^2*x^3*e^(-d*x - c) + a^2*d^3*e^(d*x + c) - 2*a*b*d^2*x*e^(d*
x + c) + 2*b^2*d*x^2*e^(d*x + c) + a^2*d^3*e^(-d*x - c) + 2*a*b*d^2*x*e^(-d
*x - c) + 2*b^2*d*x^2*e^(-d*x - c) - 2*b^2*x*e^(d*x + c) + 2*b^2*x*e^(-d*x
- c))/(d^3*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^2}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^2,x)
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^2, x)
```

3.54 $\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^3}\,dx = -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{2x}+\frac{b^2x\sinh(c+dx)}{d}+2ab\sinh(c)\mathrm{Shi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)$$

```
[Out] 2*a*b*Chi(d*x)*cosh(c)+1/2*a^2*d^2*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2-1/2
*a^2*cosh(d*x+c)/x^2+2*a*b*Shi(d*x)*sinh(c)+1/2*a^2*d^2*Shi(d*x)*sinh(c)-1/
2*a^2*d*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2718}

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2ab\cosh(c)\mathrm{Chi}(dx)+2ab\sinh(c)\mathrm{Shi}(dx)-\frac{b^2\cosh(c+dx)}{d^2}+\frac{b^2x\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(2*x^2) + 2*a*b*Cosh[c]*Co
shIntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a^2*d*Sinh[c + d
*x])/(2*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*Sinh[c]*SinhIntegral[d*x] + (a
^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^3}+\frac{2ab\cosh(c+dx)}{x}+b^2x\cosh(c+dx)\right)dx\\
&= a^2\int\frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int\frac{\cosh(c+dx)}{x}\,dx+b^2\int x\cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}+\frac{b^2x\sinh(c+dx)}{d}\\
&\quad-\frac{b^2\int\sinh(c+dx)\,dx}{d}+\frac{1}{2}(a^2d)\int\frac{\sinh(c+dx)}{x^2}\,dx\\
&\quad+(2ab\cosh(c))\int\frac{\cosh(dx)}{x}\,dx+(2ab\sinh(c))\int\frac{\sinh(dx)}{x}\,dx\\
&= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{2x}\\
&\quad+\frac{b^2x\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)+\frac{1}{2}(a^2d^2)\int\frac{\cosh(c+dx)}{x}\,dx\\
&= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\text{Chi}(dx)\\
&\quad-\frac{a^2d\sinh(c+dx)}{2x}+\frac{b^2x\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)\\
&\quad+\frac{1}{2}(a^2d^2\cosh(c))\int\frac{\cosh(dx)}{x}\,dx+\frac{1}{2}(a^2d^2\sinh(c))\int\frac{\sinh(dx)}{x}\,dx\\
&= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)\\
&\quad-\frac{a^2d\sinh(c+dx)}{2x}+\frac{b^2x\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$
\int\frac{(a+bx^2)^2\cosh(c+dx)}{x^3}\,dx=\frac{1}{2}\left(-\frac{2b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{x^2}+a(4b+ad^2)\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{x}+\frac{2b^2x\sinh(c+dx)}{d}+a(4b+ad^2)\sinh(c)\text{Shi}(dx)\right)
$$

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] ((-2*b^2*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x^2 + a*(4*b + a*d^2)*Cos
h[c]*CoshIntegral[d*x] - (a^2*d*Sinh[c + d*x])/x + (2*b^2*x*Sinh[c + d*x])/
d + a*(4*b + a*d^2)*Sinh[c]*SinhIntegral[d*x])/2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{\mathrm{e}^{c}\,\mathrm{Ei}_{1}(-dx)a^{2}d^{4}x^{2}+\mathrm{e}^{-c}\,\mathrm{Ei}_{1}(dx)a^{2}d^{4}x^{2}+4\,\mathrm{e}^{c}\,\mathrm{Ei}_{1}(-dx)ab\,d^{2}x^{2}+4\,\mathrm{e}^{-c}\,\mathrm{Ei}_{1}(dx)ab\,d^{2}x^{2}-\mathrm{e}^{-dx-c}a^{2}d^{3}x+2\,\mathrm{e}^{-dx-c}b^{2}d\,x^{3}+\mathrm{e}^{dx+c}a^{2}}{4d^{2}x^{2}}$
meijerg	$-\frac{2b^{2}\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{2}}+\frac{b^{2}\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^{2}}+ab\cosh\left(c\right)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(}{\sqrt{\pi}}\right.$

[In] int((b*x^2+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/4/d^2*(exp(c)*Ei(1,-d*x)*a^2*d^4*x^2+exp(-c)*Ei(1,d*x)*a^2*d^4*x^2+4*exp(c)*Ei(1,-d*x)*a*b*d^2*x^2+4*exp(-c)*Ei(1,d*x)*a*b*d^2*x^2-exp(-d*x-c)*a^2*d^3*x+2*exp(-d*x-c)*b^2*d*x^3+exp(d*x+c)*a^2*d^3*x-2*exp(d*x+c)*b^2*d*x^3+d^2*exp(-d*x-c)*a^2+2*exp(-d*x-c)*b^2*x^2+d^2*exp(d*x+c)*a^2+2*exp(d*x+c)*b^2*x^2)/x^2

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int\frac{\left(a+bx^{2}\right)^{2}\cosh(c+dx)}{x^{3}}\,dx=$$
$$-\frac{2\left(a^{2}d^{2}+2\,b^{2}x^{2}\right)\cosh\left(dx+c\right)-\left(\left(a^{2}d^{4}+4\,abd^{2}\right)x^{2}\mathrm{Ei}(dx)+\left(a^{2}d^{4}+4\,abd^{2}\right)x^{2}\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+2\,(a}{4\,d^{2}x^{2}}$$

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/4*(2*(a^2*d^2 + 2*b^2*x^2)*cosh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) + (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*cosh(c) + 2*(a^2*d^3*x - 2*b^2*d*x^3)*sinh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) - (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*sinh(c))/(d^2*x^2)

Sympy [F]

$$\int\frac{\left(a+bx^{2}\right)^{2}\cosh(c+dx)}{x^{3}}\,dx=\int\frac{\left(a+bx^{2}\right)^{2}\cosh\left(c+dx\right)}{x^{3}}\,dx$$

[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**3,x)

[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx$$
$$= \frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a^2 - b^2\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}\right.\right.$$
$$+ \frac{1}{2}\left(b^2x^2 + 2\,ab\log\left(x^2\right) - \frac{a^2}{x^2}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - b^2*((d^2*x^2*
e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d
^3) - 4*a*b*cosh(d*x + c)*log(x^2)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*
b/d)*d + 1/2*(b^2*x^2 + 2*a*b*log(x^2) - a^2/x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx$$
$$= \frac{a^2d^4x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^4x^2\mathrm{Ei}(dx)\,e^{c} + 4\,abd^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,abd^2x^2\mathrm{Ei}(dx)\,e^{c} - a^2d^3xe^{(dx+c)} + 2\,b^2d}{4\,d^2x}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^4*x^2*Ei(-d*x)*e^(-c) + a^2*d^4*x^2*Ei(d*x)*e^c + 4*a*b*d^2*x^2*
Ei(-d*x)*e^(-c) + 4*a*b*d^2*x^2*Ei(d*x)*e^c - a^2*d^3*x*e^(d*x + c) + 2*b^2
*d*x^3*e^(d*x + c) + a^2*d^3*x*e^(-d*x - c) - 2*b^2*d*x^3*e^(-d*x - c) - a^
2*d^2*e^(d*x + c) - 2*b^2*x^2*e^(d*x + c) - a^2*d^2*e^(-d*x - c) - 2*b^2*x^
2*e^(-d*x - c))/(d^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^3}\,dx=\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^3}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^3,x)
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^3, x)
```

3.55 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} \, dx = -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{2ab \cosh(c+dx)}{x} - \frac{a^2 d^2 \cosh(c+dx)}{6x} + 2abd\text{Chi}(dx)\sinh(c) + \frac{1}{6}a^2 d^3 \text{Chi}(dx)\sinh(c) + \frac{b^2 \sinh(c+dx)}{d} - \frac{a^2 d \sinh(c+dx)}{6x^2} + 2abd\cosh(c)\text{Shi}(dx) + \frac{1}{6}a^2 d^3 \cosh(c)\text{Shi}(dx)$$

```
[Out] -1/3*a^2*cosh(d*x+c)/x^3-2*a*b*cosh(d*x+c)/x-1/6*a^2*d^2*cosh(d*x+c)/x+2*a*
b*d*cosh(c)*Shi(d*x)+1/6*a^2*d^3*cosh(c)*Shi(d*x)+2*a*b*d*Chi(d*x)*sinh(c)+
1/6*a^2*d^3*Chi(d*x)*sinh(c)+b^2*sinh(d*x+c)/d-1/6*a^2*d*sinh(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5395, 2717, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} \, dx = \frac{1}{6}a^2 d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2 d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2 d^2 \cosh(c+dx)}{6x} - \frac{a^2 \cosh(c+dx)}{3x^3} - \frac{a^2 d \sinh(c+dx)}{6x^2} + 2abd\sinh(c)\text{Chi}(dx) + 2abd\cosh(c)\text{Shi}(dx) - \frac{2ab\cosh(c+dx)}{x} + \frac{b^2 \sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a^2*Cosh[c + d*x])/x^3 - (2*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/(6*x) + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x
]*Sinh[c])/6 + (b^2*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/(6*x^2) + 2*a*
b*d*Cosh[c]*SinhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(b^2\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x^2}\right)\,dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int \cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}\\
&\quad+\frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^3}\,dx+(2abd)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}\\
&\quad-\frac{a^2d\sinh(c+dx)}{6x^2}+\frac{1}{6}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&\quad+(2abd\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(2abd\sinh(c))\int \frac{\cosh(dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\text{Chi}(dx)\sinh(c)\\
&\quad+\frac{b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{6x^2}+2abd\cosh(c)\text{Shi}(dx)+\frac{1}{6}\left(a^2d^3\right)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\text{Chi}(dx)\sinh(c)\\
&\quad+\frac{b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{6x^2}+2abd\cosh(c)\text{Shi}(dx)\\
&\quad+\frac{1}{6}\left(a^2d^3\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx+\frac{1}{6}\left(a^2d^3\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}\\
&\quad+2abd\text{Chi}(dx)\sinh(c)+\frac{1}{6}a^2d^3\text{Chi}(dx)\sinh(c)+\frac{b^2\sinh(c+dx)}{d}\\
&\quad-\frac{a^2d\sinh(c+dx)}{6x^2}+2abd\cosh(c)\text{Shi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(-\frac{2a^2\cosh(c+dx)}{x^3}-\frac{12ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{x}+ad\left(12b+ad^2\right)\mathrm{Chi}(dx)\sinh(c)+\frac{6b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{x^2}+ad\left(12b+ad^2\right)\cosh(c)\mathrm{Shi}(dx)\right)$$

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (12*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/x + a*d*(12*b + a*d^2)*CoshIntegral[d*x]*Sinh[c] + (6*b^2*Sinh[c + d*
x])/d - (a^2*d*Sinh[c + d*x])/x^2 + a*d*(12*b + a*d^2)*Cosh[c]*SinhIntegral
[d*x])/6
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{e^{c}\,\mathrm{Ei}_1(-dx)a^2d^4x^3-e^{-c}\,\mathrm{Ei}_1(dx)a^2d^4x^3+12\,e^{c}\,\mathrm{Ei}_1(-dx)ab\,d^2x^3-12\,e^{-c}\,\mathrm{Ei}_1(dx)ab\,d^2x^3+d^3e^{-dx-c}a^2x^2+d^3e^{dx+c}a^2x^2-d^2e^{-dx}}{12d\,x^3}$
meijerg	$\frac{b^2\cosh(c)\sinh(dx)}{d}-\frac{b^2\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{idab\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}(dx)}{\sqrt{\pi}}\right)}{2}+\frac{dba\sinh(c)\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)}{\sqrt{\pi}}\right.}{}$

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/d*(exp(c)*Ei(1,-d*x)*a^2*d^4*x^3-exp(-c)*Ei(1,d*x)*a^2*d^4*x^3+12*exp
(c)*Ei(1,-d*x)*a*b*d^2*x^3-12*exp(-c)*Ei(1,d*x)*a*b*d^2*x^3+d^3*exp(-d*x-c)
*a^2*x^2+d^3*exp(d*x+c)*a^2*x^2-d^2*exp(-d*x-c)*a^2*x+12*exp(-d*x-c)*a*b*d*
x^2+6*exp(-d*x-c)*b^2*x^3+d^2*exp(d*x+c)*a^2*x+12*exp(d*x+c)*a*b*d*x^2-6*ex
p(d*x+c)*b^2*x^3+2*exp(-d*x-c)*a^2*d+2*exp(d*x+c)*a^2*d)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\, dx = -\frac{2\,(2\,a^2d+(a^2d^3+12\,abd)x^2)\cosh\,(dx+c)-((a^2d^4+12\,abd^2)x^3\mathrm{Ei}(dx)-(a^2d^4+12\,abd^2)x^3\mathrm{Ei}(-dx))\,\mathrm{c}}{1}$$

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*(2*a^2*d + (a^2*d^3 + 12*a*b*d)*x^2)*cosh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*x) - (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(a^2*d^2*x - 6*b^2*x^3)*sinh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*x) + (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*sinh(c))/(d*x^3)

Sympy [F]

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\, dx = \int \frac{(a+bx^2)^2 \cosh\,(c+dx)}{x^4}\, dx$$

[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**4,x)

[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\, dx = \frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma(-2,dx)-a^2d^2e^c\Gamma(-2,-dx)-6\,ab\mathrm{Ei}(-dx)\,e^{(-c)}+6\,ab\mathrm{Ei}(dx)\,e^c-\frac{3\,(dxe^c-e^c)b^2e^{(dx)}}{d^2}-\frac{3}{\cdot}\right. + \frac{1}{3}\left(3\,b^2x-\frac{6\,abx^2+a^2}{x^3}\right)\cosh\,(dx+c)$$

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")

[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) - 6*a*b*Ei(-d*x)*e^(-c) + 6*a*b*Ei(d*x)*e^c - 3*(d*x*e^c - e^c)*b^2*e^(d*x)/d^2 - 3*(d*x + 1)*b^2*e^(-d*x - c)/d^2)*d + 1/3*(3*b^2*x - (6*a*b*x^2 + a^2)/x^3)*cosh(d*x + c)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx = -\frac{a^2d^4x^3\text{Ei}(-dx)\,e^{(-c)} - a^2d^4x^3\text{Ei}(dx)\,e^c + 12\,abd^2x^3\text{Ei}(-dx)\,e^{(-c)} - 12\,abd^2x^3\text{Ei}(dx)\,e^c + a^2d^3x^2e^{(dx+c)}}{}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^4*x^3*Ei(-d*x)*e^(-c) - a^2*d^4*x^3*Ei(d*x)*e^c + 12*a*b*d^2*x
^3*Ei(-d*x)*e^(-c) - 12*a*b*d^2*x^3*Ei(d*x)*e^c + a^2*d^3*x^2*e^(d*x + c) +
 a^2*d^3*x^2*e^(-d*x - c) + a^2*d^2*x*e^(d*x + c) + 12*a*b*d*x^2*e^(d*x + c
) - 6*b^2*x^3*e^(d*x + c) - a^2*d^2*x*e^(-d*x - c) + 12*a*b*d*x^2*e^(-d*x -
 c) + 6*b^2*x^3*e^(-d*x - c) + 2*a^2*d*e^(d*x + c) + 2*a^2*d*e^(-d*x - c))/
(d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^4}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^4, x)
```

3.56 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 175

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} \, dx = -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} + b^2\cosh(c)\text{Chi}(dx) + abd^2\cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx) - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{abd\sinh(c+dx)}{x} - \frac{a^2d^3\sinh(c+dx)}{24x} + b^2\sinh(c)\text{Shi}(dx) + abd^2\sinh(c)\text{Shi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)$$

```
[Out] b^2*Chi(d*x)*cosh(c)+a*b*d^2*Chi(d*x)*cosh(c)+1/24*a^2*d^4*Chi(d*x)*cosh(c)
-1/4*a^2*cosh(d*x+c)/x^4-a*b*cosh(d*x+c)/x^2-1/24*a^2*d^2*cosh(d*x+c)/x^2+b
^2*Shi(d*x)*sinh(c)+a*b*d^2*Shi(d*x)*sinh(c)+1/24*a^2*d^4*Shi(d*x)*sinh(c)-
1/12*a^2*d*sinh(d*x+c)/x^3-a*b*d*sinh(d*x+c)/x-1/24*a^2*d^3*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{5395, 3378, 3384, 3379, 3382\}$

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx = \frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx) - \frac{a^2d^3\sinh(c+dx)}{24x} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d\sinh(c+dx)}{12x^3} + abd^2\cosh(c)\mathrm{Chi}(dx) + abd^2\sinh(c)\mathrm{Shi}(dx) - \frac{ab\cosh(c+dx)}{x^2} - \frac{abd\sinh(c+dx)}{x} + b^2\cosh(c)\mathrm{Chi}(dx) + b^2\sinh(c)\mathrm{Shi}(dx)$$

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a^2*Cosh[c + d*x])/x^4 - (a*b*Cosh[c + d*x])/x^2 - (a^2*d^2*Cosh[c +
d*x])/(24*x^2) + b^2*Cosh[c]*CoshIntegral[d*x] + a*b*d^2*Cosh[c]*CoshIntegr
al[d*x] + (a^2*d^4*Cosh[c]*CoshIntegral[d*x])/24 - (a^2*d*Sinh[c + d*x])/(1
2*x^3) - (a*b*d*Sinh[c + d*x])/x - (a^2*d^3*Sinh[c + d*x])/(24*x) + b^2*Sin
h[c]*SinhIntegral[d*x] + a*b*d^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[
c]*SinhIntegral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^3}+\frac{b^2\cosh(c+dx)}{x}\right)dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^3}\,dx+b^2\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2} \\
&\quad+\frac{1}{4}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^4}\,dx+(abd)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&\quad+\left(b^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx+\left(b^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}+b^2\cosh(c)\text{Chi}(dx) \\
&\quad-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{abd\sinh(c+dx)}{x}+b^2\sinh(c)\text{Shi}(dx) \\
&\quad+\frac{1}{12}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x^3}\,dx+\left(abd^2\right)\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2} \\
&\quad+b^2\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{abd\sinh(c+dx)}{x} \\
&\quad+b^2\sinh(c)\text{Shi}(dx)+\frac{1}{24}\left(a^2d^3\right)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&\quad+\left(abd^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx+\left(abd^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}+b^2\cosh(c)\text{Chi}(dx) \\
&\quad+abd^2\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{abd\sinh(c+dx)}{x}-\frac{a^2d^3\sinh(c+dx)}{24x} \\
&\quad+b^2\sinh(c)\text{Shi}(dx)+abd^2\sinh(c)\text{Shi}(dx)+\frac{1}{24}\left(a^2d^4\right)\int \frac{\cosh(c+dx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{ab \cosh(c+dx)}{x^2} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} + b^2 \cosh(c)\text{Chi}(dx) \\
&\quad + abd^2 \cosh(c)\text{Chi}(dx) - \frac{a^2 d \sinh(c+dx)}{12x^3} - \frac{abd \sinh(c+dx)}{x} \\
&\quad - \frac{a^2 d^3 \sinh(c+dx)}{24x} + b^2 \sinh(c)\text{Shi}(dx) + abd^2 \sinh(c)\text{Shi}(dx) \\
&\quad + \frac{1}{24}\left(a^2 d^4 \cosh(c)\right) \int \frac{\cosh(dx)}{x}\,dx + \frac{1}{24}\left(a^2 d^4 \sinh(c)\right) \int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{ab \cosh(c+dx)}{x^2} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} + b^2 \cosh(c)\text{Chi}(dx) \\
&\quad + abd^2 \cosh(c)\text{Chi}(dx) + \frac{1}{24} a^2 d^4 \cosh(c)\text{Chi}(dx) - \frac{a^2 d \sinh(c+dx)}{12x^3} \\
&\quad - \frac{abd \sinh(c+dx)}{x} - \frac{a^2 d^3 \sinh(c+dx)}{24x} + b^2 \sinh(c)\text{Shi}(dx) + abd^2 \sinh(c)\text{Shi}(dx) \\
&\quad + \frac{1}{24} a^2 d^4 \sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

$$
\int \frac{\left(a+bx^2\right)^2 \cosh(c+dx)}{x^5}\,dx
= \frac{\left(24b^2 + 24abd^2 + a^2d^4\right) x^4 \cosh(c)\text{Chi}(dx) - a\left(\left(6a + 24bx^2 + ad^2x^2\right)\cosh(c+dx) + dx\left(2a + 24bx^2 + ad^2\right.\right.}{24x^4}
$$

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] ((24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Cosh[c]*CoshIntegral[d*x] - a*((6*a +
24*b*x^2 + a*d^2*x^2)*Cosh[c + d*x] + d*x*(2*a + 24*b*x^2 + a*d^2*x^2)*Sinh
[c + d*x]) + (24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Sinh[c]*SinhIntegral[d*x])
/(24*x^4)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{e^{-c}\,\text{Ei}_1(dx)a^2d^4x^4 + e^c\,\text{Ei}_1(-dx)a^2d^4x^4 + 24\,e^{-c}\,\text{Ei}_1(dx)ab\,d^2x^4 + 24\,e^c\,\text{Ei}_1(-dx)ab\,d^2x^4 + e^{dx+c}a^2d^3x^3 - e^{-dx-c}a^2d^3x^3 + 24\,e^{-c}\,\text{I}}{}$
meijerg	$\frac{b^2\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}} + \frac{2\,\text{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)}{2} + b^2\,\text{Shi}\,(dx)\sinh\,(c) - \frac{d^2ab\cosh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}\,x^2d^2} - \frac{2(2\gamma-3+2\ln}{\sqrt{}}\right.}{}$

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(exp(-c)*Ei(1,d*x)*a^2*d^4*x^4+exp(c)*Ei(1,-d*x)*a^2*d^4*x^4+24*exp(-
c)*Ei(1,d*x)*a*b*d^2*x^4+24*exp(c)*Ei(1,-d*x)*a*b*d^2*x^4+exp(d*x+c)*a^2*d^
3*x^3-exp(-d*x-c)*a^2*d^3*x^3+24*exp(-c)*Ei(1,d*x)*b^2*x^4+24*exp(c)*Ei(1,-
d*x)*b^2*x^4+exp(d*x+c)*a^2*d^2*x^2+24*exp(d*x+c)*a*b*d*x^3+exp(-d*x-c)*a^2
*d^2*x^2-24*exp(-d*x-c)*a*b*d*x^3+2*exp(d*x+c)*a^2*d*x+24*exp(d*x+c)*a*b*x^
2-2*exp(-d*x-c)*a^2*d*x+24*exp(-d*x-c)*a*b*x^2+6*exp(d*x+c)*a^2+6*exp(-d*x-
c)*a^2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx =$$
$$-\frac{2\left(\left(a^2d^2+24\,ab\right)x^2+6\,a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^4+24\,abd^2+24\,b^2\right)x^4\mathrm{Ei}(dx)+\left(a^2d^4+24\,abd^2+24\,b^2\right)\right.}{}$$

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a^2*d^2 + 24*a*b)*x^2 + 6*a^2)*cosh(d*x + c) - ((a^2*d^4 + 24*a*
b*d^2 + 24*b^2)*x^4*Ei(d*x) + (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x))
*cosh(c) + 2*(2*a^2*d*x + (a^2*d^3 + 24*a*b*d)*x^3)*sinh(d*x + c) - ((a^2*d
^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) - (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4
*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)^2\cosh\left(c+dx\right)}{x^5}\,dx$$

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx$$
$$=\frac{1}{8}\left(\left(d^3e^{(-c)}\Gamma(-3,dx)+d^3e^c\Gamma(-3,-dx)\right)a^2+4\left(de^{(-c)}\Gamma(-1,dx)+de^c\Gamma(-1,-dx)\right)ab-\frac{4\,b^2\cosh\left(dx+\right.}{d}\right.$$
$$+\frac{1}{4}\left(2\,b^2\log\left(x^2\right)-\frac{4\,abx^2+a^2}{x^4}\right)\cosh\left(dx+c\right)$$

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")

[Out] 1/8*((d^3*e^(-c)*gamma(-3, d*x) + d^3*e^c*gamma(-3, -d*x))*a^2 + 4*(d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a*b - 4*b^2*cosh(d*x + c)*log(x^2)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/d)*d + 1/4*(2*b^2*log(x^2) - (4*a*b*x^2 + a^2)/x^4)*cosh(d*x + c)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx$$
$$=\frac{a^2d^4x^4\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^4x^4\mathrm{Ei}(dx)\,e^c+24\,abd^2x^4\mathrm{Ei}(-dx)\,e^{(-c)}+24\,abd^2x^4\mathrm{Ei}(dx)\,e^c-a^2d^3x^3e^{(dx+c)}+}{}$$

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")

[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c + 24*a*b*d^2*x^4*Ei(-d*x)*e^(-c) + 24*a*b*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) + a^2*d^3*x^3*e^(-d*x - c) + 24*b^2*x^4*Ei(-d*x)*e^(-c) + 24*b^2*x^4*Ei(d*x)*e^c - a^2*d^2*x^2*e^(d*x + c) - 24*a*b*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c) + 24*a*b*d*x^3*e^(-d*x - c) - 2*a^2*d*x*e^(d*x + c) - 24*a*b*x^2*e^(d*x + c) + 2*a^2*d*x*e^(-d*x - c) - 24*a*b*x^2*e^(-d*x - c) - 6*a^2*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^5}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^5,x)
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^5, x)
```

3.57 $\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 273

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = -\frac{2x\cosh(c+dx)}{bd^2} + \frac{(-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}} + \frac{2\sinh(c+dx)}{bd^3} - \frac{a\sinh(c+dx)}{b^2d} + \frac{x^2\sinh(c+dx)}{bd} - \frac{(-a)^{3/2}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}$$

```
[Out] -2*x*cosh(d*x+c)/b/d^2-1/2*(-a)^(3/2)*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-
d*(-a)^(1/2)/b^(1/2))/b^(5/2)+1/2*(-a)^(3/2)*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))
*cosh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)+2*sinh(d*x+c)/b/d^3-a*sinh(d*x+c)/b^2
/d+x^2*sinh(d*x+c)/b/d-1/2*(-a)^(3/2)*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-
d*(-a)^(1/2)/b^(1/2))/b^(5/2)+1/2*(-a)^(3/2)*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*
sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5401, 2717, 3377, 5389, 3384, 3379, 3382}

$$\int \frac{x^4\cosh(c+dx)}{a+bx^2}\,dx = \frac{(-a)^{3/2}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{a\sinh(c+dx)}{b^2d} + \frac{2\sinh(c+dx)}{bd^3} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{x^2\sinh(c+dx)}{bd}$$

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (-2*x*Cosh[c + d*x])/(b*d^2) + ((-a)^(3/2)*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2)
) + (2*Sinh[c + d*x])/(b*d^3) - (a*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d
*x])/(b*d) - ((-a)^(3/2)*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Sinh[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a\cosh(c+dx)}{b^2}+\frac{x^2\cosh(c+dx)}{b}+\frac{a^2\cosh(c+dx)}{b^2\left(a+bx^2\right)}\right)dx \\
&= -\frac{a\int\cosh(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\cosh(c+dx)}{a+bx^2}\,dx}{b^2}+\frac{\int x^2\cosh(c+dx)\,dx}{b} \\
&= -\frac{a\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd} \\
&\quad +\frac{a^2\int\left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b^2}-\frac{2\int x\sinh(c+dx)\,dx}{bd} \\
&= -\frac{2x\cosh(c+dx)}{bd^2}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd} \\
&\quad -\frac{(-a)^{3/2}\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^2}-\frac{(-a)^{3/2}\int\frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^2}+\frac{2\int\cosh(c+dx)\,dx}{bd^2}
\end{aligned}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2} + \frac{2\sinh(c+dx)}{bd^3} - \frac{a\sinh(c+dx)}{b^2d} + \frac{x^2\sinh(c+dx)}{bd} - \frac{\left((-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2b^2} - \frac{\left((-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2b^2} - \frac{\left((-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2b^2} + \frac{\left((-a)^{3/2}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2b^2}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2} + \frac{(-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}} + \frac{2\sinh(c+dx)}{bd^3} - \frac{a\sinh(c+dx)}{b^2d} + \frac{x^2\sinh(c+dx)}{bd} - \frac{(-a)^{3/2}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01

$$\int\frac{x^4\cosh(c+dx)}{a+bx^2}\,dx = \frac{-ia^{3/2}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+ia^{3/2}e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i}{}}\right.}{}$$

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] ((-I)*a^(3/2)*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d*((I*Sqrt[a]
)/Sqrt[b] + x)]) + I*a^(3/2)*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt
[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegral
Ei[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*Sqrt[b]*Cosh[d*x]*(-2*b*d*x*Cosh[c] +
 (-(a*d^2) + b*(2 + d^2*x^2))*Sinh[c]))/d^3 + (4*Sqrt[b]*((-(a*d^2) + b*(2
+ d^2*x^2))*Cosh[c] - 2*b*d*x*Sinh[c])*Sinh[d*x])/d^3)/(4*b^(5/2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{\mathrm{e}^{\frac{d\sqrt{-ab}+cb}{b}} \mathrm{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)a^2}{4b^2\sqrt{-ab}} - \frac{\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}} \mathrm{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)a^2}{4b^2\sqrt{-ab}} + \frac{\mathrm{e}^{dx+c}x^2}{2db} - \frac{\mathrm{e}^{-dx-c}x^2}{2db} + \frac{\mathrm{e}^{\frac{-d\sqrt{-ab}}{b}}}{}$

```
[In] int(x^4*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x
+c)*b+c*b)/b)*a^2-1/4/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(
d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2+1/2/d/b*exp(d*x+c)*x^2-1/2/d/b*exp(-d*
x-c)*x^2+1/4/b^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^
(1/2)+(d*x+c)*b-c*b)/b)*a^2+1/4/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)
/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2-1/2/d/b^2*a*exp(d*x+c)-1/d^2
/b*exp(d*x+c)*x+1/2/d/b^2*exp(-d*x-c)*a-1/d^2/b*exp(-d*x-c)*x+1/d^3/b*exp(d
*x+c)-1/d^3/b*exp(-d*x-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(217) = 434$.

Time = 0.27 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.22

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = -\frac{8\,bdx\cosh\left(dx+c\right) + \left(\left(ad^2\cosh\left(dx+c\right)^2 - ad^2\sinh\left(dx+c\right)^2\right)\sqrt{-\frac{ad^2}{b}}\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right) + \left(ad^2\cosh\right.\right.}{}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(8*b*d*x*cosh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)
^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^
2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b
)*Ei(d*x + sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2
)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(
b*d^2*x^2 - a*d^2 + 2*b)*sinh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*si
nh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x +
c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sin
h(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh
(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^
2/b)))/(b^2*d^3*cosh(d*x + c)^2 - b^2*d^3*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx$$

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh(dx+c)}{bx^2+a}\,dx$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b*d^2*x^4*e^(2*c) - 2*b*d*x^3*e^(2*c) - 2*a*d*x*e^(2*c) + 2*b*x^2*e^(
2*c))*e^(d*x) - (b*d^2*x^4 + 2*b*d*x^3 + 2*a*d*x + 2*b*x^2)*e^(-d*x))/(b^2*
d^3*x^2*e^c + a*b*d^3*e^c) + 1/2*integrate(2*(a*b*d*x^2*e^c + a^2*d*e^c + (
a^2*d^2*e^c - 2*a*b*e^c)*x)*e^(d*x)/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*
d^3), x) + 1/2*integrate(2*(a*b*d*x^2 + a^2*d - (a^2*d^2 - 2*a*b)*x)*e^(-d*
x)/(b^3*d^3*x^4*e^c + 2*a*b^2*d^3*x^2*e^c + a^2*b*d^3*e^c), x)
```

Giac [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh(dx+c)}{bx^2+a}\,dx$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{b\,x^2+a}\,dx$$

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^2),x)
```

```
[Out] int((x^4*cosh(c + d*x))/(a + b*x^2), x)
```

3.58 $\int \frac{x^3 \cosh(c+dx)}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2} \, dx = -\frac{\cosh(c+dx)}{bd^2} - \frac{a\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} + \frac{x\sinh(c+dx)}{bd} + \frac{a\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}$$

```
[Out] -cosh(d*x+c)/b/d^2-1/2*a*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/
b^(1/2))/b^2-1/2*a*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/
2))/b^2+x*sinh(d*x+c)/b/d-1/2*a*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)
^(1/2)/b^(1/2))/b^2-1/2*a*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)
/b^(1/2))/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{5401, 3377, 2718, 3384, 3379, 3382\}$

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2}\,dx = -\frac{a\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{a\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} - \frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) - (a*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2) + (x*Sinh[c + d*x])/(b*d) +
(a*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(2*b^2) - (a*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x\cosh(c+dx)}{b}-\frac{ax\cosh(c+dx)}{b\,(a+bx^2)}\right)dx\\
&= \frac{\int x\cosh(c+dx)\,dx}{b}-\frac{a\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{b}\\
&= \frac{x\sinh(c+dx)}{bd}-\frac{a\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b}-\frac{\int \sinh(c+dx)\,dx}{bd}\\
&= -\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}+\frac{a\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}-\frac{a\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}}\\
&= -\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}-\frac{\left(a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}}\\
&\quad+\frac{\left(a\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}\\
&\quad-\frac{\left(a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}}\\
&\quad-\frac{\left(a\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{bd^2} - \frac{a\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} + \frac{x\sinh(c+dx)}{bd} + \frac{a\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx = -\frac{ae^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+ae^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{Ex}\right.}{}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] -1/4*(a*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpInt
egralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sqrt
[b] + x)]) + a*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b]
)*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*
d)/Sqrt[b] - d*x]) + (4*b*Cosh[d*x]*(Cosh[c] - d*x*Sinh[c]))/d^2 - (4*b*(d*
x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2)/b^2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.28

method	result
risch	$\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\text{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)a}{4b^2}+\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\text{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)a}{4b^2}+\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\text{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)a}{4b^2}$

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*
a+1/4/b^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)
/b)*a+1/4/b^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+
c*b)/b)*a+1/4/b^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c
)*b-c*b)/b)*a-1/2/d/b*exp(-d*x-c)*x+1/2/d/b*exp(d*x+c)*x-1/2/d^2/b*exp(-d*x
-c)-1/2/d^2/b*exp(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(169) = 338.

Time = 0.26 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.40

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx = \frac{4\,bdx\sinh(dx+c) - 4\,b\cosh(dx+c) - \left(\left(ad^2\cosh(dx+c)^2 - ad^2\sinh(dx+c)^2\right)\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right) + \right.}{}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x*sinh(d*x + c) - 4*b*cosh(d*x + c) - ((a*d^2*cosh(d*x + c)^2 -
a*d^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 -
a*d^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b))
 + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)
))*cosh(-c + sqrt(-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x
 + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x
 + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(
-a*d^2/b)))/(b^2*d^2*cosh(d*x + c)^2 - b^2*d^2*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^3\cosh(dx+c)}{bx^2+a}\,dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^3*e^(2*c) - x^2*e^(2*c))*e^(d*x) - (d*x^3 + x^2)*e^(-d*x))/(b*d^2
*x^2*e^c + a*d^2*e^c) - 1/2*integrate(2*(a*d*x^2*e^c - a*x*e^c)*e^(d*x)/(b^
2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2), x) + 1/2*integrate(2*(a*d*x^2 + a*x)*
e^(-d*x)/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c), x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^3 \cosh\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{b\,x^2+a}\,dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^2),x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^2), x)
```

3.59 $\int \frac{x^2 \cosh(c+dx)}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2} \, dx = \frac{\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}} + \frac{\sinh(c+dx)}{bd} - \frac{\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}$$

```
[Out] sinh(d*x+c)/b/d-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/
2))*(-a)^(1/2)/b^(3/2)+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/
2)/b^(1/2))*(-a)^(1/2)/b^(3/2)-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(
-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(3/2)+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sin
h(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{5401, 2717, 5389, 3384, 3379, 3382\}$

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sinh(c+dx)}{bd}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sinh[c + d*x]/(b*d) - (Sqrt[-a]
*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b^(3/2)) - (Sqrt[-a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*b^(3/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b\,(a+bx^2)}\right) dx \\
&= \frac{\int \cosh(c+dx)\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{b} \\
&= \frac{\sinh(c+dx)}{bd} - \frac{a\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{b} \\
&= \frac{\sinh(c+dx)}{bd} - \frac{\sqrt{-a}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} - \frac{\sqrt{-a}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} \\
&= \frac{\sinh(c+dx)}{bd} - \frac{\left(\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} \\
&\quad - \frac{\left(\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} \\
&\quad - \frac{\left(\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} \\
&\quad + \frac{\left(\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} \\
&= \frac{\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}} \\
&\quad + \frac{\sinh(c+dx)}{bd} - \frac{\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} \\
&\quad - \frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.98

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \frac{i\sqrt{a}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{4b^{3/2}} - \frac{i\sqrt{a}e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)-\text{ExpIntegralEi}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)\right)}{4b^{3/2}} + \frac{\cosh(dx)\sinh(c)}{bd}+\frac{\cosh(c)\sinh(dx)}{bd}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] ((I/4)*Sqrt[a]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])
*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d*((I*Sqrt[a
])/Sqrt[b] + x)]))/b^(3/2) - ((I/4)*Sqrt[a]*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*
(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x
] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/b^(3/2) + (Cosh[d*x]*Sinh[
c])/(b*d) + (Cosh[c]*Sinh[d*x])/(b*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.15

method	result
risch	$\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)a}{4b\sqrt{-ab}} - \frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)a}{4b\sqrt{-ab}} + \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)a}{4b\sqrt{-ab}}$

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)
*b+c*b)/b)*a-1/4/b/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b
)^(1/2)+(d*x+c)*b-c*b)/b)*a+1/4/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a-1/4/b/(-a*b)^(1/2)*exp(-(-d*(-a*
b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a-1/2/d/b*exp(-d*x-
c)+1/2/d/b*exp(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(170) = 340$.

Time = 0.26 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.19

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx$$
$$= \frac{\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh(dx+c)^2 - \sinh(dx+c)^2\right)\mathrm{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \sqrt{-\frac{ad^2}{b}}\left(\cosh(dx+c)^2 - \sinh(dx+c)^2\right)\mathrm{I}\right.}{}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d
^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(
-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - s
inh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2
- sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (
sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b))
 - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2
/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*
x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh
(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)) + 4*sinh(
d*x + c))/(b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \cosh(dx+c)}{bx^2+a}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a*integrate(x*e^(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + a*integr
ate(x*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x) + 1/2*(x^2
*e^(d*x + 2*c) - x^2*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \cosh(dx+c)}{bx^2+a}\,dx$$

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{b\,x^2+a}\,dx$$

[In] int((x^2*cosh(c + d*x))/(a + b*x^2),x)

[Out] int((x^2*cosh(c + d*x))/(a + b*x^2), x)

3.60 $\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 177

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}$$

```
[Out] 1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Chi(-d
*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b+1/2*Shi(d*x+d*(-a)^
(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1
/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used

$= \{5401, 3384, 3379, 3382\}$

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]
*d)/Sqrt[b] + d*x])/(2*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2\sqrt{b}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2\sqrt{b}}$$

$$= \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{2\sqrt{b}} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{2\sqrt{b}} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{2\sqrt{b}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{2\sqrt{b}}$$

$$= \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x\cosh(c+dx)}{a+bx^2} dx = \frac{e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{2c+\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+e^{2c}\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegr}\right.}{4b}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2),x]
[Out] (E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(2*c + ((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpInt
egralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + E^(2*c)*ExpIntegralEi[d*((I*Sqrt[
a])/Sqrt[b] + x)] + E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[
a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/(4*b)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4b}-\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4b}-\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4b}-$

```
[In] int(x*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1
/4/b*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1
/4/b*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1
/4/b*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.24

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \frac{\left(\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\operatorname{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\operatorname{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\operatorname{Ei}\left(-dx-\sqrt{-\right.\right.}{}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-
a*d^2/b)) + (Ei(d*x + sqrt(-a*d^2/b)) + Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c
+ sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*
sinh(c + sqrt(-a*d^2/b)) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^
2/b)))*sinh(-c + sqrt(-a*d^2/b)))/b
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh\left(c+dx\right)}{a+bx^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c) + 1/2*integrate((b*x^2*e^c - a*e^c)*e^(d*x)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 1/2*integrate((b*x^2 - a)*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x)

Giac [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh(c+d\,x)}{b\,x^2+a}\,dx$$

[In] int((x*cosh(c + d*x))/(a + b*x^2),x)

[Out] int((x*cosh(c + d*x))/(a + b*x^2), x)

3.61 $\int \frac{\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+bx^2}\,dx =& \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}\\ &-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}\\ &-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}\\ &-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}\end{aligned}$$

```
[Out] -1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(1/2)/
b^(1/2)+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a
)^(1/2)/b^(1/2)-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/
2))/(-a)^(1/2)/b^(1/2)+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2
)/b^(1/2))/(-a)^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5389, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x^2),x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx\\
&= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}\\
&= -\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}-\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}\\
&\quad -\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}+\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}\\
&= \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}\\
&\quad -\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = -\frac{ie^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{2c+\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-e^{2c}\,\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpInte}\right.}{4\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2),x]
[Out] ((-1/4*I)*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(2*c + ((2*I)*Sqrt[a]*d)/Sqrt[b
])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - E^(2*c)*ExpIntegralEi[d*
((I*Sqrt[a])/Sqrt[b] + x)] - E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((
-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x])
)/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4\sqrt{-ab}}+\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4\sqrt{-ab}}-\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4\sqrt{-ab}}$

```
[In] int(cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*
b+c*b)/b)+1/4/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/
2)+(d*x+c)*b-c*b)/b)-1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d
*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)
/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.

Time = 0.26 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx =$$
$$-\frac{\left(\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-\left(\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx+\right.\right.}{}$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/
b)) + (sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/
b)))/(a*d)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{a+bx^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh(c+d\,x)}{b\,x^2+a}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*x^2),x)
[Out] int(cosh(c + d*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a} + \frac{\sinh(c)\text{Shi}(dx)}{a} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a}$$

```
[Out] Chi(d*x)*cosh(c)/a-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^
(1/2))/a-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a+
Shi(d*x)*sinh(c)/a-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^
(1/2))/a-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

$= \{5401, 3384, 3379, 3382\}$

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = -\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} + \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a) + (Sinh[c]*SinhIntegral[d*x])/a
 + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(2*a) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(2*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx\cosh(c+dx)}{a(a+bx^2)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a}$$

$$= -\frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a}$$

$$= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a}$$

$$= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\left(\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a} + \frac{\left(\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a} - \frac{\left(\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a} - \frac{\left(\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a}$$

$$= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a} + \frac{\sinh(c)\text{Shi}(dx)}{a} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = -\frac{-4\cosh(c)\text{Chi}(dx) + e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{2c+\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right) + e^{2c}\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+\right.\right.\right.}{4a}$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] -1/4*(-4*Cosh[c]*CoshIntegral[d*x] + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(2*c
 + ((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]
 + E^(2*c)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)] + E^(((2*I)*Sqrt[a]*d
)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I
*Sqrt[a]*d)/Sqrt[b] - d*x]) - 4*Sinh[c]*SinhIntegral[d*x])/a
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)}{2a}-\frac{\mathrm{e}^{c}\,\mathrm{Ei}_1(-dx)}{2a}+\frac{\mathrm{e}^{\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4a}+\frac{\mathrm{e}^{\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4a}+\frac{\mathrm{e}^{-\frac{d\sqrt{-ab}+}{b}}}{}$

```
[In] int(cosh(d*x+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)-1/2/a*exp(c)*Ei(1,-d*x)+1/4/a*exp((d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a*exp((-d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/4/a*exp(-(d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a*exp(-(-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.26

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)}\,dx = -\frac{\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-2\left(\mathrm{Ei}(dx)+\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+\left(\mathrm{E}\right.}{}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) - 2*(Ei(d*x) + Ei(-d*x))*cosh(c) + (Ei(d*x + sqrt(-a*d^2/b)) + E
i(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2
/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 2*(Ei(d*x) - E
i(-d*x))*sinh(c) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^2/b)))*s
inh(-c + sqrt(-a*d^2/b)))/a
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\cosh\,(c+dx)}{x\,(a+bx^2)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\cosh\,(dx+c)}{(bx^2+a)x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\cosh\,(dx+c)}{(bx^2+a)x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\cosh(c+d\,x)}{x\,(b\,x^2+a)}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x^2)),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)), x)
```

3.63 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 249

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = -\frac{\cosh(c+dx)}{ax} + \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}$$

```
[Out] -cosh(d*x+c)/a/x+d*cosh(c)*Shi(d*x)/a+d*Chi(d*x)*sinh(c)/a-1/2*Chi(d*x+d*(-
a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)+1/2*Chi(-
d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)-1
/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^
(3/2)+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2
)/(-a)^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \frac{\sqrt{b}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{d\sinh(c)\text{Chi}(dx)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{\cosh(c+dx)}{ax}$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) + (Sqrt[b]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) - (Sqrt[b]*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2)) + (d
*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - (Sqrt[b]*
Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2
*(-a)^(3/2)) - (Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b\cosh(c+dx)}{a\left(a+bx^2\right)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} \\
&\quad + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a} + \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{a}
\end{aligned}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{\left(b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{\left(b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{\left(b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} + \frac{\left(b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98

$$\int\frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = -\frac{\cosh(c)\cosh(dx)}{ax} + \frac{i\sqrt{b}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{4a^{3/2}} - \frac{i\sqrt{b}e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)-\text{ExpIntegralEi}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)\right)}{4a^{3/2}} - \frac{\sinh(c)\sinh(dx)}{ax} + \frac{d(\text{Chi}(dx)\sinh(c)+\cosh(c)\text{Shi}(dx))}{a}$$

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
[Out] -((Cosh[c]*Cosh[d*x])/(a*x)) + ((I/4)*Sqrt[b]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])
*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x
```

```
)] - ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/a^(3/2) - ((I/4)*Sqrt[b]*
E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi
[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*
x]))/a^(3/2) - (Sinh[c]*Sinh[d*x])/(a*x) + (d*(CoshIntegral[d*x]*Sinh[c] +
Cosh[c]*SinhIntegral[d*x]))/a
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{e^{-dx-c}}{2ax}+\frac{b\,e^{-\frac{d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4a\sqrt{-ab}}-\frac{b\,e^{-\frac{-d\sqrt{-ab}+cb}{b}}\operatorname{Ei}_1\left(\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4a\sqrt{-ab}}+\frac{d\,e^{-c}\operatorname{Ei}_1(dx)}{2a}-\frac{e^{dx+c}}{2ax}$

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-d*x-c)/a/x+1/4*b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1
/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/2*d/a*exp(-c)*Ei(1,d*x
)-1/2*exp(d*x+c)/a/x-1/4*b/a/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/4*b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/2*d/a*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(193) = 386.

Time = 0.27 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.41

$$\int \frac{\cosh(c+dx)}{x^2\,(a+bx^2)}\,dx = -\frac{4\,ad\cosh\left(dx+c\right)-\left(\left(bx\cosh\left(dx+c\right)^2-bx\sinh\left(dx+c\right)^2\right)\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\left(bx\cosh\left(dx+\right.\right.\right.}{}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*cosh(d*x + c) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 - 2*(a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*cosh(c) + ((b*x*cosh(d*x + c)^2 -
 b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x*cosh(d
*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*
cosh(-c + sqrt(-a*d^2/b)) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x
```

```
 + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b))
 - 2*(a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c) - ((b*x*cosh(d*x + c)^2 -
 b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x*cosh(d
*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*
sinh(-c + sqrt(-a*d^2/b)))/(a^2*d*x*cosh(d*x + c)^2 - a^2*d*x*sinh(d*x + c)
^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,\left(b\,x^2+a\right)}\,dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)), x)
```

3.64 $\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx =& -\frac{\cosh(c+dx)}{2ax^2}-\frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2}+\frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a}\\&+\frac{b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\&+\frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}-\frac{d\sinh(c+dx)}{2ax}\\&-\frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2}+\frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a}\\&-\frac{b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\&+\frac{b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}\end{aligned}$$

```
[Out] -b*Chi(d*x)*cosh(c)/a^2+1/2*d^2*Chi(d*x)*cosh(c)/a-1/2*cosh(d*x+c)/a/x^2+1/
2*b*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*Ch
i(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2-b*Shi(d*x)*si
nh(c)/a^2+1/2*d^2*Shi(d*x)*sinh(c)/a-1/2*d*sinh(d*x+c)/a/x+1/2*b*Shi(d*x+d*
(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*Shi(d*x-d*(-a)^(
1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5401, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = -\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} +\frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} -\frac{b\sinh(c)\text{Shi}(dx)}{a^2}-\frac{b\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} +\frac{b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a} +\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a*x^2) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (d^2*Cosh[
c]*CoshIntegral[d*x])/(2*a) + (b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*Sinh[c + d*x])/(2*a*
x) - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2
*a) - (b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(2*a^2) + (b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2x\cosh(c+dx)}{a^2(a+bx^2)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b^2\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a^2} \\
&\quad + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a} - \frac{(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^2} - \frac{(b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\text{Chi}(dx)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\text{Shi}(dx)}{a^2} \\
&\quad - \frac{b^{3/2}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} + \frac{b^{3/2}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} + \frac{d^2\int \frac{\cosh(c+dx)}{x}\,dx}{2a}
\end{aligned}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\text{Chi}(dx)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\text{Shi}(dx)}{a^2} + \frac{(d^2\cosh(c))\int\frac{\cosh(dx)}{x}dx}{2a} + \frac{\left(b^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^2} - \frac{\left(b^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^2} + \frac{(d^2\sinh(c))\int\frac{\sinh(dx)}{x}dx}{2a} + \frac{\left(b^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^2} + \frac{\left(b^{3/2}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^2}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\text{Chi}(dx)}{a^2} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a} + \frac{b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} + \frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\text{Shi}(dx)}{a^2} + \frac{d^2\sinh(c)\text{Shi}(dx)}{2a} - \frac{b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} + \frac{b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.86

$$\int\frac{\cosh(c+dx)}{x^3(a+bx^2)}dx = \frac{be^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+be^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIn}\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] (b*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegral
Ei[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] +
 x)]) + b*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*Exp
IntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d)/Sq
rt[b] - d*x]) - (2*a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/x^2 - (2*a*(d*x*Cos
h[c] + Sinh[c])*Sinh[d*x])/x^2 + 2*(-2*b + a*d^2)*(Cosh[c]*CoshIntegral[d*x
] + Sinh[c]*SinhIntegral[d*x]))/(4*a^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.22

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}}{4ax}-\frac{\mathrm{e}^{-dx-c}}{4a\,x^{2}}-\frac{b\,\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_{1}\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4a^{2}}-\frac{b\,\mathrm{e}^{-\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_{1}\left(\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4a^{2}}-\frac{d^{2}\mathrm{e}^{-c}\,\mathrm{Ei}_{1}(d}{4a}$

```
[In] int(cosh(d*x+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2-1/4*b/a^2*exp(-(d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a^2*exp(-(-d*(-a*b)^(
1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4*d^2/a*exp(-c)*Ei(1,
d*x)+1/2/a^2*exp(-c)*Ei(1,d*x)*b-1/4*d/a/x*exp(d*x+c)-1/4/a/x^2*exp(d*x+c)-
1/4*b/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b
)-1/4*b/a^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*
b)/b)-1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/2/a^2*exp(c)*Ei(1,-d*x)*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(222) = 444$.

Time = 0.28 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.16

$$\int \frac{\cosh(c+dx)}{x^3\,(a+bx^2)}\,dx =$$

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\,a\cosh\left(dx+c\right)-\left(\left(bx^{2}\cosh\left(dx+c\right)^{2}-bx^{2}\sinh\left(dx+c\right)^{2}\right)\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^{2}}{b}}\right)\right.}{}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((b*x^2*cosh(d*x + c)^2 -
 b*x^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x^2*cosh(d*x + c)^2 -
 b*x^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 - ((a*d^2 - 2*b)*x^2*Ei(d*x) + (a*d^2 - 2*b)*x^2*Ei(-d*x))*cosh(c) - ((b*x
^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x
^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh
(-c + sqrt(-a*d^2/b)) - ((b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei
(d*x - sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei
(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((a*d^2 - 2*b)*x^2*Ei(d
*x) - (a*d^2 - 2*b)*x^2*Ei(-d*x))*sinh(c) + ((b*x^2*cosh(d*x + c)^2 - b*x^2
*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2
*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/(a^
2*x^2*cosh(d*x + c)^2 - a^2*x^2*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}\,dx = \int \frac{\cosh\left(c+dx\right)}{x^3\left(a+bx^2\right)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^2+a)x^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^2+a)x^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,\left(b\,x^2+a\right)}\,dx$$

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x^2)),x)
[Out] int(cosh(c + d*x)/(x^3*(a + b*x^2)), x)
```

3.65 $\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 449

$$\begin{aligned}\int \frac{x^4\cosh(c+dx)}{(a+bx^2)^2}\,dx &= \frac{x\cosh(c+dx)}{2b^2}-\frac{x^3\cosh(c+dx)}{2b(a+bx^2)} \\ &+\frac{3\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ &-\frac{3\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} \\ &-\frac{ad\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} \\ &-\frac{ad\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3}+\frac{\sinh(c+dx)}{b^2d} \\ &+\frac{ad\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} \\ &-\frac{3\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ &-\frac{ad\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3} \\ &-\frac{3\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}\end{aligned}$$

```
[Out] 1/2*x*cosh(d*x+c)/b^2-1/2*x^3*cosh(d*x+c)/b/(b*x^2+a)-1/4*a*d*cosh(c+d*(-a)
^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/b^3-1/4*a*d*cosh(c-d*(-a)^(1/
2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/b^3+sinh(d*x+c)/b^2/d-1/4*a*d*Chi
```

```
(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^3-1/4*a*d*Chi(-d*
x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^3-3/4*Chi(d*x+d*(-a)
^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)+3/4*Chi(-d*
x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-3/4
*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5
/2)+3/4*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/
2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5399, 5401, 2717, 5389, 3384, 3379, 3382, 5400, 3377}

$$\begin{aligned}\int \frac{x^4\cosh(c+dx)}{(a+bx^2)^2}\,dx = {} & \frac{3\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ & -\frac{3\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} \\ & -\frac{3\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\ & -\frac{3\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} \\ & -\frac{ad\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} \\ & -\frac{ad\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} \\ & +\frac{ad\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} \\ & -\frac{ad\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} \\ & -\frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)}+\frac{\sinh(c+dx)}{b^2d}+\frac{x\cosh(c+dx)}{2b^2}\end{aligned}$$

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (x*Cosh[c + d*x])/(2*b^2) - (x^3*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (3*Sqrt
[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(4*b^(5/2)) - (3*Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) - (a*d*CoshIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) - (a*d*CoshIntegral[(Sq
```

```
rt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) + Sinh[c +
 d*x]/(b^2*d) + (a*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-
a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(4*b^(5/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
```

```
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{3\int\frac{x^2\cosh(c+dx)}{a+bx^2}\,dx}{2b}+\frac{d\int\frac{x^3\sinh(c+dx)}{a+bx^2}\,dx}{2b}\\
&=-\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{3\int\left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b(a+bx^2)}\right)dx}{2b}+\frac{d\int\left(\frac{x\sinh(c+dx)}{b}-\frac{ax\sinh(c+dx)}{b(a+bx^2)}\right)dx}{2b}\\
&=-\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{3\int\cosh(c+dx)\,dx}{2b^2}-\frac{(3a)\int\frac{\cosh(c+dx)}{a+bx^2}\,dx}{2b^2}\\
&\quad+\frac{d\int x\sinh(c+dx)\,dx}{2b^2}-\frac{(ad)\int\frac{x\sinh(c+dx)}{a+bx^2}\,dx}{2b^2}\\
&=\frac{x\cosh(c+dx)}{2b^2}-\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{3\sinh(c+dx)}{2b^2d}\\
&\quad-\frac{\int\cosh(c+dx)\,dx}{2b^2}-\frac{(3a)\int\left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2}\\
&\quad-\frac{(ad)\int\left(-\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2}\\
&=\frac{x\cosh(c+dx)}{2b^2}-\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{\sinh(c+dx)}{b^2d}-\frac{\left(3\sqrt{-a}\right)\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}\\
&\quad-\frac{\left(3\sqrt{-a}\right)\int\frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}+\frac{(ad)\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{5/2}}-\frac{(ad)\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{5/2}}
\end{aligned}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{\sinh(c+dx)}{b^2d} - \frac{\left(3\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2} - \frac{\left(ad\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{5/2}} - \frac{\left(3\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} - \frac{\left(ad\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{5/2}} - \frac{\left(3\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2} - \frac{\left(ad\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{5/2}} + \frac{\left(3\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} + \frac{\left(ad\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{5/2}}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{3\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{3\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} - \frac{ad\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} - \frac{ad\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} + \frac{\sinh(c+dx)}{b^2d} + \frac{ad\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} - \frac{3\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{ad\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3} - \frac{3\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.68

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} dx$$
$$= \frac{-\sqrt{a}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-3i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(3i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\right.\right.}{}$$

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (-(Sqrt[a]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^((
(2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + (
(3*I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)])) + S
qrt[a]*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*
I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((3*
I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + 4*b*C
osh[d*x]*((a*x*Cosh[c])/(a + b*x^2) + (2*Sinh[c])/d) + 4*b*((2*Cosh[c])/d +
 (a*x*Sinh[c])/(a + b*x^2))*Sinh[d*x])/(8*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. 2(349) = 698.

Time = 0.42 (sec) , antiderivative size = 1038, normalized size of antiderivative = 2.31

method	result	size
risch	Expression too large to display	1038

```
[In] int(x^4*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*
x+c)*b+c*b)/b)*a*b*d^2*x^2-(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b*d^2*x^2-(-a*b)^(1/2)*exp((-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b*d^2*x^2+(-a*b)^(1/2)
*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b*d
^2*x^2-3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/
b)*a*b^2*d*x^2-3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b
+c*b)/b)*a*b^2*d*x^2+3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(
d*x+c)*b-c*b)/b)*a*b^2*d*x^2+3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)
^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*d*x^2+exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1
/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*d^2-(-a*b)^(1/2)*exp((d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*d^2-(-a*b)^(1/
2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2
```

```
*d^2+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c
)*b-c*b)/b)*a^2*d^2-3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d
*x+c)*b+c*b)/b)*a^2*b*d-2*(-a*b)^(1/2)*exp(d*x+c)*a*b*d*x-4*(-a*b)^(1/2)*ex
p(d*x+c)*b^2*x^2-2*(-a*b)^(1/2)*exp(-d*x-c)*a*b*d*x+4*(-a*b)^(1/2)*exp(-d*x
-c)*b^2*x^2-3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*
b)/b)*a^2*b*d+3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*
b-c*b)/b)*a^2*b*d+3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x
+c)*b-c*b)/b)*a^2*b*d-4*(-a*b)^(1/2)*exp(d*x+c)*a*b+4*(-a*b)^(1/2)*exp(-d*x
-c)*a*b)/b^3/(b*x^2+a)/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(349) = 698$.

Time = 0.27 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.63

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
(a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^
2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b
)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sin
h(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*
x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b
)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*si
nh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d
*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d
^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2
 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 8*(b^2*x^2 + a*b)*si
nh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^
2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*
b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^
2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3*
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x
^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3
*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-
a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)
^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x
+ c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a
*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((b^4*d*x^2 + a*b^3*d)*cosh(d*x + c)^2
 - (b^4*d*x^2 + a*b^3*d)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4 \cosh{(c+dx)}}{(a+bx^2)^2}\,dx$$

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a)**2,x)
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4 \cosh{(dx+c)}}{(bx^2+a)^2}\,dx$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*((b*d*x^4*e^(2*c) - 4*a*x*e^(2*c))*e^(d*x) - (b*d*x^4 + 4*a*x)*e^(-d*x)
)/(b^3*d^2*x^4*e^c + 2*a*b^2*d^2*x^2*e^c + a^2*b*d^2*e^c) - 1/2*integrate(-
4*(a^2*d*x*e^c - 3*a*b*x^2*e^c + a^2*e^c)*e^(d*x)/(b^4*d^2*x^6 + 3*a*b^3*d^
2*x^4 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 1/2*integrate(4*(a^2*d*x + 3*a
*b*x^2 - a^2)*e^(-d*x)/(b^4*d^2*x^6*e^c + 3*a*b^3*d^2*x^4*e^c + 3*a^2*b^2*d
^2*x^2*e^c + a^3*b*d^2*e^c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.66 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 431

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}\,dx &= \frac{\cosh(c+dx)}{2b^2}-\frac{x^2\cosh(c+dx)}{2b(a+bx^2)}+\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}\\
&+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}\\
&-\frac{\sqrt{-a}d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}\\
&+\frac{\sqrt{-a}d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}\\
&-\frac{\sqrt{-a}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}\\
&-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}\\
&-\frac{\sqrt{-a}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}\\
&+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}\end{aligned}$$

```
[Out] 1/2*cosh(d*x+c)/b^2-1/2*x^2*cosh(d*x+c)/b/(b*x^2+a)+1/2*Chi(d*x+d*(-a)^(1/2
)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b^2+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/
2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b^2+1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh
(c-d*(-a)^(1/2)/b^(1/2))/b^2+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a
)^(1/2)/b^(1/2))/b^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2
```

```
)/b^(1/2))*(-a)^(1/2)/b^(5/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*
(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-1/4*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*
sinh(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)+1/4*d*Chi(-d*x+d*(-a)^(1/2)
/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5399, 5401, 3384, 3379, 3382, 5400, 2718, 5388}

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}\,dx = &-\frac{\sqrt{-a}d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}\\&+\frac{\sqrt{-a}d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}\\&-\frac{\sqrt{-a}d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}\\&-\frac{\sqrt{-a}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}\\&+\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}\\&+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}\\&-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}\\&+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}\\&-\frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)}+\frac{\cosh(c+dx)}{2b^2}\end{aligned}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] Cosh[c + d*x]/(2*b^2) - (x^2*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (C
osh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
b^2) - (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[
-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) - (Sqrt[-a]*d*Cosh[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)
) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
```

```
])/(2*b^2) - (Sqrt[-a]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{b} + \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b} + \frac{d\int \left(\frac{\sinh(c+dx)}{b} - \frac{a\sinh(c+dx)}{b(a+bx^2)}\right)dx}{2b}$$

$$= -\frac{x^2\cosh(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{d\int \sinh(c+dx)\,dx}{2b^2} - \frac{(ad)\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{2b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} - \frac{(ad)\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} - \frac{\left(\sqrt{-a}d\right)\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} - \frac{\left(\sqrt{-a}d\right)\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} - \frac{\left(\sqrt{-a}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2} + \frac{\left(\sqrt{-a}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} - \frac{\left(\sqrt{-a}d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2} - \frac{\left(\sqrt{-a}d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} - \frac{\sqrt{-a}d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\sqrt{-a}d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} - \frac{\sqrt{-a}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{\sqrt{-a}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.67

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}\,dx$$

$$= \frac{\frac{4a\sqrt{b}\cosh(c)\cosh(dx)}{a+bx^2} + e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(2\sqrt{b}+i\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\mathrm{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right) + \left(2\sqrt{b}-i\sqrt{a}d\right)\mathrm{ExpI}\right.}{}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((4*a*Sqrt[b]*Cosh[c]*Cosh[d*x])/(a + b*x^2) + E^(c - (I*Sqrt[a]*d)/Sqrt[b]
)*((2*Sqrt[b] + I*Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*
(((-I)*Sqrt[a])/Sqrt[b] + x)] + (2*Sqrt[b] - I*Sqrt[a]*d)*ExpIntegralEi[d*(
(I*Sqrt[a])/Sqrt[b] + x)]) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((2*Sqrt[b] + I
*Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sq
rt[b] - d*x] + (2*Sqrt[b] - I*Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b
] - d*x]) + (4*a*Sqrt[b]*Sinh[c]*Sinh[d*x])/(a + b*x^2))/(8*b^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(331) = 662$.

Time = 0.30 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{-\mathrm{e}^{\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}_{1}\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)abd\,x^{2}+\mathrm{e}^{\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{Ei}_{1}\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)abd\,x^{2}+\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}_{1}\left(-\frac{d\sqrt{-ab}-(dx+c)}{b}\right.}{}$

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a
*b*d*x^2+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/
b)*a*b*d*x^2+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c
*b)/b)*a*b*d*x^2-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)
*b-c*b)/b)*a*b*d*x^2+2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*
x+c)*b+c*b)/b)*(-a*b)^(1/2)*b*x^2+2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(
-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*(-a*b)^(1/2)*b*x^2+2*exp(-(d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*b*x^2+2*(-a*b)^(1
/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b*
x^2-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*
d+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2*
d+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*
d-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2*
d+2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-a*
b)^(1/2)*a+2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c
*b)/b)*(-a*b)^(1/2)*a+2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*a+2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/
b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a-2*a*(-a*b)^(1/2)*exp(-d*x-c)-2*
a*(-a*b)^(1/2)*exp(d*x+c))/b^2/(b*x^2+a)/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(331) = 662$.

Time = 0.28 (sec) , antiderivative size = 931, normalized size of antiderivative = 2.16

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(d*x + sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)
) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d
*x - sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(
d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((2*(b*
x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cos
h(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2
+ ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((b^3*x^2 + a*b^2)
*cosh(d*x + c)^2 - (b^3*x^2 + a*b^2)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^2} \, dx$$

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((d^2*x^3*e^(2*c) + d*x^2*e^(2*c) + 2*x*e^(2*c))*e^(d*x) - (d^2*x^3 - d*x^2 + 2*x)*e^(-d*x))/(b^2*d^3*x^4*e^c + 2*a*b*d^3*x^2*e^c + a^2*d^3*e^c) - 1/2*integrate(2*(2*a*d*x*e^c + (2*a*d^2*e^c - 3*b*e^c)*x^2 + a*e^c)*e^(d*x)/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3), x) + 1/2*integrate(-2*(2*a*d*x - (2*a*d^2 - 3*b)*x^2 - a)*e^(-d*x)/(b^3*d^3*x^6*e^c + 3*a*b^2*d^3*x^4*e^c + 3*a^2*b*d^3*x^2*e^c + a^3*d^3*e^c), x)

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^2} \, dx$$

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} \, dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^2+a)^2} \, dx$$

[In] int((x^3*cosh(c + d*x))/(a + b*x^2)^2,x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x^2)^2, x)

3.67 $\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx = & -\frac{x\cosh(c+dx)}{2b(a+bx^2)}+\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} \\ & -\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} \\ & +\frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} \\ & +\frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} \\ & -\frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} \\ & -\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} \\ & +\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2} \\ & -\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}\end{aligned}$$

```
[Out] -1/2*x*cosh(d*x+c)/b/(b*x^2+a)+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d
*(-a)^(1/2)/b^(1/2))/b^2+1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^
(1/2)/b^(1/2))/b^2+1/4*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/
b^(1/2))/b^2+1/4*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/
```

```
2))/b^2-1/4*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b^(3
/2)/(-a)^(1/2)+1/4*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/
2))/b^(3/2)/(-a)^(1/2)-1/4*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2
)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-
a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5399, 5389, 3384, 3379, 3382, 5400}

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx = \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} + \frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} - \frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} - \frac{x\cosh(c+dx)}{2b\,(a+bx^2)}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(x*Cosh[c + d*x])/(b*(a + b*x^2)) + (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) - (Cosh[c - (S
qrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b
^(3/2)) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)
/Sqrt[b]])/(4*b^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (
Sqrt[-a]*d)/Sqrt[b]])/(4*b^2) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*
SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Cosh[c
```

```
- (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2) -
 (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(4*Sqrt[-a]*b^(3/2))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{x\cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x\cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b} + \frac{d\int \left(-\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}$$

$$= -\frac{x\cosh(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{3/2}} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{3/2}}$$

$$= -\frac{x\cosh(c+dx)}{2b(a+bx^2)} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b} + \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{3/2}} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b} + \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{3/2}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b} + \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{3/2}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{3/2}}$$

$$= -\frac{x\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} + \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.69

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx =$$

$$-\frac{\frac{4bx\cosh(c)\cosh(dx)}{a+bx^2} - \frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{a}} + \cdots}{8b^2}$$

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] -1/8*((4*b*x*Cosh[c]*Cosh[d*x])/(a + b*x^2) - (E^(c - (I*Sqrt[a]*d)/Sqrt[b]
)*(((-I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d
*(((-I)*Sqrt[a])/Sqrt[b] + x)] + (I*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[d*((
I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[a] + (E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((-I)
*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqr
t[a]*d)/Sqrt[b] - d*x] + (I*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d
)/Sqrt[b] - d*x]))/Sqrt[a] + (4*b*x*Sinh[c]*Sinh[d*x])/(a + b*x^2))/b^2

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(318) = 636.

Time = 0.25 (sec) , antiderivative size = 901, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{\mathrm{e}^{\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)\sqrt{-ab}\,bd\,x^2+\mathrm{e}^{\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)\sqrt{-ab}\,bd\,x^2-\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(-\cdots\right.}{\cdots}$

[In] int(x^2*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*(exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-
a*b)^(1/2)*b*d*x^2+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)*(-a*b)^(1/2)*b*d*x^2-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a
*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*b*d*x^2-(-a*b)^(1/2)*exp(-(-d*(-a*
b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b*d*x^2+exp((d*(-a*
b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^2*x^2-exp((-d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b^2*x^2+exp(-(d*(
-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^2*x^2-exp(-(-
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b^2*x^2+exp((
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*
a*d+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*(-
a*b)^(1/2)*a*d-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b

```
+c*b)/b)*(-a*b)^(1/2)*a*d-(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*d+exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-
a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b-exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*
b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b+2*exp(-d*x-c)*x*b*(-a*b)^(1/2)+exp(-(d*(-a*b
)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b+2*exp(d*x+c)*x*
b*(-a*b)^(1/2)-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b
-c*b)/b)*a*b)/b^2/(b*x^2+a)/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(318) = 636$.

Time = 0.27 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.79

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^2)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
 (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2
 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)
) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh
(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
 c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(
d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x
 - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
 a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*
x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 -
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^3*d*x^
2 + a^2*b^2*d)*cosh(d*x + c)^2 - (a*b^3*d*x^2 + a^2*b^2*d)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \cosh\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**2,x)
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \cosh\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*((d*x^2*e^(2*c) + 2*x*e^(2*c))*e^(d*x) - (d*x^2 - 2*x)*e^(-d*x))/(b^2*d
^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c) + 1/2*integrate(-2*(2*a*d*x*e
^c - 3*b*x^2*e^c + a*e^c)*e^(d*x)/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*
d^2*x^2 + a^3*d^2), x) + 1/2*integrate(2*(2*a*d*x + 3*b*x^2 - a)*e^(-d*x)/(
b^3*d^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*e^c),
 x)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \cosh\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{\left(b\,x^2+a\right)^2}\,dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^2*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.68 $\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$

```
[Out] -1/2*cosh(d*x+c)/b/(b*x^2+a)+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(
-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Sh
i(d*x+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)-1/4*d*Chi(d*x+d*(-a)^(1/2)/b
^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*d*Chi(-d*x+d*(-
a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5397, 5388, 3384, 3379, 3382}

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cosh(c+dx)}{2b(a+bx^2)}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Cosh[c + d*x]/(b*(a + b*x^2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (d*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b
^(3/2)) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[
b] - d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{d\int\frac{\sinh(c+dx)}{a+bx^2}\,dx}{2b}\\
&= -\frac{\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{d\int\left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}\\
&= -\frac{\cosh(c+dx)}{2b\,(a+bx^2)}-\frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b}-\frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}\\
&= -\frac{\cosh(c+dx)}{2b\,(a+bx^2)}-\frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}\\
&\quad+\frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b}\\
&\quad-\frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}\\
&\quad-\frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b}\\
&= -\frac{\cosh(c+dx)}{2b\,(a+bx^2)}-\frac{d\,\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\,\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}\\
&\quad-\frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \frac{-\frac{4\sqrt{b}\cosh(c)\cosh(dx)}{a+bx^2} - \frac{ide^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{a}} - \frac{ide^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpI}\right.}{}}{8b^{3/2}}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((-4*Sqrt[b]*Cosh[c]*Cosh[d*x])/(a + b*x^2) - (I*d*E^(c - (I*Sqrt[a]*d)/Sqr
t[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b
] + x)] - ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[a] - (I*d*E^(-c
 - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I
)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/
Sqrt[a] - (4*Sqrt[b]*Sinh[c]*Sinh[d*x])/(a + b*x^2))/(8*b^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(181) = 362.

Time = 0.23 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.88

method	result
risch	$-\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\text{Ei}_1\left(\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)bd\,x^2-e^{\frac{-d\sqrt{-ab}+cb}{b}}\text{Ei}_1\left(-\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)bd\,x^2-e^{-\frac{d\sqrt{-ab}+cb}{b}}\text{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+}{b}\right.}{}$

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b*
d*x^2-exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*
b*d*x^2-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b
)*b*d*x^2+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)
/b)*b*d*x^2+exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)
/b)*a*d-exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b
)*a*d-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*
a*d+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*
d+2*(-a*b)^(1/2)*exp(-d*x-c)+2*(-a*b)^(1/2)*exp(d*x+c))/(b*x^2+a)/b/(-a*b)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(181) = 362.

Time = 0.25 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.68

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{4\,a\cosh\left(dx+c\right)+\left(\left(\left(bx^2+a\right)\cosh\left(dx+c\right)^2-\left(bx^2+a\right)\sinh\left(dx+c\right)^2\right)\sqrt{-\frac{ad^2}{b}}\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)-(}{}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*cosh(d*x + c) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x
 + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2
/b)))*cosh(c + sqrt(-a*d^2/b)) - (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - ((b*x^2 + a)*c
osh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt
(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b
*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + ((b*x^
2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*
x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)
^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) +
 ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)
*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^2*x^2 + a^2*b)
*cosh(d*x + c)^2 - (a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\cosh\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d
*e^c) + 1/2*integrate((3*b*x^2*e^c - a*e^c)*e^(d*x)/(b^3*d*x^6 + 3*a*b^2*d*
x^4 + 3*a^2*b*d*x^2 + a^3*d), x) - 1/2*integrate((3*b*x^2 - a)*e^(-d*x)/(b^
3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a^2*b*d*x^2*e^c + a^3*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\cosh(c+d\,x)}{\left(b\,x^2+a\right)^2}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.69 $\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 476

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}$$

```
[Out] -1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d
*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d*Chi(d
```

```
*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d*Chi(-d*x+d*
(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a/b+1/4*Chi(d*x+d*(-a)^(1/
2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*Chi(-d*x+d*
(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*Shi
(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-
1/4*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b
^(1/2)-1/4*cosh(d*x+c)/a/b^(1/2)/((-a)^(1/2)-x*b^(1/2))+1/4*cosh(d*x+c)/a/b
^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5389, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} - \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^2,x]

[Out] -1/4*Cosh[c + d*x]/(a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*
Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshInteg
```

```
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-
a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt
[b]) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sq
rt[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqr
t[-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) +
(Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(
4*(-a)^(3/2)*Sqrt[b])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cosh(c+dx)}{2a\left(-ab-b^2x^2\right)} \right) dx$$

$$= -\frac{b\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{2a}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{b\int \left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a} - \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a} - \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a} + \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt{a}x\cosh(c)\cosh(dx)}{a+bx^2} - \frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{b} + \frac{e^{-c-}\cdots}{}}{8a^{3/2}}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] ((4*Sqrt[a]*x*Cosh[c]*Cosh[d*x])/(a + b*x^2) - (E^(c - (I*Sqrt[a]*d)/Sqrt[b
])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(
((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[d*(
(I*Sqrt[a])/Sqrt[b] + x)]))/b + (E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b]
 + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/
Sqrt[b] - d*x] + ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqr
t[b] - d*x]))/b + (4*Sqrt[a]*x*Sinh[c]*Sinh[d*x])/(a + b*x^2))/(8*a^(3/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.06

method	result
risch	$\frac{d^{2} \mathrm{e}^{-d x -c} x}{4 a \left(b \,d^{2} x^{2}+a \,d^{2}\right)}-\frac{d \,\mathrm{e}^{-\frac{d \sqrt{-a b}+c b}{b}} \operatorname{Ei}_{1}\left(-\frac{d \sqrt{-a b}-\left(d x +c \right) b +c b}{b}\right)}{8 b a}-\frac{d \,\mathrm{e}^{-\frac{-d \sqrt{-a b}+c b}{b}} \operatorname{Ei}_{1}\left(\frac{d \sqrt{-a b}+\left(d x +c \right) b -c b}{b}\right)}{8 b a}-\frac{\mathrm{e}^{-\frac{d \sqrt{-a b}+c b}{b}} \operatorname{Ei}_{1}}{8}$

```
[In] int(cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^2*exp(-d*x-c)*x/a/(b*d^2*x^2+a*d^2)-1/8*d/b/a*exp(-(d*(-a*b)^(1/2)+c*
b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d/b/a*exp(-(-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/(-a*b)^(1/2)/a*exp(-(
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/(-a*b)^(
1/2)/a*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
+1/4*d^2*exp(d*x+c)*x/a/(b*d^2*x^2+a*d^2)+1/8*d/b/a*exp((d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/b/a*exp((-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/(-a*b)^(1/2)/a*exp((d*(
-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/(-a*b)^(1/2)
/a*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(365) = 730.

Time = 0.35 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
(a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2
- (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
 - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(
d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) -
 (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d
*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*co
sh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*
cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x
- sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*co
sh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*
cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x -
```

```
sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + s
qrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 +
 a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a
*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x
^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + (
(b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*
d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^2*d*x
^2 + a^3*b*d)*cosh(d*x + c)^2 - (a^2*b^2*d*x^2 + a^3*b*d)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**2,x)
[Out] Integral(cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\cosh(c+d\,x)}{{\left(b\,x^2+a\right)}^2}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*x^2)^2,x)
[Out] int(cosh(c + d*x)/(a + b*x^2)^2, x)
```

3.70 $\int \frac{\cosh(c+dx)}{x(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 435

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx &= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}-\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\
&-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}\\
&-\frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}\\
&+\frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}\\
&-\frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}\\
&+\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\
&-\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}\\
&-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}\end{aligned}$$

```
[Out] Chi(d*x)*cosh(c)/a^2+1/2*cosh(d*x+c)/a/(b*x^2+a)-1/2*Chi(d*x+d*(-a)^(1/2)/b
^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))
*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2+Shi(d*x)*sinh(c)/a^2-1/2*Shi(d*x+d*(-a)^(
```

```
1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Shi(d*x-d*(-a)^(1/2)/b^(
1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*S
hi(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b
^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*Chi(d*x+d*(-
a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Chi
(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5401, 3384, 3379, 3382, 5397, 5388}

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = &-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\ &-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}\\ &+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\ &-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}\\ &+\frac{\sinh(c)\text{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}\\ &+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}\\ &-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}\\ &-\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh(c+dx)}{2a\left(a+bx^2\right)}\end{aligned}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c
 + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2)
- (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d
)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*Sin
hIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
```

```
-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqr
t[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*a^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{a^2x} - \frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)^2} - \frac{bx\cosh(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{2a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^2} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a^2}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} - \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} - \frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\left(\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} + \frac{\left(\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} - \frac{\left(\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} - \frac{\left(\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{3/2}} + \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{3/2}} - \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{3/2}} - \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{3/2}}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.95

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx$$

$$= \frac{\frac{4a\cosh(c)\cosh(dx)}{a+bx^2} + \frac{i\sqrt{a}de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{b}} - 2e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpI}\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((4*a*Cosh[c]*Cosh[d*x])/(a + b*x^2) + (I*Sqrt[a]*d*E^(c - (I*Sqrt[a]*d)/Sq
rt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[
b] + x)] - ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[b] - 2*E^(c -
(I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I
)*Sqrt[a])/Sqrt[b] + x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]) + (I
*Sqrt[a]*d*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*Ex
pIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/S
qrt[b] - d*x]))/Sqrt[b] - 2*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[
a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralE
i[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*a*Sinh[c]*Sinh[d*x])/(a + b*x^2) + 8*(
Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/(8*a^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.26

method	result
risch	$\frac{\mathrm{e}^{-dx-c}d^2}{4a\left((dx+c)^2b-2b(dx+c)c+a\,d^2+c^2b\right)} + \frac{\mathrm{e}^{-\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(\frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)d}{8a\sqrt{-ab}} - \frac{\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_1\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)d}{8a\sqrt{-ab}}$

```
[In] int(cosh(d*x+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(-d*x-c)*d^2/a/((d*x+c)^2*b-2*b*(d*x+c)*c+a*d^2+c^2*b)+1/8/a/(-a*b)^
(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*
d-1/8/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*
x+c)*b+c*b)/b)*d+1/4/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)
+(d*x+c)*b-c*b)/b)+1/4/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/
2)-(d*x+c)*b+c*b)/b)-1/2/a^2*exp(-c)*Ei(1,d*x)+1/4*exp(d*x+c)*d^2/a/((d*x+c
)^2*b-2*b*(d*x+c)*c+a*d^2+c^2*b)+1/8/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*d-1/8/a/(-a*b)^(1/2)*exp((-d*(-
a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d+1/4/a^2*exp((d
```

```
*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a^2*exp((-
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2/a^2*exp(
c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(337) = 674$.

Time = 0.27 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.28

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) - ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) + (b*x^2 + a)*Ei(-d*x))*cosh(c) - ((2*
(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*
cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqr
t(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - ((2*(b*x^2 + a
)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x +
 c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b
)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-
d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) -
(b*x^2 + a)*Ei(-d*x))*sinh(c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x
+ c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*
x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-
c + sqrt(-a*d^2/b)))/((a^2*b*x^2 + a^3)*cosh(d*x + c)^2 - (a^2*b*x^2 + a^3)
*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{x\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**2,x)
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^2 x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^2+a\right)^2}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x^2)^2),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)^2), x)
```

3.71 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 500

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx = &-\frac{\cosh(c+dx)}{a^2x}+\frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&-\frac{3\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}\\
&+\frac{3\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}}\\
&+\frac{d\mathrm{Chi}(dx)\sinh(c)}{a^2}+\frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}\\
&+\frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2}\\
&-\frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}\\
&+\frac{3\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}\\
&+\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}\\
&+\frac{3\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}}\end{aligned}$$

```
[Out] -cosh(d*x+c)/a^2/x+d*cosh(c)*Shi(d*x)/a^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2)
)*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(
```

```
d*x+d*(-a)^(1/2)/b^(1/2))/a^2+d*Chi(d*x)*sinh(c)/a^2+1/4*d*Chi(d*x+d*(-a)^(
1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*Chi(-d*x+d*(-a)^(1/2)/
b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2+3/4*Chi(d*x+d*(-a)^(1/2)/b^(1/2))
*cosh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)-3/4*Chi(-d*x+d*(-a)^(1/2)/
b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+3/4*Shi(d*x+d*(-a)
^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)-3/4*Shi(d*x
-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+1/4*
cosh(d*x+c)*b^(1/2)/a^2/((-a)^(1/2)-x*b^(1/2))-1/4*cosh(d*x+c)*b^(1/2)/a^2/
((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = {} & \frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} \\ & +\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} \\ & -\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} \\ & +\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} \\ & -\frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{d\sinh(c)\text{Chi}(dx)}{a^2}+\frac{d\cosh(c)\text{Shi}(dx)}{a^2} \\ & -\frac{\cosh(c+dx)}{a^2x}-\frac{3\sqrt{b}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} \\ & +\frac{3\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}} \\ & +\frac{3\sqrt{b}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} \\ & +\frac{3\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}}\end{aligned}$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(Cosh[c + d*x]/(a^2*x)) + (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] - Sqrt[
b]*x)) - (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt[b]*x)) - (3*Sqrt[b
]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*(-a)^(5/2)) + (3*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2)) + (d*CoshIntegral[d*x]*Sinh[c])/a^
2 + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[
b]])/(4*a^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-
a]*d)/Sqrt[b]])/(4*a^2) + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (3
*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(4*(-a)^(5/2)) + (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{b\cosh(c+dx)}{a\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^2\left(a+bx^2\right)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a^2}$$
$$- \frac{b\int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cosh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right) dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}}$$
$$+ \frac{b^2\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{2a^2}$$
$$+ \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^2} + \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^2}$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&+ \frac{d\mathrm{Chi}(dx)\sinh(c)}{a^2} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2} \\
&+ \frac{b^2\int\left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a^2} - \frac{(bd)\int\frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}dx}{4a^2} \\
&+ \frac{(bd)\int\frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}dx}{4a^2} + \frac{\left(b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2(-a)^{5/2}} \\
&+ \frac{\left(b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2(-a)^{5/2}} \\
&+ \frac{\left(b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2(-a)^{5/2}} \\
&- \frac{\left(b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2(-a)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&- \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{5/2}} + \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{5/2}} \\
&+ \frac{d\mathrm{Chi}(dx)\sinh(c)}{a^2} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2} + \frac{\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{5/2}} \\
&+ \frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{5/2}} + \frac{b\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}} \\
&+ \frac{b\int\frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{\left(bd\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}dx}{4a^2} \\
&+ \frac{\left(bd\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}dx}{4a^2} \\
&+ \frac{\left(bd\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}dx}{4a^2} \\
&- \frac{\left(bd\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}dx}{4a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&\quad - \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{5/2}} + \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{5/2}} \\
&\quad + \frac{d\mathrm{Chi}(dx)\sinh(c)}{a^2} + \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} \\
&\quad + \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2} \\
&\quad - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{5/2}} \\
&\quad + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2} + \frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{5/2}} \\
&\quad + \frac{\left(b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{5/2}} \\
&\quad + \frac{\left(b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}} \\
&\quad + \frac{\left(b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{5/2}} \\
&\quad - \frac{\left(b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{3\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} + \frac{3\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}}$$
$$+ \frac{d\text{Chi}(dx)\sinh(c)}{a^2} + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}$$
$$+ \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d\cosh(c)\text{Shi}(dx)}{a^2}$$
$$- \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{3\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}$$
$$+ \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2} + \frac{3\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.67

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$$
$$= \frac{-\frac{4\sqrt{a}(2a+3bx^2)\cosh(c)\cosh(dx)}{x(a+bx^2)} + e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(3i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-3i\sqrt{b}+\sqrt{\right.\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] ((-4*Sqrt[a]*(2*a + 3*b*x^2)*Cosh[c]*Cosh[d*x])/(x*(a + b*x^2)) + E^(c - (I
*Sqrt[a]*d)/Sqrt[b])*(((3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt
[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-3*I)*Sqrt[b] + Sqrt
[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]) - E^(-c - (I*Sqrt[a]*d)/
Sqrt[b])*(((3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpInte
gralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((-3*I)*Sqrt[b] + Sqrt[a]*d)*ExpIn
tegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) - (4*Sqrt[a]*(2*a + 3*b*x^2)*Sinh[c]
*Sinh[d*x])/(x*(a + b*x^2)) + 8*Sqrt[a]*d*(CoshIntegral[d*x]*Sinh[c] + Cosh
[c]*SinhIntegral[d*x]))/(8*a^(5/2))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{3\,\mathrm{e}^{-dx-c} x\,d^{2} b}{4a^{2}(b\,d^{2}x^{2}+a\,d^{2})}-\frac{\mathrm{e}^{-dx-c} d^{2}}{2a(b\,d^{2}x^{2}+a\,d^{2})x}+\frac{d\,\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_{1}\left(-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8a^{2}}+\frac{d\,\mathrm{e}^{-\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{Ei}_{1}\left(\frac{d\sqrt{-ab}+(dx+c)b}{b}\right.}{8a^{2}}$

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*exp(-d*x-c)/a^2/(b*d^2*x^2+a*d^2)*x*d^2*b-1/2*exp(-d*x-c)/a/(b*d^2*x^2
+a*d^2)/x*d^2+1/8*d/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)+1/8*d/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1
/2)+(d*x+c)*b-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/8/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)
^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+1/2*d/a^2*exp(-c)*E
i(1,d*x)-3/4*exp(d*x+c)/a^2/(b*d^2*x^2+a*d^2)*x*d^2*b-1/2*exp(d*x+c)/a/(b*d
^2*x^2+a*d^2)/x*d^2-1/8*d/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1
/2)-(d*x+c)*b+c*b)/b)-1/8*d/a^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b
)^(1/2)+(d*x+c)*b-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*
Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/8/a^2/(-a*b)^(1/2)*exp((-d*(-a*b
)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/2*d/a^2*exp(c)*
Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. 2(389) = 778.

Time = 0.29 (sec) , antiderivative size = 1310, normalized size of antiderivative = 2.62

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(3*a*b*d*x^2 + 2*a^2*d)*cosh(d*x + c) - (((a*b*d^2*x^3 + a^2*d^2*x)
*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3
+ a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 -
(a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x +
 c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-
a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x)
- (a*b*d^2*x^3 + a^2*d^2*x)*Ei(-d*x))*cosh(c) - (((a*b*d^2*x^3 + a^2*d^2*x)
*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3
+ a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 -
```

```
(a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x +
 c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x
 + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*
cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
 - sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*
x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (
b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x) + (a*b*d^
2*x^3 + a^2*d^2*x)*Ei(-d*x))*sinh(c) + (((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x
 + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*
cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
 + sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*
x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (
b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*sinh(-c + sqrt(-a*d^2/b)))/((a^3*b*d*x^3 + a^4*d*x)*cosh(d*x + c)^2 - (a^
3*b*d*x^3 + a^4*d*x)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{x^2\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**2,x)
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2 (a+bx^2)^2} \, dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^2 x^2} \, dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2 (a+bx^2)^2} \, dx = \int \frac{\cosh(c+d\,x)}{x^2\,(b\,x^2+a)^2} \, dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)^2),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

3.72 $\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 476

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^3}\,dx &= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2}-\frac{\cosh(c+dx)}{4b^2(a+bx^2)}\\
&+\frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}\\
&+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3}\\
&-\frac{3d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}\\
&+\frac{3d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}-\frac{dx\sinh(c+dx)}{8b^2(a+bx^2)}\\
&-\frac{3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}\\
&-\frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}\\
&-\frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}\\
&+\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3}\end{aligned}$$

```
[Out] -1/4*x^2*cosh(d*x+c)/b/(b*x^2+a)^2-1/4*cosh(d*x+c)/b^2/(b*x^2+a)+1/16*d^2*C
hi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b^3+1/16*d^2*Chi(
```

```
-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b^3-1/8*d*x*sinh(d*
x+c)/b^2/(b*x^2+a)+1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/
2)/b^(1/2))/b^3+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/
b^(1/2))/b^3+3/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/
2))/b^(5/2)/(-a)^(1/2)-3/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(
1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-3/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(
c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)+3/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(
1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5399, 5397, 5388, 3384, 3379, 3382, 5398, 5401}

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^3}\,dx = &-\frac{3d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}\\&+\frac{3d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}\\&-\frac{3d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}\\&-\frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}\\&+\frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}\\&+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3}\\&-\frac{d^2\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}\\&+\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3}\\&-\frac{dx\sinh(c+dx)}{8b^2(a+bx^2)}-\frac{\cosh(c+dx)}{4b^2(a+bx^2)}-\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2}\end{aligned}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(x^2*Cosh[c + d*x])/(b*(a + b*x^2)^2) - Cosh[c + d*x]/(4*b^2*(a + b*x^
2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(16*b^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[
```

```
-a]*d)/Sqrt[b] + d*x])/(16*b^3) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) + (3*d*CoshInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a
]*b^(5/2)) - (d*x*Sinh[c + d*x])/(8*b^2*(a + b*x^2)) - (3*d*Cosh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5
/2)) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x])/(16*b^3) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt
[-a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d^2*Sinh[c - (Sqrt[-a]*d)/
Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*b^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
```

```
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{\int\frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{2b}+\frac{d\int\frac{x^2\sinh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)}\\
&\quad+\frac{d\int\frac{\sinh(c+dx)}{a+bx^2}\,dx}{8b^2}+\frac{d\int\frac{\sinh(c+dx)}{a+bx^2}\,dx}{4b^2}+\frac{d^2\int\frac{x\cosh(c+dx)}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)}\\
&\quad+\frac{d\int\left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}+\frac{d\int\left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{4b^2}\\
&\quad+\frac{d^2\int\left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}
\end{aligned}
$$

$$= -\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} - \frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} - \frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2} - \frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{8\sqrt{-a}b^2} - \frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{8\sqrt{-a}b^2} - \frac{d^2\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16b^{5/2}} + \frac{d^2\int\frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16b^{5/2}}$$

$$= -\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} - \frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)}$$

$$- \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$

$$- \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}$$

$$+ \frac{\left(d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16b^{5/2}}$$

$$+ \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$

$$+ \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}$$

$$- \frac{\left(d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16b^{5/2}}$$

$$- \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$

$$- \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}$$

$$+ \frac{\left(d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16b^{5/2}}$$

$$- \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$

$$- \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{8\sqrt{-a}b^2}$$

$$+ \frac{\left(d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16b^{5/2}}$$

$$= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}$$
$$+ \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3} - \frac{3d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}$$
$$+ \frac{3d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{dx\sinh(c+dx)}{8b^2(a+bx^2)}$$
$$- \frac{3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}$$
$$- \frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.70

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{\frac{de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-3i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(3i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{a}} + \frac{de^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-3i\sqrt{b}+\sqrt{a}d\right)\right.}{}}{}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((d*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*S
qrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((3*I)*S
qrt[b] + Sqrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[a] +
(d*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*S
qrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((3*I)*S
qrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/Sqrt[a] -
(4*b*Cosh[d*x]*(2*(a + 2*b*x^2)*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*
x^2)^2 - (4*b*(d*x*(a + b*x^2)*Cosh[c] + 2*(a + 2*b*x^2)*Sinh[c])*Sinh[d*x]
)/(a + b*x^2)^2)/(32*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(374) = 748.

Time = 0.55 (sec) , antiderivative size = 1547, normalized size of antiderivative = 3.25

method	result	size
risch	Expression too large to display	1547

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-6*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)
/b)*a*b^2*d*x^2-2*(-a*b)^(1/2)*exp(-d*x-c)*a*b*d*x+6*exp(-(-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*d*x^2+exp(-(d*(-a*b)^(
1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^2*d^2*x
^4+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*
b-c*b)/b)*b^2*d^2*x^4+8*(-a*b)^(1/2)*exp(-d*x-c)*b^2*x^2+4*(-a*b)^(1/2)*exp
(-d*x-c)*a*b+(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)*a^2*d^2+(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2*d^2+3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*b*d-3*exp((-d*(-a*b)^(1/2)+c*b)/b)*E
i(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2*b*d-3*exp(-(d*(-a*b)^(1/2)+c*b)/
b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^3*d*x^4+3*exp(-(-d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b^3*d*x^4-2*(-a*b)^(1/2)*e
xp(-d*x-c)*b^2*d*x^3+2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(
-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b*d^2*x^2+2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1
/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b*d^2*x^2+3*exp((d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^3*d*x^4-3*exp((-
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b^3*d*x^4+2*
(-a*b)^(1/2)*exp(d*x+c)*b^2*d*x^3+2*(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b*d^2*x^2+2*(-a*b)^(1/2)*exp((-d
*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b*d^2*x^2+(
-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)
/b)*b^2*d^2*x^4+(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(
1/2)+(d*x+c)*b-c*b)/b)*b^2*d^2*x^4-6*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*d*x^2+6*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei
(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b^2*d*x^2+2*(-a*b)^(1/2)*exp(d*x+c)*
a*b*d*x+8*(-a*b)^(1/2)*exp(d*x+c)*b^2*x^2+4*(-a*b)^(1/2)*exp(d*x+c)*a*b+exp
(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)
/b)*a^2*d^2+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)
+(d*x+c)*b-c*b)/b)*a^2*d^2-3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(
1/2)-(d*x+c)*b+c*b)/b)*a^2*b*d+3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*
b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2*b*d)/(b^2*x^4+2*a*b*x^2+a^2)/b^3/(-a*b)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(374) = 748$.

Time = 0.29 (sec) , antiderivative size = 1620, normalized size of antiderivative = 3.40

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(2*a*b^2*x^2 + a^2*b)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*
d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 -
 (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x -
sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c
)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3
*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*
b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqr
t(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^
2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh
(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/
b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a*b^2*d*x^3 +
 a^2*b*d*x)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*c
osh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^
2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2
*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (
(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x^4 + 2*a*b^2*x^2
+ a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*s
inh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(
-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^
3)*cosh(d*x + c)^2 - (a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^3} \, dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^3} \, dx = \int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^3} \, dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3
- 3*d*x^2 + 12*x)*e^(-d*x))/(b^3*d^3*x^6*e^c + 3*a*b^2*d^3*x^4*e^c + 3*a^2*
b*d^3*x^2*e^c + a^3*d^3*e^c) - 1/2*integrate(6*(3*a*d*x*e^c + (a*d^2*e^c -
10*b*e^c)*x^2 + 2*a*e^c)*e^(d*x)/(b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2
*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) + 1/2*integrate(-6*(3*a*d*x - (a*
d^2 - 10*b)*x^2 - 2*a)*e^(-d*x)/(b^4*d^3*x^8*e^c + 4*a*b^3*d^3*x^6*e^c + 6*
a^2*b^2*d^3*x^4*e^c + 4*a^3*b*d^3*x^2*e^c + a^4*d^3*e^c), x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^3} \, dx = \int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^3} \, dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^2)^3, x)
```

3.73 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 746

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b\,(a+bx^2)^2}-\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\sinh(c+dx)}{8b^2\,(a+bx^2)} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}$$

```
[Out] -1/4*x*cosh(d*x+c)/b/(b*x^2+a)^2+1/16*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-
d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*Chi(-d*x+d*(-a)^(1/2)/b^(1/2)
)*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d*cosh(c+d*(-a)^(1/2
)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*cosh(c-d*(-a)^(1/2)/b
```

```
^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/8*d*sinh(d*x+c)/b^2/(b*x^2+a)
-1/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a/b^2+1/
16*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^
(3/2)-1/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a/
b^2-1/16*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3
/2)/b^(3/2)-1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1
/2))/b^(5/2)/(-a)^(1/2)+1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-
a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))
*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)+1/16*d^2*Shi(d*x-d*(-a)^(1
/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-1/16*cosh(d*x+
c)/a/b^(3/2)/((-a)^(1/2)-x*b^(1/2))+1/16*cosh(d*x+c)/a/b^(3/2)/((-a)^(1/2)+
x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{5399, 5389, 3378, 3384, 3379, 3382, 5396\}$

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = \frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} + \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/16*Cosh[c + d*x]/(a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*
a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Cosh[c + d*x])/(4*b*(a + b*x^2)^2) -
 (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
```

```
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) + (Cosh[c - (Sqrt[-a]*d)
/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2))
 - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(16*Sqrt[-a]*b^(5/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
inh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*Sinh[c + d*x])
/(8*b^2*(a + b*x^2)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt
[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-
a]*b^(5/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x])/(16*a*b^2) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2
))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

```
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5396

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x\cosh(c+dx)}{4b(a+bx^2)^2}+\frac{\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx}{4b}+\frac{d\int \frac{x\sinh(c+dx)}{(a+bx^2)^2}\,dx}{4b}\\
&= -\frac{x\cosh(c+dx)}{4b(a+bx^2)^2}-\frac{d\sinh(c+dx)}{8b^2(a+bx^2)}\\
&\quad+\frac{\int\left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\cosh(c+dx)}{2a(-ab-b^2x^2)}\right)dx}{4b}+\frac{d^2\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{x\cosh(c+dx)}{4b(a+bx^2)^2}-\frac{d\sinh(c+dx)}{8b^2(a+bx^2)}-\frac{\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a}-\frac{\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a}\\
&\quad-\frac{\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{8a}+\frac{d^2\int\left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}\\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{x\cosh(c+dx)}{4b(a+bx^2)^2}\\
&\quad-\frac{d\sinh(c+dx)}{8b^2(a+bx^2)}-\frac{\int\left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a}\\
&\quad+\frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}-\frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}-\frac{d^2\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}-\frac{d^2\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b}$$
$$+ \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}$$
$$- \frac{\left(d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$
$$- \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}$$
$$- \frac{\left(d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$
$$- \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}$$
$$- \frac{\left(d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$
$$+ \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}$$
$$+ \frac{\left(d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&\quad -\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\
&\quad -\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}-\frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} \\
&\quad -\frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2}-\frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} \\
&\quad +\frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} \\
&\quad -\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}-\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} \\
&\quad +\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b}+\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b} \\
&\quad +\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b}-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.73 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.51

$$\int \frac{x^2\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx$$

$$= \frac{-ie^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(b-i\sqrt{a}\sqrt{b}d+ad^2\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\left(b+i\sqrt{a}\sqrt{b}d+ad^2\right)\text{ExpInteg}\right.}{}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-I)*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*((b - I*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^((
(2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - (
b + I*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]
) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((I*b + Sqrt[a]*Sqrt[b]*d + I*a*d^2)*E^(
((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] -
I*(b + I*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d
*x]) - (4*Sqrt[a]*Sqrt[b]*Cosh[d*x]*(-(b*x*(-a + b*x^2)*Cosh[c]) + a*d*(a +
 b*x^2)*Sinh[c]))/(a + b*x^2)^2 - (4*Sqrt[a]*Sqrt[b]*(a*d*(a + b*x^2)*Cosh[
c] + b*x*(a - b*x^2)*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2)/(32*a^(3/2)*b^(5/2)
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. 2(574) = 1148.

Time = 0.36 (sec) , antiderivative size = 2172, normalized size of antiderivative = 2.91

method	result	size
risch	Expression too large to display	2172

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a*(2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)
/b)*a^2*b*d^2*x^2-2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x
+c)*b+c*b)/b)*(-a*b)^(1/2)*a*b*d*x^2-2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b*d*x^2-2*exp(d*x+c)*a*b*d*
x^2*(-a*b)^(1/2)+2*exp(-d*x-c)*a*b*d*x^2*(-a*b)^(1/2)+2*b^2*x^3*(-a*b)^(1/2
)*exp(d*x+c)-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b
)/b)*a*b^2*d^2*x^4+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)*a*b^2*d^2*x^4+exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)
-(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*b^2*d*x^4+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*(-a*b)^(1/2)*b^2*d*x^4-2*exp((d*(-a*b)^
(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*b*d^2*x^2+2*b^2*x^
3*(-a*b)^(1/2)*exp(-d*x-c)+2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/
2)-(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*a*b*d*x^2+2*exp((-d*(-a*b)^(1/2)+c*b)/b)*
Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*(-a*b)^(1/2)*a*b*d*x^2-2*exp(-(d*(-
a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b^2*x^2+2*exp(
-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*x^2-
exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-a*b)
^(1/2)*a^2*d-(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2
)+(d*x+c)*b-c*b)/b)*a^2*d-2*exp(d*x+c)*a^2*d*(-a*b)^(1/2)+2*exp(-d*x-c)*a^2
*d*(-a*b)^(1/2)-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+
c*b)/b)*b^3*x^4+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*
b-c*b)/b)*b^3*x^4-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*
b+c*b)/b)*a^3*d^2+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c
)*b-c*b)/b)*a^3*d^2-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c
)*b+c*b)/b)*a^2*b+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c
)*b-c*b)/b)*a^2*b-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c
)*b+c*b)/b)*b^3*x^4+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x
+c)*b-c*b)/b)*b^3*x^4-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d
*x+c)*b+c*b)/b)*a^3*d^2+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+
(d*x+c)*b-c*b)/b)*a^3*d^2-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)*a^2*b+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2
)+(d*x+c)*b-c*b)/b)*a^2*b-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)*a*b^2*d^2*x^4+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a
*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*d^2*x^4-exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*(-a*b)^(1/2)*b^2*d*x^4-(-a*b)^(1/2)*exp
```

```
(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b^2*d*x^4
-2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2
*b*d^2*x^2+2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c
*b)/b)*a^2*b*d^2*x^2-2*exp(d*x+c)*a*b*x*(-a*b)^(1/2)-2*exp(-d*x-c)*a*b*x*(-
a*b)^(1/2)-2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b
)/b)*a*b^2*x^2+2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)
*b-c*b)/b)*a*b^2*x^2+exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+
c)*b+c*b)/b)*(-a*b)^(1/2)*a^2*d+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b
)^(1/2)+(d*x+c)*b-c*b)/b)*(-a*b)^(1/2)*a^2*d)/b^2/(b^2*x^4+2*a*b*x^2+a^2)/(
-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2047 vs. 2(575) = 1150.

Time = 0.28 (sec) , antiderivative size = 2047, normalized size of antiderivative = 2.74

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3}\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(a*b^2*d*x^3 - a^2*b*d*x)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*
b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 +
a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x +
 c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3
*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x
+ c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x
^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sq
rt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)
^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^
2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c
)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)
*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2
*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b
^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a^2*b*d^2*x^2
+ a^3*d^2)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*co
sh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2
 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*c
```

```
osh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 +
a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3
)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*
b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b^
2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2
*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x
^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2
*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(
d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b
^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b
^2*d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)*sinh(
d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^2 \cosh(dx+c)}{(bx^2+a)^3}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 4*x*e^(2*c))*e^(d*x) - (d*x^2 - 4*x)*e^(-d*x))/(b^3*d
^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*e^c) + 1/2
*integrate(-2*(3*a*d*x*e^c - 10*b*x^2*e^c + 2*a*e^c)*e^(d*x)/(b^4*d^2*x^8 +
 4*a*b^3*d^2*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2), x) + 1/2
*integrate(2*(3*a*d*x + 10*b*x^2 - 2*a)*e^(-d*x)/(b^4*d^2*x^8*e^c + 4*a*b^3
*d^2*x^6*e^c + 6*a^2*b^2*d^2*x^4*e^c + 4*a^3*b*d^2*x^2*e^c + a^4*d^2*e^c),
x)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^2 \cosh\left(dx+c\right)}{\left(bx^2+a\right)^3}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{\left(b\,x^2+a\right)^3}\,dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x^2*cosh(c + d*x))/(a + b*x^2)^3, x)
```

3.74 $\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 512

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx &= -\frac{\cosh(c+dx)}{4b\,(a+bx^2)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} \\ &\quad - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} \\ &\quad + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} \\ &\quad - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} \\ &\quad + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} \\ &\quad + \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} \\ &\quad + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} \\ &\quad - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}\end{aligned}$$

```
[Out] -1/4*cosh(d*x+c)/b/(b*x^2+a)^2-1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(
c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(
c+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d
```

```
*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))
*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d*Chi(d*x+d*(-a)^(1/
2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d^2*Shi(d*
x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*Chi(-d*x+
d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*
d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d
*sinh(d*x+c)/a/b^(3/2)/((-a)^(1/2)-x*b^(1/2))+1/16*d*sinh(d*x+c)/a/b^(3/2)/
((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5397, 5388, 3378, 3384, 3379, 3382}

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} + \frac{d^2\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/4*Cosh[c + d*x]/(b*(a + b*x^2)^2) - (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d^2*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (d*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16
```

```
*(-a)^(3/2)*b^(3/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c +
 (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d*Sinh[c + d*x])/(16*a*b
^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/(16*a*b^(3/2)*(Sqrt[-a]
+ Sqrt[b]*x)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (d*Cosh[c - (Sqr
t[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*
b^(3/2)) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(16*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{d\int \frac{\sinh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{d\int \left(-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\sinh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4b}\\
&= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a}-\frac{d\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a}-\frac{d\int \frac{\sinh(c+dx)}{-ab-b^2x^2}\,dx}{8a}\\
&= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&\quad-\frac{d\int \left(-\frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a}\\
&\quad+\frac{d^2\int \frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}-\frac{d^2\int \frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}\\
&= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&\quad+\frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b}+\frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{3/2}b}-\frac{\left(d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}\\
&\quad+\frac{\left(d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}\\
&\quad-\frac{\left(d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16ab}\\
&\quad-\frac{\left(d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16ab}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{16(-a)^{3/2}b} - \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{16(-a)^{3/2}b} + \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{16(-a)^{3/2}b} + \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{16(-a)^{3/2}b}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.63

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \frac{-de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}\right.\right.\right.}{}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] (-(d*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[
a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-I)*Sqrt[b
] + Sqrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)])) - d*E^(-c - (I*
Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((-I)*Sqrt[b] + Sqrt[a]*d)*
ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*Sqrt[a]*b*Cosh[d*x]*(-2*a*
Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (4*Sqrt[a]*b*(d*x*(a +
b*x^2)*Cosh[c] - 2*a*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2)/(32*a^(3/2)*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. 2(398) = 796.

Time = 0.31 (sec) , antiderivative size = 1503, normalized size of antiderivative = 2.94

method	result	size
risch	Expression too large to display	1503

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a*(2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)
/b)*a*b^2*d*x^2-2*(-a*b)^(1/2)*exp(-d*x-c)*a*b*d*x-2*exp(-(-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*d*x^2+exp(-(d*(-a*b)^(
1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^2*d^2*x
^4+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*
b-c*b)/b)*b^2*d^2*x^4-4*(-a*b)^(1/2)*exp(-d*x-c)*a*b+(-a*b)^(1/2)*exp((d*(-
a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2*d^2+(-a*b)^(1
/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^
2*d^2-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^
2*b*d+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*
a^2*b*d+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b
)*b^3*d*x^4-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*
b)/b)*b^3*d*x^4-2*(-a*b)^(1/2)*exp(-d*x-c)*b^2*d*x^3+2*exp(-(d*(-a*b)^(1/2)
```

```
+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b*d^2*x^2+2
*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-
c*b)/b)*a*b*d^2*x^2-exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c
)*b+c*b)/b)*b^3*d*x^4+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d
*x+c)*b-c*b)/b)*b^3*d*x^4+2*(-a*b)^(1/2)*exp(d*x+c)*b^2*d*x^3+2*(-a*b)^(1/2
)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b*d^
2*x^2+2*(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*
x+c)*b-c*b)/b)*a*b*d^2*x^2+(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b^2*d^2*x^4+(-a*b)^(1/2)*exp((-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b^2*d^2*x^4+2*exp((-d*(-
a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a*b^2*d*x^2-2*ex
p((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a*b^2*d*x^
2+2*(-a*b)^(1/2)*exp(d*x+c)*a*b*d*x-4*(-a*b)^(1/2)*exp(d*x+c)*a*b+exp(-(d*(
-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^
2*d^2+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)*a^2*d^2+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*
x+c)*b+c*b)/b)*a^2*b*d-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(
d*x+c)*b-c*b)/b)*a^2*b*d)/b^2/(b^2*x^4+2*a*b*x^2+a^2)/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1607 vs. $2(399) = 798$.

Time = 0.26 (sec) , antiderivative size = 1607, normalized size of antiderivative = 3.14

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*a^2*b*cosh(d*x + c) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2
)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x +
c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^
2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) +
((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x
^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + (((a*
b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 +
 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2
*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt
(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh
(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 +
 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
```

```
2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x + c
) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*
d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*
x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*
d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (
b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x
^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)
*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 -
(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^
2/b)))/((a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*cosh(d*x + c)^2 - (a^2*b^4*
x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx^2+a\right)^3}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a
^2*b*d*x^2*e^c + a^3*d*e^c) + 1/2*integrate((5*b*x^2*e^c - a*e^c)*e^(d*x)/(
b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x) -
1/2*integrate((5*b*x^2 - a)*e^(-d*x)/(b^4*d*x^8*e^c + 4*a*b^3*d*x^6*e^c + 6
*a^2*b^2*d*x^4*e^c + 4*a^3*b*d*x^2*e^c + a^4*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\cosh\left(dx+c\right)}{(bx^2+a)^3}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\cosh(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x*cosh(c + d*x))/(a + b*x^2)^3, x)
```

3.75 $\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 856

$$\begin{aligned}\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = &-\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\\
&+\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&+\frac{3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}\\
&+\frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}\\
&-\frac{3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}\\
&-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}\\
&-\frac{3d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}\\
&-\frac{3d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}+\frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}\\
&+\frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}\\
&-\frac{3\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}\\
&-\frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}\\
&-\frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}\\
&-\frac{3\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}\\
&-\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}\end{aligned}$$

```
[Out] -1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(
3/2)/b^(3/2)+1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^
(1/2))/(-a)^(3/2)/b^(3/2)-3/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a
)^(1/2)/b^(1/2))/a^2/b-3/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(
1/2)/b^(1/2))/a^2/b-3/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2
)/b^(1/2))/a^2/b-1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)
/b^(1/2))/(-a)^(3/2)/b^(3/2)-3/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d
*(-a)^(1/2)/b^(1/2))/a^2/b+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*
(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-3/16*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*c
osh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+3/16*Chi(-d*x+d*(-a)^(1/2)/b
^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-3/16*Shi(d*x+d*(-a)
^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+3/16*Shi(d*
x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/1
6*cosh(d*x+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)-x*b^(1/2))^2+1/16*d*sinh(d*x+c
)/(-a)^(3/2)/b/((-a)^(1/2)-x*b^(1/2))-3/16*cosh(d*x+c)/a^2/b^(1/2)/((-a)^(1
/2)-x*b^(1/2))+1/16*cosh(d*x+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)+x*b^(1/2))^2
+1/16*d*sinh(d*x+c)/(-a)^(3/2)/b/((-a)^(1/2)+x*b^(1/2))+3/16*cosh(d*x+c)/a^
2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used

$= \{5389, 3378, 3384, 3379, 3382\}$

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}} - \frac{3\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b} - \frac{3\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b} + \frac{\sinh(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sinh(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{b}x+\sqrt{-a}\right)} + \frac{3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^2b} - \frac{3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)} - \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)^2} + \frac{3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{3\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{3\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] -1/16*Cosh[c + d*x]/((-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cosh
[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-a)
```

```
^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/(16*a^2*Sqrt[b
]*(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (Sqrt[
-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^
(3/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b
] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*CoshI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2
*b) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/S
qrt[b]])/(16*a^2*b) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b
]*x)) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) + (3*d*C
osh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16
*a^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[
b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^
2*b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3}-\frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\right.\\
&\qquad\left.-\frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}+bx\right)^3}-\frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{3b\cosh(c+dx)}{8a^2\left(-ab-b^2x^2\right)}\right)dx\\
&= -\frac{(3b)\int\frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^2}-\frac{(3b)\int\frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^2}-\frac{(3b)\int\frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{8a^2}\\
&\quad-\frac{b^{3/2}\int\frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^3}\,dx}{8(-a)^{3/2}}-\frac{b^{3/2}\int\frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^3}\,dx}{8(-a)^{3/2}}\\
&= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\\
&\quad+\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&\quad-\frac{(3b)\int\left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^2}+\frac{(3d)\int\frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}\\
&\quad-\frac{(3d)\int\frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2}+\frac{\left(\sqrt{b}d\right)\int\frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16(-a)^{3/2}}-\frac{\left(\sqrt{b}d\right)\int\frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16(-a)^{3/2}}
\end{aligned}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}$$
$$+ \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$- \frac{3\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}} - \frac{3\int\frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}} - \frac{d^2\int\frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16(-a)^{3/2}\sqrt{b}} - \frac{d^2\int\frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16(-a)^{3/2}\sqrt{b}}$$
$$- \frac{\left(3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} - \frac{\left(3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}$$
$$- \frac{\left(3d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} + \frac{\left(3d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}$$

$$
\begin{aligned}
= &-\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\\
&+\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&-\frac{3d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}-\frac{3d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}\\
&+\frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}\\
&+\frac{3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}\\
&-\frac{\left(3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}}\\
&-\frac{\left(d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16(-a)^{3/2}\sqrt{b}}\\
&-\frac{\left(3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}\\
&-\frac{\left(d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16(-a)^{3/2}\sqrt{b}}\\
&-\frac{\left(3\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}}\\
&-\frac{\left(d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16(-a)^{3/2}\sqrt{b}}\\
&+\frac{\left(3\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}\\
&+\frac{\left(d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16(-a)^{3/2}\sqrt{b}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} \\
&+ \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&+ \frac{3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} \\
&- \frac{3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} \\
&- \frac{3d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} - \frac{3d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} \\
&+ \frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&+ \frac{3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} - \frac{3\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
&- \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} \\
&- \frac{3\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.46

$$
\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx
= \frac{-e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(3ib+3\sqrt{a}\sqrt{b}d-iad^2\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-3ib+3\sqrt{a}\sqrt{b}d+iad^2\right)\mathrm{Ex}
$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] (-(E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((3*I)*b + 3*Sqrt[a]*Sqrt[b]*d - I*a*d^2)
*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)
] + ((-3*I)*b + 3*Sqrt[a]*Sqrt[b]*d + I*a*d^2)*ExpIntegralEi[d*((I*Sqrt[a])
/Sqrt[b] + x)])) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((3*I)*b + 3*Sqrt[a]*Sqr
t[b]*d - I*a*d^2)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]
```

```
*d)/Sqrt[b] - d*x] + ((-3*I)*b + 3*Sqrt[a]*Sqrt[b]*d + I*a*d^2)*ExpIntegral
Ei[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*Sqrt[a]*Sqrt[b]*Cosh[d*x]*(b*x*(5*a +
 3*b*x^2)*Cosh[c] + a*d*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (4*Sqrt[a]*Sq
rt[b]*(a*d*(a + b*x^2)*Cosh[c] + b*x*(5*a + 3*b*x^2)*Sinh[c])*Sinh[d*x])/(a
 + b*x^2)^2)/(32*a^(5/2)*b^(3/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.24

method	result	size
risch	Expression too large to display	1064

```
[In] int(cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*ex
p(-d*x-c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(-d*x-c
)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*exp(-d*x-c)/a/(b^2*d^4*x^4
+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*
b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d^2/b/a/(-a*b)^(1/2)*exp
(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32*d/b/
a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-3/
32*d/b/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b
)/b)-3/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/
2)-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei
(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*d^4*x^4+2*a
*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*exp(d*x+c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^
2+a^2*d^4)*x^3+1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/
16*d^4*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*
b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)
-1/32*d^2/b/a/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/
2)+(d*x+c)*b-c*b)/b)+3/32*d/b/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b
)^(1/2)-(d*x+c)*b+c*b)/b)+3/32*d/b/a^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp((-
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. 2(655) = 1310.

Time = 0.29 (sec) , antiderivative size = 2116, normalized size of antiderivative = 2.47

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*cosh(d*x + c) - ((3*(a*b^2*d^2*x^4 +
2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2
*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a
^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2
 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqr
t(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)
*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b
*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 -
 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei
(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - ((3*(a*b^2*d^2*x^4 + 2*
a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x
^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2
*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 -
 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
 a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*s
inh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d
^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3
*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-
d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^2 + a^3*d
^2)*sinh(d*x + c) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*
x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 -
((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2
)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b
*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/
b)) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*
b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)
^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)
*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x
+ c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((
a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*
cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d
```

```
^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)
) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b
^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^
2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2
 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x
^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + s
qrt(-a*d^2/b)))/((a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*cosh(d*x + c)^
2 - (a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*x^2)^3,x)
[Out] int(cosh(c + d*x)/(a + b*x^2)^3, x)
```

3.76 $\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 730

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2}+\frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^3}-\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}+\frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}+\frac{5d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{5d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^3}+\frac{5d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}-\frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{5d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}+\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$

```
[Out] Chi(d*x)*cosh(c)/a^3+1/4*cosh(d*x+c)/a/(b*x^2+a)^2+1/2*cosh(d*x+c)/a^2/(b*x
^2+a)-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^3+1/
16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-1/2
```

```
*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d^2*C
hi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2/b+Shi(d*x)*s
inh(c)/a^3-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a
^3+1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2/
b-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d
^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2/b-5/16*d*
cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/
2)+5/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(
5/2)/b^(1/2)+5/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/
2))/(-a)^(5/2)/b^(1/2)-5/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^
(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2
)-x*b^(1/2))-1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{5401, 3384, 3379, 3382, 5397, 5388, 3378\}$

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = -\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} + \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} + \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} - \frac{d^2\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} + \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{5d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{5d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{5d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{5d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Co
sh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]
*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[
```

```
c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2
*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/S
qrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d*Sinh[c +
 d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x])/(16*a^2*
Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 + (5*d*Co
sh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c - (Sqrt[-a]*d)/S
qrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) -
 (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{a^3x} - \frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)^3} - \frac{bx\cosh(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{bx\cosh(c+dx)}{a^3\left(a+bx^2\right)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a} \\
&= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a^3} \\
&\quad - \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{2a^2} - \frac{d\int \frac{\sinh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^3} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a^3} \\
&= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} \\
&\quad + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^3} - \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^3} - \frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{2a^2} \\
&\quad - \frac{d\int \left(-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sinh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right) dx}{4a}
\end{aligned}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}}$$
$$+ \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{(bd)\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}dx}{16a^2} + \frac{(bd)\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}dx}{16a^2} + \frac{(bd)\int \frac{\sinh(c+dx)}{-ab-b^2x^2}dx}{8a^2}$$
$$- \frac{\left(\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^3} + \frac{\left(\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^3}$$
$$- \frac{\left(\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{2a^3} - \frac{\left(\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^3}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}$$
$$- \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} + \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$+ \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}$$
$$+ \frac{(bd)\int \left(-\frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^2} - \frac{d^2\int \frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}dx}{16a^2} + \frac{d^2\int \frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}dx}{16a^2}$$
$$+ \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} - \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}}$$
$$+ \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4(-a)^{5/2}} + \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}}$$

$$
\begin{aligned}
&= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} \\
&- \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} \\
&+ \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}\sqrt{b}} - \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}\sqrt{b}} \\
&+ \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a^3} \\
&+ \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} \\
&+ \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}\sqrt{b}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} \\
&+ \frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}} + \frac{d\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}} + \frac{\left(d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} \\
&- \frac{\left(d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2} \\
&+ \frac{\left(d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^2} \\
&+ \frac{\left(d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} \\
&- \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} \\
&- \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} + \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} \\
&+ \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}\sqrt{b}} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}\sqrt{b}} \\
&+ \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
&+ \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}} \\
&+ \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} \\
&+ \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}\sqrt{b}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} \\
&+ \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} + \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}} \\
&- \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}} \\
&+ \frac{\left(d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{5/2}} \\
&+ \frac{\left(d\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} \\
&- \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} \\
&- \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} + \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b} \\
&+ \frac{5d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} - \frac{5d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}} \\
&+ \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a^3} \\
&+ \frac{5d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} \\
&- \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b} + \frac{5d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
&- \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3} + \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.92

$$
\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx
$$

$$
= \frac{\frac{8a\left(3a+2bx^2\right)\cosh(c+dx)}{\left(a+bx^2\right)^2} + 32\cosh(c)\mathrm{Chi}(dx) + \frac{4i\sqrt{a}de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\mathrm{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\mathrm{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right.\right.}{\sqrt{b}}}{}
$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] ((8*a*(3*a + 2*b*x^2)*Cosh[c + d*x])/(a + b*x^2)^2 + 32*Cosh[c]*CoshIntegra
l[d*x] + ((4*I)*Sqrt[a]*d*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*
d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d
*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[b] - 8*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^
(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] +
 ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]) + (Sqrt[a]*d*E^(c - (I*Sqrt[a]
*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpInte
gralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpInte
gralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/b + ((4*I)*Sqrt[a]*d*E^(-c - (I*Sqrt[
```

```
a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d
)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/Sqrt[b] - 8
*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralE
i[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d
*x]) + (Sqrt[a]*d*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E
^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x]
+ ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/b
 - (4*a*d*x*Sinh[c + d*x])/(a + b*x^2) + 32*Sinh[c]*SinhIntegral[d*x])/(32*
a^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 1090, normalized size of antiderivative = 1.49

method	result	size
risch	Expression too large to display	1090

```
[In] int(cosh(d*x+c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*exp(-d*x-c)*d^2*(b*(d*x+c)^3-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*b
*c^2-d^2*c*a-b*c^3+4*(d*x+c)^2*b-8*b*(d*x+c)*c+6*a*d^2+4*c^2*b)/a^2/((d*x+c
)^4*b^2-4*(d*x+c)^3*c*b^2+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*a*b*(d*
x+c)*c*d^2-4*b^2*(d*x+c)*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/32/b/a^2*exp(
-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*d^2-1/32/b
/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d
^2-5/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)
-(d*x+c)*b+c*b)/b)*d+5/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei
(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d+1/4/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a^3*exp(-(-d*(-a*b)^(1/2)+c*b)
/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2/a^3*exp(-c)*Ei(1,d*x)-1/16*e
xp(d*x+c)*d^2*(b*(d*x+c)^3-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*b*c^2-d^
2*c*a-b*c^3-4*(d*x+c)^2*b+8*b*(d*x+c)*c-6*a*d^2-4*c^2*b)/a^2/((d*x+c)^4*b^2
-4*(d*x+c)^3*c*b^2+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*a*b*(d*x+c)*c*
d^2-4*b^2*(d*x+c)*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/32/b/a^2*exp((d*(-a*
b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*d^2-1/32/b/a^2*exp(
(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d^2+5/32/a
^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+
c*b)/b)*d-5/32/a^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b
)^(1/2)+(d*x+c)*b-c*b)/b)*d+1/4/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a
*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a^3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(
-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2/a^3*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2076 vs. 2(575) = 1150.

Time = 0.29 (sec) , antiderivative size = 2076, normalized size of antiderivative = 2.84

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(2*a*b^2*x^2 + 3*a^2*b)*cosh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 - 8
*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d
^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(
d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4
+ 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^
2/b)) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*
b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + 16*((b
^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) + (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(
-d*x))*cosh(c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*
d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 -
8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a^3*d^2 + (a*b^2*d^2
- 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^
3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*si
nh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x
 + c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a
*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*
x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (
a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x +
c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) -
(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(-d*x))*sinh(c) - (((a^3*d^2 + (a*b^2*d^2
 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a
^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*s
inh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
```

```
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 -
8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*
b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^
2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^
2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((
a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*cosh(d*x + c)^2 - (a^3*b^3*x^4 + 2*a^4
*b^2*x^2 + a^5*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^3x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^3x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x\,(b\,x^2+a)^3}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x^2)^3),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)^3), x)
```

3.77 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 874

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2\,(a+bx^2)^3}\,dx = & -\frac{\cosh(c+dx)}{a^3x}-\frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} \\
& +\frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} \\
& -\frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{15\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} \\
& +\frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
& -\frac{15\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}} \\
& -\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
& +\frac{d\text{Chi}(dx)\sinh(c)}{a^3}+\frac{7d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} \\
& +\frac{7d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} \\
& +\frac{d\sinh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\sinh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)} \\
& +\frac{d\cosh(c)\text{Shi}(dx)}{a^3}-\frac{7d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} \\
& -\frac{15\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} \\
& -\frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} \\
& +\frac{7d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} \\
& -\frac{15\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}} \\
& -\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}
\end{aligned}
$$

```
[Out] -cosh(d*x+c)/a^3/x+d*cosh(c)*Shi(d*x)/a^3+7/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2
))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a^3+7/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Sh
i(d*x+d*(-a)^(1/2)/b^(1/2))/a^3+d*Chi(d*x)*sinh(c)/a^3+7/16*d*Chi(d*x+d*(-a
)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^3+7/16*d*Chi(-d*x+d*(-a)^(1
/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*Chi(d*x+d*(-a)^(1/2)
/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*Chi(-d*x
+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16
*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/
b^(1/2)+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))
/(-a)^(5/2)/b^(1/2)-15/16*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)
/b^(1/2))*b^(1/2)/(-a)^(7/2)+15/16*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*
(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-15/16*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*
sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+15/16*Shi(d*x-d*(-a)^(1/2)/
b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-1/16*cosh(d*x+c)*b
^(1/2)/(-a)^(5/2)/((-a)^(1/2)-x*b^(1/2))^2+1/16*d*sinh(d*x+c)/(-a)^(5/2)/((
-a)^(1/2)-x*b^(1/2))+7/16*cosh(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/
16*cosh(d*x+c)*b^(1/2)/(-a)^(5/2)/((-a)^(1/2)+x*b^(1/2))^2+1/16*d*sinh(d*x+
c)/(-a)^(5/2)/((-a)^(1/2)+x*b^(1/2))-7/16*cosh(d*x+c)*b^(1/2)/a^3/((-a)^(1/
2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{5401, 3378, 3384, 3379, 3382, 5389\}$

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}$$

$$-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}$$

$$-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}$$

$$-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}+\frac{\text{Chi}(dx)\sinh(c)d}{a^3}$$

$$+\frac{7\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3}$$

$$+\frac{7\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3}$$

$$+\frac{\sinh(c+dx)d}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sinh(c+dx)d}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)}$$

$$+\frac{\cosh(c)\text{Shi}(dx)d}{a^3}-\frac{7\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^3}$$

$$+\frac{7\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3}-\frac{\cosh(c+dx)}{a^3x}$$

$$+\frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{b}x+\sqrt{-a}\right)}$$

$$-\frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2}+\frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)^2}$$

$$+\frac{15\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$

$$-\frac{15\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

$$-\frac{15\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}$$

$$-\frac{15\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a]
 - Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)
) + (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*S
qrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^
(7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b
])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{a^3x^2} - \frac{b\cosh(c+dx)}{a\,(a+bx^2)^3} - \frac{b\cosh(c+dx)}{a^2\,(a+bx^2)^2} - \frac{b\cosh(c+dx)}{a^3\,(a+bx^2)} \right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx}{a^3}$$
$$- \frac{b\int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cosh(c+dx)}{2a(-ab-b^2x^2)} \right) dx}{a^2}$$
$$- \frac{b\int \left(-\frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3} - \frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}+bx\right)^3} - \frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{3b\cosh(}{8a^2(-ab} \right.}{a}$$
$$+ \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^3}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{(3b^2)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^3}$$
$$+ \frac{(3b^2)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a^3}$$
$$+ \frac{(3b^2)\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{8a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{2a^3} - \frac{b^{5/2}\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^3}\,dx}{8(-a)^{5/2}}$$
$$- \frac{b^{5/2}\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^3}\,dx}{8(-a)^{5/2}} + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^3} + \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^3}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}$$
$$+ \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\operatorname{Chi}(dx)\sinh(c)}{a^3}$$
$$+ \frac{d\cosh(c)\operatorname{Shi}(dx)}{a^3} + \frac{(3b^2)\int \left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^3}$$
$$+ \frac{b^2\int \left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a^3} - \frac{(3bd)\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3}$$
$$+ \frac{(3bd)\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3} - \frac{(bd)\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{4a^3} + \frac{(bd)\int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{4a^3}$$
$$+ \frac{\left(b^{3/2}d\right)\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16(-a)^{5/2}} - \frac{\left(b^{3/2}d\right)\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16(-a)^{5/2}}$$
$$- \frac{\left(b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$- \frac{\left(b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$- \frac{\left(b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$+ \frac{\left(b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.70

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx$$

$$= \frac{8i\sqrt{b}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+\frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(7ib+7\sqrt{a}\sqrt{\right.\right.}{}}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] ((8*I)*Sqrt[b]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])
*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d*((I*Sqrt[a
])/Sqrt[b] + x)]) + (E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((7*I)*b + 7*Sqrt[a]*Sq
rt[b]*d - I*a*d^2)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqr
t[a])/Sqrt[b] + x)] + ((-7*I)*b + 7*Sqrt[a]*Sqrt[b]*d + I*a*d^2)*ExpIntegra
lEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[b] - (8*I)*Sqrt[b]*E^(-c - (I*Sqrt[
a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d
)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) - (I*E^(-c -
 (I*Sqrt[a]*d)/Sqrt[b])*((7*b - (7*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*E^(((2*I)*
Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + (-7*b -
 (7*I)*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x
]))/Sqrt[b] - (4*Sqrt[a]*Cosh[d*x]*((8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)*Cosh[
c] + a*d*x*(a + b*x^2)*Sinh[c]))/(x*(a + b*x^2)^2) - (4*Sqrt[a]*(a*d*x*(a +
 b*x^2)*Cosh[c] + (8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)*Sinh[c])*Sinh[d*x])/(x*
(a + b*x^2)^2) + 32*Sqrt[a]*d*(CoshIntegral[d*x]*Sinh[c] + Cosh[c]*SinhInte
gral[d*x]))/(32*a^(7/2))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 1178, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	1178

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*d^5*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x^2-15/16*ex
p(-d*x-c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+1/16*d^5*exp(
-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(-d*x-c)*d^4/a^2/(b^
2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x-1/2*exp(-d*x-c)/a/x*d^4/(b^2*d^4*x^4+2
*a*b*d^4*x^2+a^2*d^4)-1/32/a^2*d^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b
```

```
)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32/a^2*d^2/(-a*b)^(1/2)*exp(-(-
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+7/32*d/a^3*ex
p(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+7/32*d/a
^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+15/
32/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x
+c)*b+c*b)/b)*b-15/32/a^3/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+1/2*d/a^3*exp(-c)*Ei(1,d*x)-1/16*d^5*exp
(d*x+c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x^2-15/16*exp(d*x+c)/a^3*
x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/16*d^5*exp(d*x+c)/a/(b^2*
d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(d*x+c)*d^4/a^2/(b^2*d^4*x^4+2*a*b*
d^4*x^2+a^2*d^4)*b*x-1/2*exp(d*x+c)/a/x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)-1/32/a^2*d^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(
1/2)-(d*x+c)*b+c*b)/b)+1/32/a^2*d^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/
b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-7/32*d/a^3*exp((d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-7/32*d/a^3*exp((-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+15/32/a^3/(-a*b)^(1/2)*e
xp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-15/32/a
^3/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*
b-c*b)/b)*b-1/2*d/a^3*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2346 vs. $2(673) = 1346$.

Time = 0.29 (sec) , antiderivative size = 2346, normalized size of antiderivative = 2.68

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*cosh(d*x + c) - ((7*(a
*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d^2*
x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*d^2 - 15*b^3)
*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x + c)
^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (7
*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d
^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2 - 15*b
^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x +
 c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))
*cosh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*
x)*Ei(d*x) - (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(-d*x))*cosh(c
) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(
```

```
a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2
 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cos
h(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (
a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^
2/b)) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 -
7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*
d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*
cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3
+ (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d
*x + c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2
 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b
^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*
x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x
^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt
(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c
)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((
a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*
b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2
)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x +
sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^
2*x^3 + a^3*d^2*x)*Ei(d*x) + (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*
Ei(-d*x))*sinh(c) + ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(
d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^
2 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 1
5*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(
d*x + sqrt(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*co
sh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x +
c)^2 - (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
 - 15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2
- 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*d*x^5 + 2*a
^5*b*d*x^3 + a^6*d*x)*cosh(d*x + c)^2 - (a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^
6*d*x)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,(b\,x^2+a)^3}\,dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)^3),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)^3), x)
```

3.78 $\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 791

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{3b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} + \frac{3b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} + \frac{9\sqrt{b}d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} - \frac{9\sqrt{b}d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3b\sinh(c)\text{Shi}(dx)}{a^4} + \frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3} + \frac{9\sqrt{b}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} - \frac{3b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} + \frac{9\sqrt{b}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}} + \frac{3b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3}$$

```
[Out] -3*b*Chi(d*x)*cosh(c)/a^4+1/2*d^2*Chi(d*x)*cosh(c)/a^3-1/2*cosh(d*x+c)/a^3/
x^2-1/4*b*cosh(d*x+c)/a^2/(b*x^2+a)^2-b*cosh(d*x+c)/a^3/(b*x^2+a)+3/2*b*Chi
(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Chi(d*
```

```
x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*Chi(-d*x+d*(
-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Chi(-d*x+d*(-a
)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^3-3*b*Shi(d*x)*sinh(c)/a^4+
1/2*d^2*Shi(d*x)*sinh(c)/a^3-1/2*d*sinh(d*x+c)/a^3/x+3/2*b*Shi(d*x+d*(-a)^(
1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Shi(d*x+d*(-a)^(1/2
)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*Shi(d*x-d*(-a)^(1/2)/b^(1
/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2)
)*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3-9/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(
d*x-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+9/16*d*cosh(c-d*(-a)^(1/2)/b^(
1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+9/16*d*Chi(d*x+d*(-a
)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-9/16*d*Chi
(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)
-1/16*d*sinh(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/16*d*sinh(d*x+c)*b
^(1/2)/a^3/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{5401, 3378, 3384, 3379, 3382, 5397, 5388\}$

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3}\,dx = -\frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{3b\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{3b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} - \frac{3b\sinh(c)\text{Shi}(dx)}{a^4} - \frac{3b\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{3b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^4} - \frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} + \frac{d^2\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3} - \frac{b\cosh(c+dx)}{a^3(a+bx^2)} - \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3} - \frac{\cosh(c+dx)}{2a^3x^2} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{b\cosh(c+dx)}{4a^2(a+bx^2)^2} + \frac{9\sqrt{b}d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} - \frac{9\sqrt{b}d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} + \frac{9\sqrt{b}d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} + \frac{9\sqrt{b}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b
*Cosh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (
d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]
```

```
]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sqrt[-
a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3*b*Co
sh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a
^4) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
inh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2))
 - (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a
] - Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x))
 - (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2
*a^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh[c + (Sqrt[-a]*d
)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (9*Sqrt[b]*
d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{a^3x^3} - \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2x\cosh(c+dx)}{a^2(a+bx^2)^3} + \frac{2b^2x\cosh(c+dx)}{a^3(a+bx^2)^2} + \frac{3b^2x\cosh(c+dx)}{a^4(a+bx^2)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^4} + \frac{(2b^2)\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx}{a^3} + \frac{b^2\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b\cosh(c+dx)}{a^3(a+bx^2)} + \frac{(3b^2)\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^4} + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a^3} + \frac{(bd)\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{a^3} + \frac{(bd)\int \frac{\sinh(c+dx)}{(a+bx^2)^2}\,dx}{4a^2} - \frac{(3b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^4} - \frac{(3b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{a^4}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{3b\sinh(c)\text{Shi}(dx)}{a^4} - \frac{\left(3b^{3/2}\right)\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^4} + \frac{\left(3b^{3/2}\right)\int\frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^4} + \frac{(bd)\int\left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^3} + \frac{(bd)\int\left(-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\sinh(c+dx)}{2a(-ab-b^2x^2)}\right)dx}{4a^2} + \frac{d^2\int\frac{\cosh(c+dx)}{x}\,dx}{2a^3}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{3b\sinh(c)\text{Shi}(dx)}{a^4} + \frac{(bd)\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{(bd)\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{\left(b^2d\right)\int\frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^3} - \frac{\left(b^2d\right)\int\frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^3} - \frac{\left(b^2d\right)\int\frac{\sinh(c+dx)}{-ab-b^2x^2}\,dx}{8a^3} + \frac{\left(d^2\cosh(c)\right)\int\frac{\cosh(dx)}{x}\,dx}{2a^3} + \frac{\left(3b^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^4} - \frac{\left(3b^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^4} + \frac{\left(d^2\sinh(c)\right)\int\frac{\sinh(dx)}{x}\,dx}{2a^3} + \frac{\left(3b^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^4} + \frac{\left(3b^{3/2}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^4}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b\cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4}$$
$$+ \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a^3} + \frac{3b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}$$
$$+ \frac{3b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}$$
$$+ \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4} + \frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a^3}$$
$$- \frac{3b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{3b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4}$$
$$- \frac{(b^2d)\int\left(-\frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8a^3} + \frac{(bd^2)\int\frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3}$$
$$- \frac{(bd^2)\int\frac{\cosh(c+dx)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3} + \frac{\left(bd\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$- \frac{\left(bd\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$+ \frac{\left(bd\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$
$$+ \frac{\left(bd\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}}$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} \\
&+ \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a^3} + \frac{3b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} \\
&+ \frac{3b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} + \frac{\sqrt{b}d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{7/2}} \\
&- \frac{\sqrt{b}d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{7/2}} - \frac{d\sinh(c+dx)}{2a^3x} \\
&- \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4} \\
&+ \frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a^3} + \frac{\sqrt{b}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}} \\
&- \frac{3b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{\sqrt{b}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{7/2}} \\
&+ \frac{3b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} + \frac{(bd)\int\frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{7/2}} \\
&+ \frac{(bd)\int\frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{7/2}} - \frac{\left(bd^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3} \\
&+ \frac{\left(bd^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3} \\
&- \frac{\left(bd^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}\sqrt{b}+bx}\,dx}{16a^3} \\
&- \frac{\left(bd^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}\sqrt{b}-bx}\,dx}{16a^3}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4}$$
$$+ \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a^3} + \frac{3b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4}$$
$$- \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} + \frac{3b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4}$$
$$- \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} + \frac{\sqrt{b}d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{7/2}}$$
$$- \frac{\sqrt{b}d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{7/2}} - \frac{d\sinh(c+dx)}{2a^3x}$$
$$- \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4}$$
$$+ \frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a^3} + \frac{\sqrt{b}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}}$$
$$- \frac{3b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} + \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}$$
$$+ \frac{\sqrt{b}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{7/2}} + \frac{3b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4}$$
$$- \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} + \frac{\left(bd\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{7/2}}$$
$$- \frac{\left(bd\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{7/2}}$$
$$+ \frac{\left(bd\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16(-a)^{7/2}}$$
$$+ \frac{\left(bd\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{7/2}}$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} \\
&\quad + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{3b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} \\
&\quad - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} + \frac{3b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} \\
&\quad - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3} + \frac{9\sqrt{b}d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} \\
&\quad - \frac{9\sqrt{b}d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} \\
&\quad + \frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3b\sinh(c)\text{Shi}(dx)}{a^4} + \frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3} \\
&\quad + \frac{9\sqrt{b}d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}} - \frac{3b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^4} \\
&\quad + \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3} + \frac{9\sqrt{b}d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{7/2}} \\
&\quad + \frac{3b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^4} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.55

$$
\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx
$$

$$
= \frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(24b-9i\sqrt{a}\sqrt{b}d-ad^2\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(24b+9i\sqrt{a}\sqrt{b}d-ad^2\right)\text{ExpI}\right.}{}
$$

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] (E^(c - (I*Sqrt[a]*d)/Sqrt[b])*((24*b - (9*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*E^
(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] +
 (24*b + (9*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt
[b] + x)]) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((24*b - (9*I)*Sqrt[a]*Sqrt[b]*
d - a*d^2)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqr
t[b] - d*x] + (24*b + (9*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[(I*Sqr
```

```
t[a]*d)/Sqrt[b] - d*x]) - (4*a*Cosh[d*x]*(2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)
*Cosh[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sinh[c]))/(x^2*(a + b*x^2)^2
) - (4*a*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cosh[c] + 2*(2*a^2 + 9*a*b*x^
2 + 6*b^2*x^4)*Sinh[c])*Sinh[d*x])/(x^2*(a + b*x^2)^2) + 16*(-6*b + a*d^2)*
(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/(32*a^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(629) = 1258.

Time = 0.49 (sec) , antiderivative size = 1294, normalized size of antiderivative = 1.64

method	result	size
risch	Expression too large to display	1294

```
[In] int(cosh(d*x+c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 9/32*d/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)*b-9/32*d/a^3/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*E
i(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-3/4/a^4*exp((-d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-9/8*exp(d*x+c)*d^4/a^2/(b^2*d^4
*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/4*exp(d*x+c)/a/x^2*d^4/(b^2*d^4*x^4+2*a*b*d
^4*x^2+a^2*d^4)-9/8*exp(-d*x-c)*d^4/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)
*b-1/4*exp(-d*x-c)/a/x^2*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-7/16*d^5*e
xp(d*x+c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x-3/4*exp(d*x+c)/a^3*x^
2*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-3/16*d^5*exp(d*x+c)/a^3*x^3/(
b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+3/16*d^5*exp(-d*x-c)/a^3*x^3/(b^2*d^
4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+7/16*d^5*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*
b*d^4*x^2+a^2*d^4)*b*x-3/4*exp(-d*x-c)/a^3*x^2*d^4/(b^2*d^4*x^4+2*a*b*d^4*x
^2+a^2*d^4)*b^2-1/4*d^5*exp(d*x+c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+
1/4*d^5*exp(-d*x-c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/a^3*ex
p(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2
/a^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3
/4/a^4*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)
*b-3/4/a^4*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b
)/b)*b-9/32*d/a^3/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(
1/2)-(d*x+c)*b+c*b)/b)*b+9/32*d/a^3/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/
b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+1/32*d^2/a^3*exp((d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp((-d*(-a*
b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/4/a^4*exp((d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b+3/2/a^4*b*exp(c)
*Ei(1,-d*x)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4*d^2/a^3*exp(-c)*Ei(1,d*x)-1/4*d
^2/a^3*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2363 vs. 2(630) = 1260.

Time = 0.30 (sec) , antiderivative size = 2363, normalized size of antiderivative = 2.99

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*cosh(d*x + c) + ((((a*b^2*d^2
- 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*co
sh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
(a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2
*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)
^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 +
 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 -
 ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a
^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*
x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2
/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 8*(((a*b^2*d^2 -
 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x
) + ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a
^2*b)*x^2)*Ei(-d*x))*cosh(c) + ((((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 -
24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh
(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3
*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + s
qrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
+ (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2
*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9
*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*cosh(-c + sqrt(-a*d^2/b)) + 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)
*sinh(d*x + c) + ((((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 +
2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 +
9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
 - (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 2
4*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^
2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqr
t(-a*d^2/b)) - 8*(((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 +
```

```
(a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x) - ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2
- 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(-d*x))*sinh(c) - ((((a*b^2*d^2
 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*c
osh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
 (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^
2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (((a*b^2*d^2 - 24*b^3)*x^6
+ 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2
- ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*
a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d
*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*x^6 +
 2*a^5*b*x^4 + a^6*x^2)*cosh(d*x + c)^2 - (a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx^2)^3} \, dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx^2)^3} \, dx = \int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^3} \, dx$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Giac [F]

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx^2)^3} \, dx = \int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^3} \, dx$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,(b\,x^2+a)^3}\,dx$$

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x^2)^3),x)
[Out] int(cosh(c + d*x)/(x^3*(a + b*x^2)^3), x)
```

3.79 $\int x^3(a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 154

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{720b\sinh(c+dx)}{d^7}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{360bx^2\sinh(c+dx)}{d^5}+\frac{ax^3\sinh(c+dx)}{d}+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}$$

```
[Out] -6*a*cosh(d*x+c)/d^4-720*b*x*cosh(d*x+c)/d^6-3*a*x^2*cosh(d*x+c)/d^2-120*b*
x^3*cosh(d*x+c)/d^4-6*b*x^5*cosh(d*x+c)/d^2+720*b*sinh(d*x+c)/d^7+6*a*x*sin
h(d*x+c)/d^3+360*b*x^2*sinh(d*x+c)/d^5+a*x^3*sinh(d*x+c)/d+30*b*x^4*sinh(d*
x+c)/d^3+b*x^6*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used

$= \{5395, 3377, 2718, 2717\}$

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2} +\frac{ax^3\sinh(c+dx)}{d}+\frac{720b\sinh(c+dx)}{d^7} -\frac{720bx\cosh(c+dx)}{d^6}+\frac{360bx^2\sinh(c+dx)}{d^5} -\frac{120bx^3\cosh(c+dx)}{d^4}+\frac{30bx^4\sinh(c+dx)}{d^3} -\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{bx^6\sinh(c+dx)}{d}$$

```
[In] Int[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c +
d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 + (
720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c +
d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b*x^6
*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^3\cosh(c+dx)+bx^6\cosh(c+dx)\right)\,dx\\
&= a\int x^3\cosh(c+dx)\,dx+b\int x^6\cosh(c+dx)\,dx\\
&= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^6\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(6b)\int x^5\sinh(c+dx)\,dx}{d}\\
&= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}\\
&\quad+\frac{bx^6\sinh(c+dx)}{d}+\frac{(6a)\int x\cosh(c+dx)\,dx}{d^2}+\frac{(30b)\int x^4\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}\\
&\quad+\frac{ax^3\sinh(c+dx)}{d}+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}\\
&\quad-\frac{(6a)\int\sinh(c+dx)\,dx}{d^3}-\frac{(120b)\int x^3\sinh(c+dx)\,dx}{d^3}\\
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}\\
&\quad-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{ax^3\sinh(c+dx)}{d}\\
&\quad+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}+\frac{(360b)\int x^2\cosh(c+dx)\,dx}{d^4}\\
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh(c+dx)}{d^2}\\
&\quad+\frac{6ax\sinh(c+dx)}{d^3}+\frac{360bx^2\sinh(c+dx)}{d^5}+\frac{ax^3\sinh(c+dx)}{d}\\
&\quad+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}-\frac{(720b)\int x\sinh(c+dx)\,dx}{d^5}\\
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}\\
&\quad-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{360bx^2\sinh(c+dx)}{d^5}+\frac{ax^3\sinh(c+dx)}{d}\\
&\quad+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}+\frac{(720b)\int\cosh(c+dx)\,dx}{d^6}\\
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6}-\frac{3ax^2\cosh(c+dx)}{d^2}\\
&\quad-\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{720b\sinh(c+dx)}{d^7}+\frac{6ax\sinh(c+dx)}{d^3}\\
&\quad+\frac{360bx^2\sinh(c+dx)}{d^5}+\frac{ax^3\sinh(c+dx)}{d}+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.65

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx$$
$$=\frac{-3d(ad^2(2+d^2x^2)+2bx(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(ad^4x(6+d^2x^2)+b(720+360d^2x^2+3\ldots}{d^7}$$

```
[In] Integrate[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-3*d*(a*d^2*(2 + d^2*x^2) + 2*b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a*d^4*x*(6 + d^2*x^2) + b*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6)
)*Sinh[c + d*x])/d^7
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{3dx\left(x\left(2b\,x^3+a\right)d^4+40b\,d^2x^2+240b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(\left(-b\,x^6-a\,x^3\right)d^6-6x\left(5b\,x^3+a\right)d^4-360b\,d^2x^2-720b\right)\tanh\left(\frac{dx}{2}+\ldots\right.}{d^7\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,x^6d^6-6b\,x^5d^5+a\,d^6x^3+30b\,x^4d^4-3a\,d^5x^2-120b\,d^3x^3+6a\,d^4x+360b\,d^2x^2-6d^3a-720dxb+720b\right)\mathrm{e}^{dx+c}}{2d^7}-\frac{\left(b\,x^6d^6+\ldots\right.}{}$
meijerg	$\frac{64ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{21}{8}d^4x^4+\frac{105}{2}x^2d^2+315\right)\cosh(dx)}{28\sqrt{\pi}}-\frac{i\left(\frac{7}{16}x^6d^6+\frac{105}{8}d^4x^4+\frac{315}{2}x^2d^2+315\right)\sinh(dx)}{28\sqrt{\pi}}\right)}{d^7}+\frac{64b\sinh(c)\sqrt{\pi}\ldots}{}$
parts	$\frac{b\,x^6\sinh(dx+c)}{d}+\frac{a\,x^3\sinh(dx+c)}{d}-\frac{3\left(-\frac{2b\,c^5\cosh(dx+c)}{d^5}+\frac{10b\,c^4((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^5}-\frac{20b\,c^3\left((dx+c)^2\cosh\ldots\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{b\,c^6\sinh(dx+c)}{d^3}-\frac{6b\,c^5((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{15b\,c^4\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{20b\,c^3\ldots}{}}{}$
default	$\frac{\frac{b\,c^6\sinh(dx+c)}{d^3}-\frac{6b\,c^5((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{15b\,c^4\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{20b\,c^3\ldots}{}}{}$

```
[In] int(x^3*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (3*d*x*(x*(2*b*x^3+a)*d^4+40*b*d^2*x^2+240*b)*tanh(1/2*d*x+1/2*c)^2+2*((-b*
x^6-a*x^3)*d^6-6*x*(5*b*x^3+a)*d^4-360*b*d^2*x^2-720*b)*tanh(1/2*d*x+1/2*c)
+3*d*(x^2*(2*b*x^3+a)*d^4+4*(10*b*x^3+a)*d^2+240*b*x))/d^7/(tanh(1/2*d*x+1/
2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = -\frac{3\,(2\,bd^5x^5+ad^5x^2+40\,bd^3x^3+2\,ad^3+240\,bdx)\cosh(dx+c)-(bd^6x^6+ad^6x^3+30\,bd^4x^4+6\,ad^4x+3}{d^7}$$

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
[Out] -(3*(2*b*d^5*x^5 + a*d^5*x^2 + 40*b*d^3*x^3 + 2*a*d^3 + 240*b*d*x)*cosh(d*x
 + c) - (b*d^6*x^6 + a*d^6*x^3 + 30*b*d^4*x^4 + 6*a*d^4*x + 360*b*d^2*x^2 +
 720*b)*sinh(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^3\sinh(c+dx)}{d}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{bx^6\sinh(c+dx)}{d}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{30bx^4\sinh(c+dx)}{d^3} \\ \left(\frac{ax^4}{4}+\frac{bx^7}{7}\right)\cosh(c) \end{cases}$$

```
[In] integrate(x**3*(b*x**3+a)*cosh(d*x+c),x)
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**6*sinh(c + d*x)/d - 6*b*x**
5*cosh(c + d*x)/d**2 + 30*b*x**4*sinh(c + d*x)/d**3 - 120*b*x**3*cosh(c + d
*x)/d**4 + 360*b*x**2*sinh(c + d*x)/d**5 - 720*b*x*cosh(c + d*x)/d**6 + 720
*b*sinh(c + d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.74

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = -\frac{1}{56}\,d\left(\frac{7\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ae^{(dx)}}{d^5}+\frac{7\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+2}{d^5}\right. + \frac{1}{28}\,(4\,bx^7+7\,ax^4)\cosh(dx+c)$$

[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] -1/56*d*(7*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*e^(d*x)/d^5 + 7*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e^(-d*x - c)/d^5 + 4*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 42*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040*d*x*e^c - 5040*e^c)*b*e^(d*x)/d^8 + 4*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b*e^(-d*x - c)/d^8) + 1/28*(4*b*x^7 + 7*a*x^4)*cosh(d*x + c)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int x^3\left(a+bx^3\right)\cosh(c+dx)\,dx$$
$$= \frac{\left(bd^6x^6 - 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 - 3\,ad^5x^2 - 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 - 6\,ad^3 - 720\,bdx + 720\right.}{2\,d^7}$$
$$- \frac{\left(bd^6x^6 + 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 + 3\,ad^5x^2 + 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 + 6\,ad^3 + 720\,bdx + 7\right.}{2\,d^7}$$

[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")

[Out] 1/2*(b*d^6*x^6 - 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 - 3*a*d^5*x^2 - 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 - 6*a*d^3 - 720*b*d*x + 720*b)*e^(d*x + c)/d^7 - 1/2*(b*d^6*x^6 + 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 + 3*a*d^5*x^2 + 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 + 6*a*d^3 + 720*b*d*x + 720*b)*e^(-d*x - c)/d^7

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int x^3\left(a+bx^3\right)\cosh(c+dx)\,dx = \frac{30\,b\,x^4\sinh(c+d\,x)+6\,a\,x\sinh(c+d\,x)}{d^3} - \frac{3\,a\,x^2\cosh(c+d\,x)+6\,b\,x^5\cosh(c+d\,x)}{d^2} + \frac{a\,x^3\sinh(c+d\,x)+b\,x^6\sinh(c+d\,x)}{d} - \frac{6\,a\cosh(c+d\,x)+120\,b\,x^3\cosh(c+d\,x)}{d^4} + \frac{720\,b\sinh(c+d\,x)}{d^7} - \frac{720\,b\,x\cosh(c+d\,x)}{d^6} + \frac{360\,b\,x^2\sinh(c+d\,x)}{d^5}$$

```
[In] int(x^3*cosh(c + d*x)*(a + b*x^3),x)

[Out] (30*b*x^4*sinh(c + d*x) + 6*a*x*sinh(c + d*x))/d^3 - (3*a*x^2*cosh(c + d*x)
 + 6*b*x^5*cosh(c + d*x))/d^2 + (a*x^3*sinh(c + d*x) + b*x^6*sinh(c + d*x))
/d - (6*a*cosh(c + d*x) + 120*b*x^3*cosh(c + d*x))/d^4 + (720*b*sinh(c + d*
x))/d^7 - (720*b*x*cosh(c + d*x))/d^6 + (360*b*x^2*sinh(c + d*x))/d^5
```

3.80 $\int x^2(a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 124

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = -\frac{120b\cosh(c+dx)}{d^6}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{120bx\sinh(c+dx)}{d^5}+\frac{ax^2\sinh(c+dx)}{d}+\frac{20bx^3\sinh(c+dx)}{d^3}+\frac{bx^5\sinh(c+dx)}{d}$$

```
[Out] -120*b*cosh(d*x+c)/d^6-2*a*x*cosh(d*x+c)/d^2-60*b*x^2*cosh(d*x+c)/d^4-5*b*x
^4*cosh(d*x+c)/d^2+2*a*sinh(d*x+c)/d^3+120*b*x*sinh(d*x+c)/d^5+a*x^2*sinh(d
*x+c)/d+20*b*x^3*sinh(d*x+c)/d^3+b*x^5*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5395, 3377, 2717, 2718}

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{120b\cosh(c+dx)}{d^6}+\frac{120bx\sinh(c+dx)}{d^5}-\frac{60bx^2\cosh(c+dx)}{d^4}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{bx^5\sinh(c+dx)}{d}$$

```
[In] Int[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c +
 d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120*b
*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x])/
d^3 + (b*x^5*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^2\cosh(c+dx)+bx^5\cosh(c+dx)\right)\,dx \\
&= a\int x^2\cosh(c+dx)\,dx+b\int x^5\cosh(c+dx)\,dx \\
&= \frac{ax^2\sinh(c+dx)}{d}+\frac{bx^5\sinh(c+dx)}{d}-\frac{(2a)\int x\sinh(c+dx)\,dx}{d}-\frac{(5b)\int x^4\sinh(c+dx)\,dx}{d} \\
&= -\frac{2ax\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d} \\
&\quad +\frac{bx^5\sinh(c+dx)}{d}+\frac{(2a)\int\cosh(c+dx)\,dx}{d^2}+\frac{(20b)\int x^3\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{2ax\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+dx)}{d} \\
&\quad +\frac{20bx^3\sinh(c+dx)}{d^3}+\frac{bx^5\sinh(c+dx)}{d}-\frac{(60b)\int x^2\sinh(c+dx)\,dx}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} \\
&\quad + \frac{ax^2\sinh(c+dx)}{d} + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{bx^5\sinh(c+dx)}{d} + \frac{(120b)\int x\cosh(c+dx)\,dx}{d^4} \\
&= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{5bx^4\cosh(c+dx)}{d^2} \\
&\quad + \frac{2a\sinh(c+dx)}{d^3} + \frac{120bx\sinh(c+dx)}{d^5} + \frac{ax^2\sinh(c+dx)}{d} \\
&\quad + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{bx^5\sinh(c+dx)}{d} - \frac{(120b)\int\sinh(c+dx)\,dx}{d^5} \\
&= -\frac{120b\cosh(c+dx)}{d^6} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} \\
&\quad - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{120bx\sinh(c+dx)}{d^5} \\
&\quad + \frac{ax^2\sinh(c+dx)}{d} + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{bx^5\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$
\int x^2(a+bx^3)\cosh(c+dx)\,dx
$$

$$
= \frac{-((2ad^4x+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx)) + d(ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\sinh(c}{d^6}
$$

```
[In] Integrate[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-((2*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*(a*d^2*
(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\left(\left(5b\,x^{4}+2ax\right)d^{4}+60b\,d^{2}x^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2d\left(x^{2}\left(b\,x^{3}+a\right)d^{4}+2\left(10b\,x^{3}+a\right)d^{2}+120bx\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(5b\,x^{4}+2ax\right)d^{4}+}{d^{6}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,x^{5}d^{5}-5b\,x^{4}d^{4}+a\,d^{5}x^{2}+20b\,d^{3}x^{3}-2a\,d^{4}x-60b\,d^{2}x^{2}+2d^{3}a+120dxb-120b\right)\mathrm{e}^{dx+c}}{2d^{6}}-\frac{\left(b\,x^{5}d^{5}+5b\,x^{4}d^{4}+a\,d^{5}x^{2}+20b\,d^{3}\right.}{}$
meijerg	$-\frac{32b\cosh\left(c\right)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^{4}x^{4}+\frac{45}{2}x^{2}d^{2}+45\right)\cosh\left(dx\right)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^{4}x^{4}+\frac{15}{2}x^{2}d^{2}+45\right)\sinh\left(dx\right)}{12\sqrt{\pi}}\right)}{d^{6}}+\frac{32ib\sinh\left(c\right)\sqrt{\pi}\left(-\frac{ixd}{}\right.}{}$
parts	$\frac{b\,x^{5}\sinh\left(dx+c\right)}{d}+\frac{a\,x^{2}\sinh\left(dx+c\right)}{d}-\frac{\frac{5b\,c^{4}\cosh\left(dx+c\right)}{d^{4}}-\frac{20b\,c^{3}\left(\left(dx+c\right)\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)}{d^{4}}+\frac{30b\,c^{2}\left(\left(dx+c\right)^{2}\cosh\left(dx+c\right)\right.}{}}{}$
derivativedivides	$\frac{-\frac{b\,c^{5}\sinh\left(dx+c\right)}{d^{3}}+\frac{5b\,c^{4}\left(\left(dx+c\right)\sinh\left(dx+c\right)-\cosh\left(dx+c\right)\right)}{d^{3}}-\frac{10b\,c^{3}\left(\left(dx+c\right)^{2}\sinh\left(dx+c\right)-2\left(dx+c\right)\cosh\left(dx+c\right)+2\sinh\left(dx+c\right)\right)}{d^{3}}+\frac{10b\,c^{2}}{}}{}$
default	$\frac{-\frac{b\,c^{5}\sinh\left(dx+c\right)}{d^{3}}+\frac{5b\,c^{4}\left(\left(dx+c\right)\sinh\left(dx+c\right)-\cosh\left(dx+c\right)\right)}{d^{3}}-\frac{10b\,c^{3}\left(\left(dx+c\right)^{2}\sinh\left(dx+c\right)-2\left(dx+c\right)\cosh\left(dx+c\right)+2\sinh\left(dx+c\right)\right)}{d^{3}}+\frac{10b\,c^{2}}{}}{}$

```
[In] int(x^2*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] ((((5*b*x^4+2*a*x)*d^4+60*b*d^2*x^2)*tanh(1/2*d*x+1/2*c)^2-2*d*(x^2*(b*x^3+a
)*d^4+2*(10*b*x^3+a)*d^2+120*b*x)*tanh(1/2*d*x+1/2*c)+(5*b*x^4+2*a*x)*d^4+6
0*b*d^2*x^2+240*b)/d^6/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int x^2\left(a+bx^3\right)\cosh(c+dx)\,dx = -\frac{\left(5\,bd^4x^4+2\,ad^4x+60\,bd^2x^2+120\,b\right)\cosh\left(dx+c\right)-\left(bd^5x^5+ad^5x^2+20\,bd^3x^3+2\,ad^3+120\,bdx\right)\sinh}{d^6}$$

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b*d^4*x^4 + 2*a*d^4*x + 60*b*d^2*x^2 + 120*b)*cosh(d*x + c) - (b*d^5*x
^5 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^3 + 120*b*d*x)*sinh(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx$$
$$=\begin{cases} \frac{ax^2\sinh(c+dx)}{d} - \frac{2ax\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{bx^5\sinh(c+dx)}{d} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{20bx^3\sinh(c+dx)}{d^3} - \frac{60bx^2\cosh(c+d}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^6}{6}\right)\cosh(c) \end{cases}$$

```
[In] integrate(x**2*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*x**
*3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c + d*x
)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*cosh(c
), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(124) = 248$.

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.15

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \frac{(bx^3+a)^2\cosh(dx+c)}{6\,b}$$
$$-\frac{\left(\frac{a^2e^{(dx+c)}}{d}+\frac{a^2e^{(-dx-c)}}{d}+\frac{2\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)abe^{(dx)}}{d^4}+\frac{2\,(d^3x^3+3\,d^2x^2+6\,dx+6)abe^{(-dx-c)}}{d^4}+\frac{(d^6x^6e^c-6\,d^5x^5e^c+}{}\right.}{12}$$

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^3 + a)^2*cosh(d*x + c)/b - 1/12*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
 c)/d + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4
 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d*x - c)/d^4 + (d^6*x^6*e^c
- 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*
d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 12
0*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b^2*e^(-d*x - c)/d^7)*d/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx$$
$$=\frac{(bd^5x^5-5\,bd^4x^4+ad^5x^2+20\,bd^3x^3-2\,ad^4x-60\,bd^2x^2+2\,ad^3+120\,bdx-120\,b)e^{(dx+c)}}{2\,d^6}$$
$$-\frac{(bd^5x^5+5\,bd^4x^4+ad^5x^2+20\,bd^3x^3+2\,ad^4x+60\,bd^2x^2+2\,ad^3+120\,bdx+120\,b)e^{(-dx-c)}}{2\,d^6}$$

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^5*x^5 - 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 - 2*a*d^4*x - 60*b*
d^2*x^2 + 2*a*d^3 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*(b*d^5*x^5 + 5
*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^4*x + 60*b*d^2*x^2 + 2*a*d^3
+ 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx=\frac{a\,x^2\sinh(c+d\,x)+b\,x^5\sinh(c+d\,x)}{d}-\frac{2\,a\,x\cosh(c+d\,x)+5\,b\,x^4\cosh(c+d\,x)}{d^2}+\frac{2\,a\sinh(c+d\,x)+20\,b\,x^3\sinh(c+d\,x)}{d^3}-\frac{120\,b\cosh(c+d\,x)}{d^6}+\frac{120\,b\,x\sinh(c+d\,x)}{d^5}-\frac{60\,b\,x^2\cosh(c+d\,x)}{d^4}$$

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (a*x^2*sinh(c + d*x) + b*x^5*sinh(c + d*x))/d - (2*a*x*cosh(c + d*x) + 5*b*
x^4*cosh(c + d*x))/d^2 + (2*a*sinh(c + d*x) + 20*b*x^3*sinh(c + d*x))/d^3 -
 (120*b*cosh(c + d*x))/d^6 + (120*b*x*sinh(c + d*x))/d^5 - (60*b*x^2*cosh(c
 + d*x))/d^4
```

3.81 $\int x(a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x\left(a+bx^3\right)\cosh(c+dx)\,dx=-\frac{a\cosh(c+dx)}{d^2}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{ax\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{bx^4\sinh(c+dx)}{d}$$

```
[Out] -a*cosh(d*x+c)/d^2-24*b*x*cosh(d*x+c)/d^4-4*b*x^3*cosh(d*x+c)/d^2+24*b*sinh
(d*x+c)/d^5+a*x*sinh(d*x+c)/d+12*b*x^2*sinh(d*x+c)/d^3+b*x^4*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5395, 3377, 2718, 2717}

$$\int x\left(a+bx^3\right)\cosh(c+dx)\,dx=-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d^2) - (24*b*x*Cosh[c + d*x])/d^4 - (4*b*x^3*Cosh[c + d
*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (a*x*Sinh[c + d*x])/d + (12*b*x^2*Sin
h[c + d*x])/d^3 + (b*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax\cosh(c+dx)+bx^4\cosh(c+dx)\right)\,dx\\
&= a\int x\cosh(c+dx)\,dx+b\int x^4\cosh(c+dx)\,dx\\
&= \frac{ax\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{a\int\sinh(c+dx)\,dx}{d}-\frac{(4b)\int x^3\sinh(c+dx)\,dx}{d}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}\\
&\quad+\frac{bx^4\sinh(c+dx)}{d}+\frac{(12b)\int x^2\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}\\
&\quad+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(24b)\int x\sinh(c+dx)\,dx}{d^3}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}\\
&\quad+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{bx^4\sinh(c+dx)}{d}+\frac{(24b)\int\cosh(c+dx)\,dx}{d^4}
\end{aligned}
$$

$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{24bx\cosh(c+dx)}{d^4} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c+dx)}{d^5} + \frac{ax\sinh(c+dx)}{d} + \frac{12bx^2\sinh(c+dx)}{d^3} + \frac{bx^4\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \frac{-d(ad^2+4bx(6+d^2x^2))\cosh(c+dx)+(ad^4x+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

```
[In] Integrate[x*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(a*d^2 + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^4*x + b*(24 + 12*d
^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{4dxb(x^2d^2+6)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2((-b\,x^4-ax)d^4-12b\,d^2x^2-24b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2((2b\,x^3+a)d^2+12bx)d}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b\,x^4d^4-4b\,d^3x^3+a\,d^4x+12b\,d^2x^2-d^3a-24dxb+24b)\mathrm{e}^{dx+c}}{2d^5} - \frac{(b\,x^4d^4+4b\,d^3x^3+a\,d^4x+12b\,d^2x^2+d^3a+24dxb+24b)\mathrm{e}}{2d^5}$
parts	$\frac{b\,x^4\sinh(dx+c)}{d} + \frac{ax\sinh(dx+c)}{d} - \frac{-\frac{4b\,c^3\cosh(dx+c)}{d^3}+\frac{12b\,c^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^3}-\frac{12bc\left((dx+c)^2\cosh(dx+c\right.}{}}{}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5} - \frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\frac{9}{2}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{4bc\left((dx+c)^3\sinh(dx+c\right.}{}}{}$
default	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{4bc\left((dx+c)^3\sinh(dx+c\right.}{}}{}$

```
[In] int(x*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*d*x*b*(d^2*x^2+6)*tanh(1/2*d*x+1/2*c)^2+((-b*x^4-a*x)*d^4-12*b*d^2*x^2
-24*b)*tanh(1/2*d*x+1/2*c)+((2*b*x^3+a)*d^2+12*b*x)*d)/d^5/(tanh(1/2*d*x+1/
2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x(a+bx^3)\cosh(c+dx)\,dx = -\frac{(4\,bd^3x^3 + ad^3 + 24\,bdx)\cosh(dx+c) - (bd^4x^4 + ad^4x + 12\,bd^2x^2 + 24\,b)\sinh(dx+c)}{d^5}$$

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((4*b*d^3*x^3 + a*d^3 + 24*b*d*x)*cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x + 1
2*b*d^2*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \begin{cases} \frac{ax\sinh(c+dx)}{d} - \frac{a\cosh(c+dx)}{d^2} + \frac{bx^4\sinh(c+dx)}{d} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{12bx^2\sinh(c+dx)}{d^3} - \frac{24bx\cosh(c+dx)}{d^4} + \frac{24b\sinh(c+dx)}{d^5} \\ \left(\frac{ax^2}{2} + \frac{bx^5}{5}\right)\cosh(c) \end{cases}$$

```
[In] integrate(x*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**4*sinh(c + d*x
)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*c
osh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5
/5)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(94) = 188.

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.09

$$\int x(a+bx^3)\cosh(c+dx)\,dx = -\frac{1}{20}\,d\left(\frac{5\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)ae^{(dx)}}{d^3} + \frac{5\,(d^2x^2 + 2\,dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2\,(d^5x^5e^c - 5\,d^4x^4e^c + 20\,d^3x^3e^c\right.$$
$$+ \frac{1}{10}\,(2\,bx^5 + 5\,ax^2)\cosh(dx+c)$$

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/20*d*(5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 5*(d^2*x^2 + 2
*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*
e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 2*(d^5*x^5 +
5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) +
1/10*(2*b*x^5 + 5*a*x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \frac{(bd^4x^4-4\,bd^3x^3+ad^4x+12\,bd^2x^2-ad^3-24\,bdx+24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4+4\,bd^3x^3+ad^4x+12\,bd^2x^2+ad^3+24\,bdx+24\,b)e^{(-dx-c)}}{2\,d^5}$$

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 - 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x^2 - a*d^3 - 24*b*d*x +
24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x
^2 + a*d^3 + 24*b*d*x + 24*b)*e^(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \frac{b\,x^4\sinh(c+d\,x)+a\,x\sinh(c+d\,x)}{d} - \frac{a\cosh(c+d\,x)+4\,b\,x^3\cosh(c+d\,x)}{d^2} + \frac{24\,b\sinh(c+d\,x)}{d^5} - \frac{24\,b\,x\cosh(c+d\,x)}{d^4} + \frac{12\,b\,x^2\sinh(c+d\,x)}{d^3}$$

```
[In] int(x*cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (b*x^4*sinh(c + d*x) + a*x*sinh(c + d*x))/d - (a*cosh(c + d*x) + 4*b*x^3*co
sh(c + d*x))/d^2 + (24*b*sinh(c + d*x))/d^5 - (24*b*x*cosh(c + d*x))/d^4 +
(12*b*x^2*sinh(c + d*x))/d^3
```

3.82 $\int (a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a+bx^3)\cosh(c+dx)\,dx = -\frac{6b\cosh(c+dx)}{d^4} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{bx^3\sinh(c+dx)}{d}$$

```
[Out] -6*b*cosh(d*x+c)/d^4-3*b*x^2*cosh(d*x+c)/d^2+a*sinh(d*x+c)/d+6*b*x*sinh(d*x
+c)/d^3+b*x^3*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5385, 2717, 3377, 2718}

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{a\sinh(c+dx)}{d} - \frac{6b\cosh(c+dx)}{d^4} + \frac{6bx\sinh(c+dx)}{d^3} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{bx^3\sinh(c+dx)}{d}$$

```
[In] Int[(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/
d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a\cosh(c+dx)+bx^3\cosh(c+dx)\right)\,dx\\
&= a\int \cosh(c+dx)\,dx+b\int x^3\cosh(c+dx)\,dx\\
&= \frac{a\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}-\frac{(3b)\int x^2\sinh(c+dx)\,dx}{d}\\
&= -\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}+\frac{(6b)\int x\cosh(c+dx)\,dx}{d^2}\\
&= -\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}+\frac{6bx\sinh(c+dx)}{d^3}\\
&\quad+\frac{bx^3\sinh(c+dx)}{d}-\frac{(6b)\int \sinh(c+dx)\,dx}{d^3}\\
&= -\frac{6b\cosh(c+dx)}{d^4}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}\\
&\quad+\frac{6bx\sinh(c+dx)}{d^3}+\frac{bx^3\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int (a+bx^3)\cosh(c+dx)\,dx$$
$$=\frac{-3b(2+d^2x^2)\cosh(c+dx)+d(ad^2+bx(6+d^2x^2))\sinh(c+dx)}{d^4}$$

```
[In] Integrate[(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-3*b*(2 + d^2*x^2)*Cosh[c + d*x] + d*(a*d^2 + b*x*(6 + d^2*x^2))*Sinh[c +
d*x])/d^4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{3b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2x^2d^2-2d\left((b\,x^3+a)d^2+6bx\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+3b\,d^2x^2+12b}{d^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b\,d^3x^3-3b\,d^2x^2+d^3a+6dxb-6b)\,\mathrm{e}^{dx+c}}{2d^4}-\frac{(b\,d^3x^3+3b\,d^2x^2+d^3a+6dxb+6b)\,\mathrm{e}^{-dx-c}}{2d^4}$
parts	$\frac{b\,x^3\sinh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}-\frac{3b\left(c^2\cosh(dx+c)-2c((dx+c)\cosh(dx+c)-\sinh(dx+c))+(dx+c)^2\cosh(dx+c)-2(\right.}{d^4}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i\left(\frac{15x}{}\right.}{}\right.}{d^4}$
derivativedivides	$\frac{-\frac{b\,c^3\sinh(dx+c)}{d^3}+\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{b\left((dx+c)\right.}{}}{d}$
default	$\frac{-\frac{b\,c^3\sinh(dx+c)}{d^3}+\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{b\left((dx+c)\right.}{}}{d}$

```
[In] int((b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (3*b*tanh(1/2*d*x+1/2*c)^2*x^2*d^2-2*d*((b*x^3+a)*d^2+6*b*x)*tanh(1/2*d*x+1
/2*c)+3*b*d^2*x^2+12*b)/d^4/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int (a+bx^3)\cosh(c+dx)\,dx = -\frac{3\,(bd^2x^2+2\,b)\cosh(dx+c)-(bd^3x^3+ad^3+6\,bdx)\sinh(dx+c)}{d^4}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
[Out] -(3*(b*d^2*x^2 + 2*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3 + 6*b*d*x)*sinh(d*
x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int (a+bx^3)\cosh(c+dx)\,dx = \begin{cases} \frac{a\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(ax+\frac{bx^4}{4}\right)\cosh(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x**3+a)*cosh(d*x+c),x)
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d
*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((
a*x + b*x**4/4)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{ae^{(dx+c)}}{2\,d}-\frac{ae^{(-dx-c)}}{2\,d}+\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)be^{(dx)}}{2\,d^4}-\frac{(d^3x^3+3\,d^2x^2+6\,dx+6)be^{(-dx-c)}}{2\,d^4}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^3*x^3*e^c - 3*d^2*x^2*e
^c + 6*d*x*e^c - 6*e^c)*b*e^(d*x)/d^4 - 1/2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x +
6)*b*e^(-d*x - c)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{(bd^3x^3 - 3\,bd^2x^2 + ad^3 + 6\,bdx - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + 3\,bd^2x^2 + ad^3 + 6\,bdx + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 - 3*b*d^2*x^2 + a*d^3 + 6*b*d*x - 6*b)*e^(d*x + c)/d^4 - 1/2
*(b*d^3*x^3 + 3*b*d^2*x^2 + a*d^3 + 6*b*d*x + 6*b)*e^(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{a\sinh(c+d\,x) + b\,x^3\sinh(c+d\,x)}{d} - \frac{6\,b\cosh(c+d\,x)}{d^4} + \frac{6\,b\,x\sinh(c+d\,x)}{d^3} - \frac{3\,b\,x^2\cosh(c+d\,x)}{d^2}$$

```
[In] int(cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (a*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d - (6*b*cosh(c + d*x))/d^4 + (6*b*
x*sinh(c + d*x))/d^3 - (3*b*x^2*cosh(c + d*x))/d^2
```

3.83 $\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = -\frac{2bx\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{2b\sinh(c+dx)}{d^3} + \frac{bx^2\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)$$

```
[Out] a*Chi(d*x)*cosh(c)-2*b*x*cosh(d*x+c)/d^2+a*Shi(d*x)*sinh(c)+2*b*sinh(d*x+c)
/d^3+b*x^2*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5395, 3384, 3379, 3382, 3377, 2717}

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx) + a\sinh(c)\text{Shi}(dx) + \frac{2b\sinh(c+dx)}{d^3} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{bx^2\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*
x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x} + bx^2\cosh(c+dx)\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x}\,dx + b\int x^2\cosh(c+dx)\,dx \\
&= \frac{bx^2\sinh(c+dx)}{d} - \frac{(2b)\int x\sinh(c+dx)\,dx}{d} \\
&\quad + (a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + (a\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{2bx\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{bx^2\sinh(c+dx)}{d} \\
&\quad + a\sinh(c)\text{Shi}(dx) + \frac{(2b)\int \cosh(c+dx)\,dx}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2bx\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{2b\sinh(c+dx)}{d^3} \\
&\quad + \frac{bx^2\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = {} & a\cosh(c)\text{Chi}(dx) \\
& + \frac{b(-2dx\cosh(c+dx) + (2+d^2x^2)\sinh(c+dx))}{d^3} \\
& + a\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x,x]
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*(-2*d*x*Cosh[c + d*x] + (2 + d^2*x^2)*Sinh
[c + d*x]))/d^3 + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{a\,\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)}{2} - \frac{\mathrm{e}^{-dx-c}b\,x^2}{2d} - \frac{a\,\mathrm{e}^{c}\,\mathrm{Ei}_1(-dx)}{2} + \frac{\mathrm{e}^{dx+c}b\,x^2}{2d} - \frac{\mathrm{e}^{-dx-c}bx}{d^2} - \frac{\mathrm{e}^{dx+c}bx}{d^2} - \frac{\mathrm{e}^{-dx-c}b}{d^3} + \frac{\mathrm{e}^{dx+c}b}{d^3}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}} - \frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3} + \frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}} - \frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3} + \frac{a\cosh(c)\sqrt{\pi}}{}$

```
[In] int((b*x^3+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*a*exp(-c)*Ei(1,d*x)-1/2/d*exp(-d*x-c)*b*x^2-1/2*a*exp(c)*Ei(1,-d*x)+1/
2/d*exp(d*x+c)*b*x^2-1/d^2*exp(-d*x-c)*b*x-1/d^2*exp(d*x+c)*b*x-1/d^3*exp(-
d*x-c)*b+1/d^3*exp(d*x+c)*b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = -\frac{4\,bdx\cosh\left(dx+c\right)-\left(ad^3\mathrm{Ei}(dx)+ad^3\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\left(bd^2x^2+2\,b\right)\sinh\left(dx+c\right)-\left(ad^3\mathrm{Ei}(dx)-\right.}{2\,d^3}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x*cosh(d*x + c) - (a*d^3*Ei(d*x) + a*d^3*Ei(-d*x))*cosh(c) - 2*
(b*d^2*x^2 + 2*b)*sinh(d*x + c) - (a*d^3*Ei(d*x) - a*d^3*Ei(-d*x))*sinh(c))
/d^3
```

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = a\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a\cosh\left(c\right)\mathrm{Chi}\left(dx\right) + b\left(\begin{cases}\frac{x^2\sinh\left(c+dx\right)}{d}-\frac{2x\cosh\left(c+dx\right)}{d^2}+\frac{2\sinh\left(c+dx\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{x^3\cosh\left(c\right)}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x**2*sinh(c + d*x)/d
 - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**3*cosh(c)/
3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = -\frac{1}{6}\left(b\left(\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)e^{(dx)}}{d^4}+\frac{(d^3x^3+3\,d^2x^2+6\,dx+6)e^{(-dx-c)}}{d^4}\right)+\frac{2\,a\cosh\left(dx+c\right.}{d}\right. + \frac{1}{3}\left(bx^3+a\log\left(x^3\right)\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/6*(b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d
^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + 2*a*cosh(d*x + c)*log(x
^3)/d - 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + 1/3*(b*x^3 + a*log(x^3))
*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = \frac{bd^2x^2e^{(dx+c)} - bd^2x^2e^{(-dx-c)} + ad^3\mathrm{Ei}(-dx)\,e^{(-c)} + ad^3\mathrm{Ei}(dx)\,e^{c} - 2\,bdxe^{(dx+c)} - 2\,bdxe^{(-dx-c)} + 2\,be^{(dx+c}}{2\,d^3}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2*e^(d*x + c) - b*d^2*x^2*e^(-d*x - c) + a*d^3*Ei(-d*x)*e^(-c)
 + a*d^3*Ei(d*x)*e^c - 2*b*d*x*e^(d*x + c) - 2*b*d*x*e^(-d*x - c) + 2*b*e^(
d*x + c) - 2*b*e^(-d*x - c))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = a\,\mathrm{coshint}(d\,x)\,\cosh(c) + a\,\mathrm{sinhint}(d\,x)\,\sinh(c) + \frac{b\,(2\sinh(c+d\,x) + d^2\,x^2\sinh(c+d\,x) - 2\,d\,x\cosh(c+d\,x))}{d^3}$$

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) + (b*(2*sinh(c + d*x) + d^2
*x^2*sinh(c + d*x) - 2*d*x*cosh(c + d*x)))/d^3
```

3.84 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{bx\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)$$

```
[Out] -b*cosh(d*x+c)/d^2-a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c
)+b*x*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2718}

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^2} + bx\cosh(c+dx)\right) dx \\
&= a\int \frac{\cosh(c+dx)}{x^2}\,dx + b\int x\cosh(c+dx)\,dx \\
&= -\frac{a\cosh(c+dx)}{x} + \frac{bx\sinh(c+dx)}{d} - \frac{b\int \sinh(c+dx)\,dx}{d} + (ad)\int \frac{\sinh(c+dx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{b\cosh(c+dx)}{d^2} - \frac{a\cosh(c+dx)}{x} + \frac{bx\sinh(c+dx)}{d} + (ad\cosh(c))\int \frac{\sinh(dx)}{x}\,dx + (ad\sinh(c))\int \frac{\cosh(dx)}{x}\,dx$$

$$= -\frac{b\cosh(c+dx)}{d^2} - \frac{a\cosh(c+dx)}{x} + ad\mathrm{Chi}(dx)\sinh(c) + \frac{bx\sinh(c+dx)}{d} + ad\cosh(c)\mathrm{Shi}(dx)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{b\cosh(c+dx)}{d^2} - \frac{a\cosh(c+dx)}{x} + ad\mathrm{Chi}(dx)\sinh(c) + \frac{bx\sinh(c+dx)}{d} + ad\cosh(c)\mathrm{Shi}(dx)$$

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{e^{c}\,\mathrm{Ei}_1(-dx)a\,d^3x - e^{-c}\,\mathrm{Ei}_1(dx)a\,d^3x + e^{-dx-c}bd\,x^2 - e^{dx+c}bd\,x^2 + d^2e^{-dx-c}a + a\,d^2e^{dx+c} + e^{-dx-c}bx + e^{dx+c}bx}{2d^2x}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{b\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2} + \frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}(dx)}{\sqrt{\pi}}\right)}{4} +$

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d^2*(exp(c)*Ei(1,-d*x)*a*d^3*x-exp(-c)*Ei(1,d*x)*a*d^3*x+exp(-d*x-c)*b
*d*x^2-exp(d*x+c)*b*d*x^2+d^2*exp(-d*x-c)*a+a*d^2*exp(d*x+c)+exp(-d*x-c)*b*
x+exp(d*x+c)*b*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = \frac{2\,bdx^2\sinh\left(dx+c\right)-2\left(ad^2+bx\right)\cosh\left(dx+c\right)+\left(ad^3x\mathrm{Ei}(dx)-ad^3x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+\left(ad^3x\mathrm{Ei}(dx)\right.}{2\,d^2x}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x^2*sinh(d*x + c) - 2*(a*d^2 + b*x)*cosh(d*x + c) + (a*d^3*x*Ei(
d*x) - a*d^3*x*Ei(-d*x))*cosh(c) + (a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*sin
h(c))/(d^2*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)\cosh\left(c+dx\right)}{x^2}\,dx$$

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{1}{4}\left(2\,a\mathrm{Ei}(-dx)\,e^{(-c)}-2\,a\mathrm{Ei}(dx)\,e^{c}+\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)be^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)be^{(-dx-c)}}{d^3}\right)d + \frac{1}{2}\left(bx^2-\frac{2\,a}{x}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*Ei(-d*x)*e^(-c) - 2*a*Ei(d*x)*e^c + (d^2*x^2*e^c - 2*d*x*e^c + 2*
e^c)*b*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3)*d + 1/2*(b*x
^2 - 2*a/x)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{ad^3x\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x\mathrm{Ei}(dx)\,e^{c} - bdx^2e^{(dx+c)} + bdx^2e^{(-dx-c)} + ad^2e^{(dx+c)} + ad^2e^{(-dx-c)} + bxe^{(dx+c)} -}{2\,d^2x}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^3*x*Ei(-d*x)*e^(-c) - a*d^3*x*Ei(d*x)*e^c - b*d*x^2*e^(d*x + c) +
 b*d*x^2*e^(-d*x - c) + a*d^2*e^(d*x + c) + a*d^2*e^(-d*x - c) + b*x*e^(d*x
 + c) + b*x*e^(-d*x - c))/(d^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^2}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3))/x^2, x)
```

3.85 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = -\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{b\sinh(c+dx)}{d} -\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

```
[Out] 1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2+1/2*a*d^2*Shi(d*x)*sinh(c)
+b*sinh(d*x+c)/d-1/2*a*d*sinh(d*x+c)/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5395, 2717, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx) -\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+\frac{b\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x])/x^2 + (a*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (b*Sinh[
c + d*x])/d - (a*d*Sinh[c + d*x])/(2*x) + (a*d^2*Sinh[c]*SinhIntegral[d*x])
/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x^3}\right)dx\\
&= a\int \frac{\cosh(c+dx)}{x^3}\,dx+b\int \cosh(c+dx)\,dx\\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx\\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a\cosh(c+dx)}{2x^2} + \frac{b\sinh(c+dx)}{d} - \frac{ad\sinh(c+dx)}{2x} \\
&\quad + \frac{1}{2}\left(ad^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx + \frac{1}{2}\left(ad^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2} + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + \frac{b\sinh(c+dx)}{d} \\
&\quad - \frac{ad\sinh(c+dx)}{2x} + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$
\begin{aligned}
\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx &= \frac{b\cosh(dx)\sinh(c)}{d} - \frac{a\cosh(dx)(\cosh(c)+dx\sinh(c))}{2x^2} \\
&\quad + \frac{b\cosh(c)\sinh(dx)}{d} - \frac{a(dx\cosh(c)+\sinh(c))\sinh(dx)}{2x^2} \\
&\quad + \frac{1}{2}ad^2(\cosh(c)\text{Chi}(dx)+\sinh(c)\text{Shi}(dx))
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

```
[Out] (b*Cosh[d*x]*Sinh[c])/d - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2) + (
b*Cosh[c]*Sinh[d*x])/d - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + (a
*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{e^{c}\,\text{Ei}_1(-dx)a\,d^3x^2+e^{-c}\,\text{Ei}_1(dx)a\,d^3x^2-d^2e^{-dx-c}ax+d^2e^{dx+c}ax+2\,e^{-dx-c}b\,x^2-2\,e^{dx+c}b\,x^2+d\,e^{-dx-c}a+ad\,e^{dx+c}}{4d\,x^2}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d}-\frac{b\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}-\frac{a\cosh(c)\sqrt{\pi}\,d^2\left(\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}-\frac{4\left(\frac{9x^2d^2}{2}+3\right)}{3\sqrt{\pi}\,x^2d^2}+\frac{4\cosh(dx)}{\sqrt{\pi}\,x^2d^2}\right.}{8}$

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(exp(c)*Ei(1,-d*x)*a*d^3*x^2+exp(-c)*Ei(1,d*x)*a*d^3*x^2-d^2*exp(-d*
x-c)*a*x+d^2*exp(d*x+c)*a*x+2*exp(-d*x-c)*b*x^2-2*exp(d*x+c)*b*x^2+d*exp(-d
*x-c)*a+a*d*exp(d*x+c))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = -\frac{2\,ad\cosh\left(dx+c\right)-\left(ad^3x^2\mathrm{Ei}(dx)+ad^3x^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+2\left(ad^2x-2\,bx^2\right)\sinh\left(dx+c\right)-\left(ad^3x^2\mathrm{Ei}\right.}{4\,dx^2}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*cosh(d*x + c) - (a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c
) + 2*(a*d^2*x - 2*b*x^2)*sinh(d*x + c) - (a*d^3*x^2*Ei(d*x) - a*d^3*x^2*Ei
(-d*x))*sinh(c))/(d*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)\cosh\left(c+dx\right)}{x^3}\,dx$$

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx &= \frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx)+ade^{c}\Gamma(-1,-dx)-\frac{2\left(dxe^{c}-e^{c}\right)be^{(dx)}}{d^2}-\frac{2\left(dx+1\right)be^{(-dx-c)}}{d^2}\right)d \\ &\quad+\frac{1}{2}\left(2\,bx-\frac{a}{x^2}\right)\cosh\left(dx+c\right)\end{aligned}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*(d*x*e^c - e^c
)*b*e^(d*x)/d^2 - 2*(d*x + 1)*b*e^(-d*x - c)/d^2)*d + 1/2*(2*b*x - a/x^2)*c
osh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{ad^3x^2\mathrm{Ei}(-dx)\,e^{(-c)}+ad^3x^2\mathrm{Ei}(dx)\,e^{c}-ad^2xe^{(dx+c)}+ad^2xe^{(-dx-c)}+2\,bx^2e^{(dx+c)}-2\,bx^2e^{(-dx-c)}-ade^{(dx}}{4\,dx^2}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^3*x^2*Ei(-d*x)*e^(-c) + a*d^3*x^2*Ei(d*x)*e^c - a*d^2*x*e^(d*x + c
) + a*d^2*x*e^(-d*x - c) + 2*b*x^2*e^(d*x + c) - 2*b*x^2*e^(-d*x - c) - a*d
*e^(d*x + c) - a*d*e^(-d*x - c))/(d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^3}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3))/x^3, x)
```

3.86 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = -\frac{a\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{6x} + b\cosh(c)\text{Chi}(dx) + \frac{1}{6}ad^3\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{6x^2} + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx) + b\sinh(c)\text{Shi}(dx)$$

```
[Out] b*Chi(d*x)*cosh(c)-1/3*a*cosh(d*x+c)/x^3-1/6*a*d^2*cosh(d*x+c)/x+1/6*a*d^3*
cosh(c)*Shi(d*x)+1/6*a*d^3*Chi(d*x)*sinh(c)+b*Shi(d*x)*sinh(c)-1/6*a*d*sinh
(d*x+c)/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}ad^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx) - \frac{ad^2\cosh(c+dx)}{6x} - \frac{a\cosh(c+dx)}{3x^3} - \frac{ad\sinh(c+dx)}{6x^2} + b\cosh(c)\text{Chi}(dx) + b\sinh(c)\text{Shi}(dx)$$

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a*Cosh[c + d*x])/x^3 - (a*d^2*Cosh[c + d*x])/(6*x) + b*Cosh[c]*CoshIn
tegral[d*x] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*
x^2) + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + b*Sinh[c]*SinhIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x}\right)dx\\
&= a\int \frac{\cosh(c+dx)}{x^4}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{3x^3}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx\\
&\quad +(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(b\sinh(c))\int \frac{\sinh(dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{3x^3}+b\cosh(c)\text{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}\\
&\quad +b\sinh(c)\text{Shi}(dx)+\frac{1}{6}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{6x} + b\cosh(c)\text{Chi}(dx) \\
&\quad - \frac{ad\sinh(c+dx)}{6x^2} + b\sinh(c)\text{Shi}(dx) + \frac{1}{6}\left(ad^3\right)\int \frac{\sinh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{6x} + b\cosh(c)\text{Chi}(dx) \\
&\quad - \frac{ad\sinh(c+dx)}{6x^2} + b\sinh(c)\text{Shi}(dx) + \frac{1}{6}\left(ad^3\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&\quad + \frac{1}{6}\left(ad^3\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{6x} + b\cosh(c)\text{Chi}(dx) + \frac{1}{6}ad^3\text{Chi}(dx)\sinh(c) \\
&\quad - \frac{ad\sinh(c+dx)}{6x^2} + \frac{1}{6}ad^3\cosh(c)\text{Shi}(dx) + b\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$
\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(\text{Chi}(dx)\left(6b\cosh(c)+ad^3\sinh(c)\right) - \frac{a((2+d^2x^2)\cosh(c+dx)+dx\sinh(c+dx))}{x^3} + \left(ad^3\cosh(c)+6b\sinh(c)\right)\text{Shi}(dx)\right)
$$

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] (CoshIntegral[d*x]*(6*b*Cosh[c] + a*d^3*Sinh[c]) - (a*((2 + d^2*x^2)*Cosh[c
 + d*x] + d*x*Sinh[c + d*x]))/x^3 + (a*d^3*Cosh[c] + 6*b*Sinh[c])*SinhInteg
ral[d*x])/6
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{e^{c}\operatorname{Ei}_1(-dx)a\,d^3x^3-e^{-c}\operatorname{Ei}_1(dx)a\,d^3x^3+6\,e^{c}\operatorname{Ei}_1(-dx)b\,x^3+e^{-dx-c}a\,d^2x^2+6\,e^{-c}\operatorname{Ei}_1(dx)b\,x^3+e^{dx+c}a\,d^2x^2-e^{-dx-c}adx+e^{dx+c}ad \cdots}{12x^3}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}+\frac{2\text{ Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}\right)}{2}+b\text{ Shi}(dx)\sinh(c)-\frac{ia\cosh(c)\sqrt{\pi}\,d^3\left(-\frac{8i\left(x^2d^2+2\right)\cosh(dx)}{3d^3x^3\sqrt{\pi}}-\frac{8i \cdots}{3 \cdots}\right.}{16}$

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(exp(c)*Ei(1,-d*x)*a*d^3*x^3-exp(-c)*Ei(1,d*x)*a*d^3*x^3+6*exp(c)*Ei(
1,-d*x)*b*x^3+exp(-d*x-c)*a*d^2*x^2+6*exp(-c)*Ei(1,d*x)*b*x^3+exp(d*x+c)*a*
d^2*x^2-exp(-d*x-c)*a*d*x+exp(d*x+c)*a*d*x+2*exp(-d*x-c)*a+2*a*exp(d*x+c))/
x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = -\frac{2\,adx\sinh\left(dx+c\right)+2\left(ad^2x^2+2\,a\right)\cosh\left(dx+c\right)-\left((ad^3+6\,b)x^3\mathrm{Ei}(dx)-(ad^3-6\,b)x^3\mathrm{Ei}(-dx)\right)\cos}{12\,x^3}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*(a*d^2*x^2 + 2*a)*cosh(d*x + c) - ((a*d^3
+ 6*b)*x^3*Ei(d*x) - (a*d^3 - 6*b)*x^3*Ei(-d*x))*cosh(c) - ((a*d^3 + 6*b)*x
^3*Ei(d*x) + (a*d^3 - 6*b)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)\cosh\left(c+dx\right)}{x^4}\,dx$$

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$$
$$= \frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma(-2,dx)-d^2e^c\Gamma(-2,-dx)\right)a-\frac{2\,b\cosh\left(dx+c\right)\log\left(x^3\right)}{d}+\frac{3\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^c\right)b}{d}\right)$$
$$+\frac{1}{3}\left(b\log\left(x^3\right)-\frac{a}{x^3}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a - 2*b*cosh(d*x
 + c)*log(x^3)/d + 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/3*(b*log(x^
3) - a/x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = -\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} - 6\,bx^3\mathrm{Ei}(-dx)\,e^{(-c)} - 6\,bx^3\mathrm{Ei}(dx)}{12\,x^3}$$

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x
 + c) + a*d^2*x^2*e^(-d*x - c) - 6*b*x^3*Ei(-d*x)*e^(-c) - 6*b*x^3*Ei(d*x)*
e^c + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*
x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^4}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3))/x^4, x)
```

3.87 $\int x(a+bx^3)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 234

$$\begin{aligned}\int x(a+bx^3)^2\cosh(c+dx)\,dx = & -\frac{5040b^2\cosh(c+dx)}{d^8}-\frac{a^2\cosh(c+dx)}{d^2}\\ & -\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}\\ & -\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{210b^2x^4\cosh(c+dx)}{d^4}\\ & -\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{48ab\sinh(c+dx)}{d^5}\\ & +\frac{5040b^2x\sinh(c+dx)}{d^7}\\ & +\frac{a^2x\sinh(c+dx)}{d}+\frac{24abx^2\sinh(c+dx)}{d^3}\\ & +\frac{840b^2x^3\sinh(c+dx)}{d^5}+\frac{2abx^4\sinh(c+dx)}{d}\\ & +\frac{42b^2x^5\sinh(c+dx)}{d^3}+\frac{b^2x^7\sinh(c+dx)}{d}\end{aligned}$$

```
[Out] -5040*b^2*cosh(d*x+c)/d^8-a^2*cosh(d*x+c)/d^2-48*a*b*x*cosh(d*x+c)/d^4-2520
*b^2*x^2*cosh(d*x+c)/d^6-8*a*b*x^3*cosh(d*x+c)/d^2-210*b^2*x^4*cosh(d*x+c)/
d^4-7*b^2*x^6*cosh(d*x+c)/d^2+48*a*b*sinh(d*x+c)/d^5+5040*b^2*x*sinh(d*x+c)
/d^7+a^2*x*sinh(d*x+c)/d+24*a*b*x^2*sinh(d*x+c)/d^3+840*b^2*x^3*sinh(d*x+c)
/d^5+2*a*b*x^4*sinh(d*x+c)/d+42*b^2*x^5*sinh(d*x+c)/d^3+b^2*x^7*sinh(d*x+c)
/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5395, 3377, 2718, 2717}

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx = -\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{48ab\sinh(c+dx)}{d^5} -\frac{48abx\cosh(c+dx)}{d^4}+\frac{24abx^2\sinh(c+dx)}{d^3} -\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d} -\frac{5040b^2\cosh(c+dx)}{d^8}+\frac{5040b^2x\sinh(c+dx)}{d^7} -\frac{2520b^2x^2\cosh(c+dx)}{d^6}+\frac{840b^2x^3\sinh(c+dx)}{d^5} -\frac{210b^2x^4\cosh(c+dx)}{d^4}+\frac{42b^2x^5\sinh(c+dx)}{d^3} -\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{b^2x^7\sinh(c+dx)}{d}$$

```
[In] Int[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c
+ d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x])/
d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 + (48
*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[c +
d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])/d^5
+ (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2*x^7*S
inh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 x \cosh(c+dx) + 2abx^4 \cosh(c+dx) + b^2 x^7 \cosh(c+dx)\right) dx$$

$$= a^2 \int x \cosh(c+dx)\, dx + (2ab) \int x^4 \cosh(c+dx)\, dx + b^2 \int x^7 \cosh(c+dx)\, dx$$

$$= \frac{a^2 x \sinh(c+dx)}{d} + \frac{2abx^4 \sinh(c+dx)}{d} + \frac{b^2 x^7 \sinh(c+dx)}{d} - \frac{a^2 \int \sinh(c+dx)\, dx}{d} - \frac{(8ab) \int x^3 \sinh(c+dx)\, dx}{d} - \frac{(7b^2) \int x^6 \sinh(c+dx)\, dx}{d}$$

$$= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{8abx^3 \cosh(c+dx)}{d^2} - \frac{7b^2 x^6 \cosh(c+dx)}{d^2} + \frac{a^2 x \sinh(c+dx)}{d} + \frac{2abx^4 \sinh(c+dx)}{d} + \frac{b^2 x^7 \sinh(c+dx)}{d} + \frac{(24ab) \int x^2 \cosh(c+dx)\, dx}{d^2} + \frac{(42b^2) \int x^5 \cosh(c+dx)\, dx}{d^2}$$

$$= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{8abx^3 \cosh(c+dx)}{d^2} - \frac{7b^2 x^6 \cosh(c+dx)}{d^2} + \frac{a^2 x \sinh(c+dx)}{d} + \frac{24abx^2 \sinh(c+dx)}{d^3} + \frac{2abx^4 \sinh(c+dx)}{d} + \frac{42b^2 x^5 \sinh(c+dx)}{d^3} + \frac{b^2 x^7 \sinh(c+dx)}{d} - \frac{(48ab) \int x \sinh(c+dx)\, dx}{d^3} - \frac{(210b^2) \int x^4 \sinh(c+dx)\, dx}{d^3}$$

$$= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{48abx \cosh(c+dx)}{d^4} - \frac{8abx^3 \cosh(c+dx)}{d^2} - \frac{210b^2 x^4 \cosh(c+dx)}{d^4} - \frac{7b^2 x^6 \cosh(c+dx)}{d^2} + \frac{a^2 x \sinh(c+dx)}{d} + \frac{24abx^2 \sinh(c+dx)}{d^3} + \frac{2abx^4 \sinh(c+dx)}{d} + \frac{42b^2 x^5 \sinh(c+dx)}{d^3} + \frac{b^2 x^7 \sinh(c+dx)}{d} + \frac{(48ab) \int \cosh(c+dx)\, dx}{d^4} + \frac{(840b^2) \int x^3 \cosh(c+dx)\, dx}{d^4}$$

$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{48abx\cosh(c+dx)}{d^4} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{210b^2x^4\cosh(c+dx)}{d^4} - \frac{7b^2x^6\cosh(c+dx)}{d^2} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{a^2x\sinh(c+dx)}{d} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{840b^2x^3\sinh(c+dx)}{d^5} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{42b^2x^5\sinh(c+dx)}{d^3} + \frac{b^2x^7\sinh(c+dx)}{d} - \frac{(2520b^2)\int x^2\sinh(c+dx)\,dx}{d^5}$$

$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{48abx\cosh(c+dx)}{d^4} - \frac{2520b^2x^2\cosh(c+dx)}{d^6} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{210b^2x^4\cosh(c+dx)}{d^4} - \frac{7b^2x^6\cosh(c+dx)}{d^2} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{a^2x\sinh(c+dx)}{d} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{840b^2x^3\sinh(c+dx)}{d^5} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{42b^2x^5\sinh(c+dx)}{d^3} + \frac{b^2x^7\sinh(c+dx)}{d} + \frac{(5040b^2)\int x\cosh(c+dx)\,dx}{d^6}$$

$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{48abx\cosh(c+dx)}{d^4} - \frac{2520b^2x^2\cosh(c+dx)}{d^6} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{210b^2x^4\cosh(c+dx)}{d^4} - \frac{7b^2x^6\cosh(c+dx)}{d^2} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{5040b^2x\sinh(c+dx)}{d^7} + \frac{a^2x\sinh(c+dx)}{d} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{840b^2x^3\sinh(c+dx)}{d^5} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{42b^2x^5\sinh(c+dx)}{d^3} + \frac{b^2x^7\sinh(c+dx)}{d} - \frac{(5040b^2)\int \sinh(c+dx)\,dx}{d^7}$$

$$= -\frac{5040b^2\cosh(c+dx)}{d^8} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{48abx\cosh(c+dx)}{d^4} - \frac{2520b^2x^2\cosh(c+dx)}{d^6} - \frac{8abx^3\cosh(c+dx)}{d^2} - \frac{210b^2x^4\cosh(c+dx)}{d^4} - \frac{7b^2x^6\cosh(c+dx)}{d^2} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{5040b^2x\sinh(c+dx)}{d^7} + \frac{a^2x\sinh(c+dx)}{d} + \frac{24abx^2\sinh(c+dx)}{d^3} + \frac{840b^2x^3\sinh(c+dx)}{d^5} + \frac{2abx^4\sinh(c+dx)}{d} + \frac{42b^2x^5\sinh(c+dx)}{d^3} + \frac{b^2x^7\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$=\frac{-((a^2d^6+8abd^4x(6+d^2x^2)+7b^2(720+360d^2x^2+30d^4x^4+d^6x^6))\cosh(c+dx))+d(a^2d^6x+2abd^2(24\ldots}{d^8}$$

```
[In] Integrate[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^6 + 8*a*b*d^4*x*(6 + d^2*x^2) + 7*b^2*(720 + 360*d^2*x^2 + 30*d^4
*x^4 + d^6*x^6))*Cosh[c + d*x]) + d*(a^2*d^6*x + 2*a*b*d^2*(24 + 12*d^2*x^2
 + d^4*x^4) + b^2*x*(5040 + 840*d^2*x^2 + 42*d^4*x^4 + d^6*x^6))*Sinh[c + d
*x])/d^8
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{8d^2\left(x^2\left(\frac{7bx^3}{8}+a\right)d^4+3\left(\frac{35bx^3}{4}+2a\right)d^2+315bx\right)xb\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2d\left(x\left(bx^3+a\right)^2d^6+6\left(7b^2x^5+4abx^2\right)d^4+24\left(35b^2x\ldots\right.\right.}{d^8\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\ldots\right.}$
risch	$\frac{\left(b^2x^7d^7-7b^2x^6d^6+2abd^7x^4+42b^2x^5d^5-8abd^6x^3+a^2d^7x-210b^2x^4d^4+24abd^5x^2-a^2d^6+840b^2d^3x^3-48abd^4x-2520\ldots\right.}{2d^8}$
meijerg	$\frac{128b^2\cosh(c)\sqrt{\pi}\left(\frac{315}{8\sqrt{\pi}}-\frac{\left(\frac{7}{16}x^6d^6+\frac{105}{8}d^4x^4+\frac{315}{2}x^2d^2+315\right)\cosh(dx)}{8\sqrt{\pi}}+\frac{xd\left(\frac{1}{16}x^6d^6+\frac{21}{8}d^4x^4+\frac{105}{2}x^2d^2+315\right)\sinh(dx)}{8\sqrt{\pi}}\right)}{d^8}-\ldots$
parts	$\frac{b^2x^7\sinh(dx+c)}{d}+\frac{2abx^4\sinh(dx+c)}{d}+\frac{a^2x\sinh(dx+c)}{d}-\frac{\frac{7b^2c^6\cosh(dx+c)}{d^6}+\frac{7b^2\left((dx+c)^6\cosh(dx+c)-6(dx+c)^5\sinh\ldots\right.}{\ldots}}{\ldots}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x*(b*x^3+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (8*d^2*(x^2*(7/8*b*x^3+a)*d^4+3*(35/4*b*x^3+2*a)*d^2+315*b*x)*x*b*tanh(1/2*
d*x+1/2*c)^2-2*d*(x*(b*x^3+a)^2*d^6+6*(7*b^2*x^5+4*a*b*x^2)*d^4+24*(35*b^2*
x^3+2*a*b)*d^2+5040*b^2*x)*tanh(1/2*d*x+1/2*c)+(7*b^2*x^6+8*a*b*x^3+2*a^2)*
d^6+6*(35*b^2*x^4+8*a*b*x)*d^4+2520*x^2*d^2*b^2+10080*b^2)/d^8/(tanh(1/2*d*
x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx = -\frac{(7\,b^2d^6x^6+8\,abd^6x^3+210\,b^2d^4x^4+a^2d^6+48\,abd^4x+2520\,b^2d^2x^2+5040\,b^2)\cosh{(dx+c)}-(b^2d^7x^7+}{d^8}$$

[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")

[Out] -((7*b^2*d^6*x^6 + 8*a*b*d^6*x^3 + 210*b^2*d^4*x^4 + a^2*d^6 + 48*a*b*d^4*x + 2520*b^2*d^2*x^2 + 5040*b^2)*cosh(d*x + c) - (b^2*d^7*x^7 + 2*a*b*d^7*x^4 + 42*b^2*d^5*x^5 + 24*a*b*d^5*x^2 + 840*b^2*d^3*x^3 + 48*a*b*d^3 + (a^2*d^7 + 5040*b^2*d)*x)*sinh(d*x + c))/d^8

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.21

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2x\sinh{(c+dx)}}{d}-\frac{a^2\cosh{(c+dx)}}{d^2}+\frac{2abx^4\sinh{(c+dx)}}{d}-\frac{8abx^3\cosh{(c+dx)}}{d^2}+\frac{24abx^2\sinh{(c+dx)}}{d^3}-\frac{48abx\cosh{(c+dx)}}{d^4}+\frac{48ab\sinh{(}}{d^5} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^5}{5}+\frac{b^2x^8}{8}\right)\cosh{(c)} \end{cases}$$

[In] integrate(x*(b*x**3+a)**2*cosh(d*x+c),x)

[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**4*sinh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)/d**2 + 24*a*b*x**2*sinh(c + d*x)/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a*b*sinh(c + d*x)/d**5 + b**2*x**7*sinh(c + d*x)/d - 7*b**2*x**6*cosh(c + d*x)/d**2 + 42*b**2*x**5*sinh(c + d*x)/d**3 - 210*b**2*x**4*cosh(c + d*x)/d**4 + 840*b**2*x**3*sinh(c + d*x)/d**5 - 2520*b**2*x**2*cosh(c + d*x)/d**6 + 5040*b**2*x*sinh(c + d*x)/d**7 - 5040*b**2*cosh(c + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8)*cosh(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.64

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx =$$
$$-\frac{1}{80}d\left(\frac{20\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2e^{(dx)}}{d^3}+\frac{20\,(d^2x^2+2\,dx+2)a^2e^{(-dx-c)}}{d^3}+\frac{16\,(d^5x^5e^c-5\,d^4x^4e^c+20\right.$$
$$+\frac{1}{40}\left(5\,b^2x^8+16\,abx^5+20\,a^2x^2\right)\cosh\left(dx+c\right)$$

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*d*(20*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 20*(d^2*x^2
 + 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 16*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d
^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^(d*x)/d^6 + 16*(
d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^(-d*x
- c)/d^6 + 5*(d^8*x^8*e^c - 8*d^7*x^7*e^c + 56*d^6*x^6*e^c - 336*d^5*x^5*e^
c + 1680*d^4*x^4*e^c - 6720*d^3*x^3*e^c + 20160*d^2*x^2*e^c - 40320*d*x*e^c
 + 40320*e^c)*b^2*e^(d*x)/d^9 + 5*(d^8*x^8 + 8*d^7*x^7 + 56*d^6*x^6 + 336*d
^5*x^5 + 1680*d^4*x^4 + 6720*d^3*x^3 + 20160*d^2*x^2 + 40320*d*x + 40320)*b
^2*e^(-d*x - c)/d^9) + 1/40*(5*b^2*x^8 + 16*a*b*x^5 + 20*a^2*x^2)*cosh(d*x
+ c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.29

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx$$
$$=\frac{(b^2d^7x^7-7\,b^2d^6x^6+2\,abd^7x^4+42\,b^2d^5x^5-8\,abd^6x^3+a^2d^7x-210\,b^2d^4x^4+24\,abd^5x^2-a^2d^6+840\,b^2d^3}{2\,d^8}$$
$$-\frac{(b^2d^7x^7+7\,b^2d^6x^6+2\,abd^7x^4+42\,b^2d^5x^5+8\,abd^6x^3+a^2d^7x+210\,b^2d^4x^4+24\,abd^5x^2+a^2d^6+840\,b^2}{2\,d^8}$$

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^7*x^7 - 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 + 42*b^2*d^5*x^5 - 8*a*b*d
^6*x^3 + a^2*d^7*x - 210*b^2*d^4*x^4 + 24*a*b*d^5*x^2 - a^2*d^6 + 840*b^2*d
^3*x^3 - 48*a*b*d^4*x - 2520*b^2*d^2*x^2 + 48*a*b*d^3 + 5040*b^2*d*x - 5040
*b^2)*e^(d*x + c)/d^8 - 1/2*(b^2*d^7*x^7 + 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 +
42*b^2*d^5*x^5 + 8*a*b*d^6*x^3 + a^2*d^7*x + 210*b^2*d^4*x^4 + 24*a*b*d^5*x
^2 + a^2*d^6 + 840*b^2*d^3*x^3 + 48*a*b*d^4*x + 2520*b^2*d^2*x^2 + 48*a*b*d
^3 + 5040*b^2*d*x + 5040*b^2)*e^(-d*x - c)/d^8
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.29

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx = -\mathrm{e}^{c+d\,x}\left(\frac{a^2\,d^6-48\,a\,b\,d^3+5040\,b^2}{2\,d^8} - \frac{x\,\left(a^2\,d^7-48\,a\,b\,d^4+5040\,b^2\,d\right)}{2\,d^8} - \frac{b^2\,x^7}{2\,d} + \frac{7\,b^2\,x^6}{2\,d^2} - \frac{21\,b^2\,x^5}{d^3} + \frac{b\,x^4\,\left(105\,b-a\,d^3\right)}{d^4} - \frac{4\,b\,x^3\,\left(105\,b-a\,d^3\right)}{d^5} + \frac{12\,b\,x^2\,\left(105\,b-a\,d^3\right)}{d^6}\right) - \mathrm{e}^{-c-d\,x}\left(\frac{a^2\,d^6+48\,a\,b\,d^3+5040\,b^2}{2\,d^8} + \frac{x\,\left(a^2\,d^7+48\,a\,b\,d^4+5040\,b^2\,d\right)}{2\,d^8} + \frac{b^2\,x^7}{2\,d} + \frac{7\,b^2\,x^6}{2\,d^2} + \frac{21\,b^2\,x^5}{d^3} + \frac{b\,x^4\,\left(a\,d^3+105\,b\right)}{d^4} + \frac{4\,b\,x^3\,\left(a\,d^3+105\,b\right)}{d^5} + \frac{12\,b\,x^2\,\left(a\,d^3+105\,b\right)}{d^6}\right)$$

```
[In] int(x*cosh(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] - exp(c + d*x)*((5040*b^2 + a^2*d^6 - 48*a*b*d^3)/(2*d^8) - (x*(5040*b^2*d
+ a^2*d^7 - 48*a*b*d^4))/(2*d^8) - (b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) -
(21*b^2*x^5)/d^3 + (b*x^4*(105*b - a*d^3))/d^4 - (4*b*x^3*(105*b - a*d^3))/
d^5 + (12*b*x^2*(105*b - a*d^3))/d^6) - exp(- c - d*x)*((5040*b^2 + a^2*d^6
 + 48*a*b*d^3)/(2*d^8) + (x*(5040*b^2*d + a^2*d^7 + 48*a*b*d^4))/(2*d^8) +
(b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) + (21*b^2*x^5)/d^3 + (b*x^4*(105*b +
a*d^3))/d^4 + (4*b*x^3*(105*b + a*d^3))/d^5 + (12*b*x^2*(105*b + a*d^3))/d^
6)
```

3.88 $\int (a+bx^3)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 186

$$\begin{aligned}\int (a+bx^3)^2 \cosh(c+dx)\,dx = & -\frac{12ab\cosh(c+dx)}{d^4}-\frac{720b^2x\cosh(c+dx)}{d^6} \\ & -\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4} \\ & -\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{720b^2\sinh(c+dx)}{d^7} \\ & +\frac{a^2\sinh(c+dx)}{d}+\frac{12abx\sinh(c+dx)}{d^3} \\ & +\frac{360b^2x^2\sinh(c+dx)}{d^5}+\frac{2abx^3\sinh(c+dx)}{d} \\ & +\frac{30b^2x^4\sinh(c+dx)}{d^3}+\frac{b^2x^6\sinh(c+dx)}{d}\end{aligned}$$

```
[Out] -12*a*b*cosh(d*x+c)/d^4-720*b^2*x*cosh(d*x+c)/d^6-6*a*b*x^2*cosh(d*x+c)/d^2
-120*b^2*x^3*cosh(d*x+c)/d^4-6*b^2*x^5*cosh(d*x+c)/d^2+720*b^2*sinh(d*x+c)/
d^7+a^2*sinh(d*x+c)/d+12*a*b*x*sinh(d*x+c)/d^3+360*b^2*x^2*sinh(d*x+c)/d^5+
2*a*b*x^3*sinh(d*x+c)/d+30*b^2*x^4*sinh(d*x+c)/d^3+b^2*x^6*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5385, 2717, 3377, 2718}

$$\begin{aligned}\int \left(a+bx^3\right)^2 \cosh(c+dx)\,dx = {} & \frac{a^2\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4} \\ & +\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2} \\ & +\frac{2abx^3\sinh(c+dx)}{d}+\frac{720b^2\sinh(c+dx)}{d^7} \\ & -\frac{720b^2x\cosh(c+dx)}{d^6}+\frac{360b^2x^2\sinh(c+dx)}{d^5} \\ & -\frac{120b^2x^3\cosh(c+dx)}{d^4}+\frac{30b^2x^4\sinh(c+dx)}{d^3} \\ & -\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{b^2x^6\sinh(c+dx)}{d}\end{aligned}$$

```
[In] Int[(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c + d*
x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*x*S
inh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c + d
*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)\,dx$$
$$= a^2\int \cosh(c+dx)\,dx+(2ab)\int x^3\cosh(c+dx)\,dx+b^2\int x^6\cosh(c+dx)\,dx$$
$$= \frac{a^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d} - \frac{(6ab)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(6b^2)\int x^5\sinh(c+dx)\,dx}{d}$$
$$= -\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d} + \frac{b^2x^6\sinh(c+dx)}{d}+\frac{(12ab)\int x\cosh(c+dx)\,dx}{d^2}+\frac{(30b^2)\int x^4\cosh(c+dx)\,dx}{d^2}$$
$$= -\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d} + \frac{12abx\sinh(c+dx)}{d^3}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{b^2x^6\sinh(c+dx)}{d}-\frac{(12ab)\int \sinh(c+dx)\,dx}{d^3}-\frac{(120b^2)\int x^3\sinh(c+dx)\,dx}{d^3}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d}+\frac{12abx\sinh(c+dx)}{d^3}+\frac{2abx^3\sinh(c+dx)}{d} + \frac{30b^2x^4\sinh(c+dx)}{d^3}+\frac{b^2x^6\sinh(c+dx)}{d}+\frac{(360b^2)\int x^2\cosh(c+dx)\,dx}{d^4}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d}+\frac{12abx\sinh(c+dx)}{d^3} + \frac{360b^2x^2\sinh(c+dx)}{d^5}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{b^2x^6\sinh(c+dx)}{d}-\frac{(720b^2)\int x\sinh(c+dx)\,dx}{d^5}$$

$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{720b^2x\cosh(c+dx)}{d^6} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{a^2\sinh(c+dx)}{d} + \frac{12abx\sinh(c+dx)}{d^3} + \frac{360b^2x^2\sinh(c+dx)}{d^5} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{b^2x^6\sinh(c+dx)}{d} + \frac{(720b^2)\int\cosh(c+dx)\,dx}{d^6}$$

$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{720b^2x\cosh(c+dx)}{d^6} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{720b^2\sinh(c+dx)}{d^7} + \frac{a^2\sinh(c+dx)}{d} + \frac{12abx\sinh(c+dx)}{d^3} + \frac{360b^2x^2\sinh(c+dx)}{d^5} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{30b^2x^4\sinh(c+dx)}{d^3} + \frac{b^2x^6\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.60

$$\int (a+bx^3)^2\cosh(c+dx)\,dx = \frac{-6bd(ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(a^2d^6+2abd^4x(6+d^2x^2)+b^2(720+360}{d^7}$$

```
[In] Integrate[(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-6*b*d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a^2*d^6 + 2*a*b*d^4*x*(6 + d^2*x^2) + b^2*(720 + 360*d^2*x^2 + 30*d^
4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{6d(x(b\,x^3+a)\,d^4+20b\,d^2x^2+120b)\,xb\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(-(b\,x^3+a)^2d^6+6\left(-5b^2x^4-2abx\right)d^4-360x^2d^2b^2-720b^2\right)\tanh}{d^7\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b^2x^6d^6-6b^2x^5d^5+2ab\,d^6x^3+30b^2x^4d^4-6ab\,d^5x^2+a^2d^6-120b^2d^3x^3+12ab\,d^4x+360x^2d^2b^2-12ab\,d^3-720b^2dx+720b^2}{2d^7}$
meijerg	$\frac{64ib^2\cosh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{21}{8}d^4x^4+\frac{105}{2}x^2d^2+315\right)\cosh(dx)}{28\sqrt{\pi}}-\frac{i\left(\frac{7}{16}x^6d^6+\frac{105}{8}d^4x^4+\frac{315}{2}x^2d^2+315\right)\sinh(dx)}{28\sqrt{\pi}}\right)}{d^7}+\frac{64b^2\sinh(c)\sqrt{}}{}$
parts	$\frac{b^2x^6\sinh(dx+c)}{d}+\frac{2ab\,x^3\sinh(dx+c)}{d}+\frac{a^2\sinh(dx+c)}{d}-\frac{6b\left(-\frac{b\,c^5\cosh(dx+c)}{d^3}+\frac{5b\,c^4((dx+c)\cosh(dx+c)-\sinh(dx+c)}{d^3}\right.}{}$
derivativedivides	$\frac{b^2c^6\sinh(dx+c)}{d^6}+\frac{b^2\left((dx+c)^6\sinh(dx+c)-6(dx+c)^5\cosh(dx+c)+30(dx+c)^4\sinh(dx+c)-120(dx+c)^3\cosh(dx+c)+360(dx+c)^2\sinh\right.}{d^6}$
default	$\frac{b^2c^6\sinh(dx+c)}{d^6}+\frac{b^2\left((dx+c)^6\sinh(dx+c)-6(dx+c)^5\cosh(dx+c)+30(dx+c)^4\sinh(dx+c)-120(dx+c)^3\cosh(dx+c)+360(dx+c)^2\sinh\right.}{d^6}$

```
[In] int((b*x^3+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3*d*(x*(b*x^3+a)*d^4+20*b*d^2*x^2+120*b)*x*b*tanh(1/2*d*x+1/2*c)^2+(-(b*
x^3+a)^2*d^6+6*(-5*b^2*x^4-2*a*b*x)*d^4-360*x^2*d^2*b^2-720*b^2)*tanh(1/2*d
*x+1/2*c)+3*d*(x^2*(b*x^3+a)*d^4+4*(5*b*x^3+a)*d^2+120*b*x)*b)/d^7/(tanh(1/
2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int (a+bx^3)^2\cosh(c+dx)\,dx = -\frac{6\,(b^2d^5x^5+abd^5x^2+20\,b^2d^3x^3+2\,abd^3+120\,b^2dx)\cosh\left(dx+c\right)-(b^2d^6x^6+2\,abd^6x^3+30\,b^2d^4x^4+a}{d^7}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d^5*x^5 + a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 2*a*b*d^3 + 120*b^2*d*x)*
cosh(d*x + c) - (b^2*d^6*x^6 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + a^2*d^6 + 1
2*a*b*d^4*x + 360*b^2*d^2*x^2 + 720*b^2)*sinh(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int \left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$=\begin{cases} \frac{a^2\sinh{(c+dx)}}{d}+\frac{2abx^3\sinh{(c+dx)}}{d}-\frac{6abx^2\cosh{(c+dx)}}{d^2}+\frac{12abx\sinh{(c+dx)}}{d^3}-\frac{12ab\cosh{(c+dx)}}{d^4}+\frac{b^2x^6\sinh{(c+dx)}}{d}-\frac{6b^2x^5\cosh{(c}}{d^2} \\ \left(a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}\right)\cosh{(c)} \end{cases}$$

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*c
osh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4
 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*x)/d**2 + 30*b**2*x**
4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**4 + 360*b**2*x**2*sin
h(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*b**2*sinh(c + d*x)/d*
*7, Ne(d, 0)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*cosh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.31

$$\int \left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$=\frac{a^2e^{(dx+c)}}{2\,d}-\frac{a^2e^{(-dx-c)}}{2\,d}+\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)abe^{(dx)}}{d^4}$$
$$-\frac{(d^3x^3+3\,d^2x^2+6\,dx+6)abe^{(-dx-c)}}{d^4}$$
$$+\frac{(d^6x^6e^c-6\,d^5x^5e^c+30\,d^4x^4e^c-120\,d^3x^3e^c+360\,d^2x^2e^c-720\,dxe^c+720\,e^c)b^2e^{(dx)}}{2\,d^7}$$
$$-\frac{(d^6x^6+6\,d^5x^5+30\,d^4x^4+120\,d^3x^3+360\,d^2x^2+720\,dx+720)b^2e^{(-dx-c)}}{2\,d^7}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^3*x^3*e^c - 3*d^2*x^2*e
^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4 - (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)
*a*b*e^(-d*x - c)/d^4 + 1/2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c -
 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7
 - 1/2*(d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*
d*x + 720)*b^2*e^(-d*x - c)/d^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.31

$$\int \left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$=\frac{(b^2d^6x^6-6\,b^2d^5x^5+2\,abd^6x^3+30\,b^2d^4x^4-6\,abd^5x^2+a^2d^6-120\,b^2d^3x^3+12\,abd^4x+360\,b^2d^2x^2-12\,a}{2\,d^7}$$
$$-\frac{(b^2d^6x^6+6\,b^2d^5x^5+2\,abd^6x^3+30\,b^2d^4x^4+6\,abd^5x^2+a^2d^6+120\,b^2d^3x^3+12\,abd^4x+360\,b^2d^2x^2+12}{2\,d^7}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^6*x^6 - 6*b^2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 - 6*a*b*d
^5*x^2 + a^2*d^6 - 120*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 - 12*a*
b*d^3 - 720*b^2*d*x + 720*b^2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 6*b^2*d
^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + 6*a*b*d^5*x^2 + a^2*d^6 + 120*b^2
*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 12*a*b*d^3 + 720*b^2*d*x + 720*
b^2)*e^(-d*x - c)/d^7
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \left(a+bx^3\right)^2\cosh(c+dx)\,dx = {} & \frac{\sinh(c+d\,x)\,\left(a^2\,d^6+720\,b^2\right)}{d^7} \\ & -\frac{6\,b^2\,x^5\cosh(c+d\,x)}{d^2}-\frac{120\,b^2\,x^3\cosh(c+d\,x)}{d^4} \\ & +\frac{b^2\,x^6\sinh(c+d\,x)}{d}+\frac{30\,b^2\,x^4\sinh(c+d\,x)}{d^3} \\ & +\frac{360\,b^2\,x^2\sinh(c+d\,x)}{d^5}-\frac{12\,a\,b\cosh(c+d\,x)}{d^4} \\ & -\frac{720\,b^2\,x\cosh(c+d\,x)}{d^6}-\frac{6\,a\,b\,x^2\cosh(c+d\,x)}{d^2} \\ & +\frac{2\,a\,b\,x^3\sinh(c+d\,x)}{d}+\frac{12\,a\,b\,x\sinh(c+d\,x)}{d^3}\end{aligned}$$

```
[In] int(cosh(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] (sinh(c + d*x)*(720*b^2 + a^2*d^6))/d^7 - (6*b^2*x^5*cosh(c + d*x))/d^2 - (
120*b^2*x^3*cosh(c + d*x))/d^4 + (b^2*x^6*sinh(c + d*x))/d + (30*b^2*x^4*si
nh(c + d*x))/d^3 + (360*b^2*x^2*sinh(c + d*x))/d^5 - (12*a*b*cosh(c + d*x))
/d^4 - (720*b^2*x*cosh(c + d*x))/d^6 - (6*a*b*x^2*cosh(c + d*x))/d^2 + (2*a
*b*x^3*sinh(c + d*x))/d + (12*a*b*x*sinh(c + d*x))/d^3
```

3.89 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} dx = -\frac{120b^2 \cosh(c+dx)}{d^6} - \frac{4abx \cosh(c+dx)}{d^2} - \frac{60b^2x^2 \cosh(c+dx)}{d^4} - \frac{5b^2x^4 \cosh(c+dx)}{d^2} + a^2 \cosh(c)\text{Chi}(dx) + \frac{4ab \sinh(c+dx)}{d^3} + \frac{120b^2x \sinh(c+dx)}{d^5} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{20b^2x^3 \sinh(c+dx)}{d^3} + \frac{b^2x^5 \sinh(c+dx)}{d} + a^2 \sinh(c)\text{Shi}(dx)$$

```
[Out] a^2*Chi(d*x)*cosh(c)-120*b^2*cosh(d*x+c)/d^6-4*a*b*x*cosh(d*x+c)/d^2-60*b^2
*x^2*cosh(d*x+c)/d^4-5*b^2*x^4*cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+4*a*b*s
inh(d*x+c)/d^3+120*b^2*x*sinh(d*x+c)/d^5+2*a*b*x^2*sinh(d*x+c)/d+20*b^2*x^3
*sinh(d*x+c)/d^3+b^2*x^5*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5395, 3384, 3379, 3382, 3377, 2717, 2718}

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = a^2\cosh(c)\text{Chi}(dx)+a^2\sinh(c)\text{Shi}(dx)+\frac{4ab\sinh(c+dx)}{d^3} - \frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d} - \frac{120b^2\cosh(c+dx)}{d^6}+\frac{120b^2x\sinh(c+dx)}{d^5} - \frac{60b^2x^2\cosh(c+dx)}{d^4}+\frac{20b^2x^3\sinh(c+dx)}{d^3} - \frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{b^2x^5\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (4*a*b*x*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Co
sh[c + d*x])/d^4 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral
[d*x] + (4*a*b*Sinh[c + d*x])/d^3 + (120*b^2*x*Sinh[c + d*x])/d^5 + (2*a*b*
x^2*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (b^2*x^5*Sinh[c + d
*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x} + 2abx^2\cosh(c+dx) + b^2x^5\cosh(c+dx)\right) dx \\
&= a^2\int \frac{\cosh(c+dx)}{x}\,dx + (2ab)\int x^2\cosh(c+dx)\,dx + b^2\int x^5\cosh(c+dx)\,dx \\
&= \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^5\sinh(c+dx)}{d} \\
&\quad - \frac{(4ab)\int x\sinh(c+dx)\,dx}{d} - \frac{(5b^2)\int x^4\sinh(c+dx)\,dx}{d} \\
&\quad + \left(a^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx + \left(a^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx \\
&= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cosh(c)\text{Chi}(dx) \\
&\quad + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^5\sinh(c+dx)}{d} + a^2\sinh(c)\text{Shi}(dx) \\
&\quad + \frac{(4ab)\int \cosh(c+dx)\,dx}{d^2} + \frac{(20b^2)\int x^3\cosh(c+dx)\,dx}{d^2} \\
&= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cosh(c)\text{Chi}(dx) \\
&\quad + \frac{4ab\sinh(c+dx)}{d^3} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{20b^2x^3\sinh(c+dx)}{d^3} \\
&\quad + \frac{b^2x^5\sinh(c+dx)}{d} + a^2\sinh(c)\text{Shi}(dx) - \frac{(60b^2)\int x^2\sinh(c+dx)\,dx}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{5b^2x^4\cosh(c+dx)}{d^2} \\
&\quad + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{4ab\sinh(c+dx)}{d^3} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{20b^2x^3\sinh(c+dx)}{d^3} \\
&\quad + \frac{b^2x^5\sinh(c+dx)}{d} + a^2\sinh(c)\mathrm{Shi}(dx) + \frac{(120b^2)\int x\cosh(c+dx)\,dx}{d^4} \\
&= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{5b^2x^4\cosh(c+dx)}{d^2} \\
&\quad + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{4ab\sinh(c+dx)}{d^3} + \frac{120b^2x\sinh(c+dx)}{d^5} \\
&\quad + \frac{2abx^2\sinh(c+dx)}{d} + \frac{20b^2x^3\sinh(c+dx)}{d^3} + \frac{b^2x^5\sinh(c+dx)}{d} \\
&\quad + a^2\sinh(c)\mathrm{Shi}(dx) - \frac{(120b^2)\int \sinh(c+dx)\,dx}{d^5} \\
&= -\frac{120b^2\cosh(c+dx)}{d^6} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} \\
&\quad - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{4ab\sinh(c+dx)}{d^3} + \frac{120b^2x\sinh(c+dx)}{d^5} \\
&\quad + \frac{2abx^2\sinh(c+dx)}{d} + \frac{20b^2x^3\sinh(c+dx)}{d^3} + \frac{b^2x^5\sinh(c+dx)}{d} + a^2\sinh(c)\mathrm{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.68

$$
\begin{aligned}
\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx &= -\frac{b(4ad^4x + 5b(24+12d^2x^2+d^4x^4))\cosh(c+dx)}{d^6} \\
&\quad + a^2\cosh(c)\mathrm{Chi}(dx) \\
&\quad + \frac{b(2ad^2(2+d^2x^2) + bx(120+20d^2x^2+d^4x^4))\sinh(c+dx)}{d^5} \\
&\quad + a^2\sinh(c)\mathrm{Shi}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*(4*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])/d^6) + a^
2*Cosh[c]*CoshIntegral[d*x] + (b*(2*a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2
*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.89

method	result
meijerg	$-\frac{32b^2\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib^2\sinh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{7}{8}d^4x^4+\frac{35}{2}\right.}{}\right.}{}$
risch	$-\frac{e^{-dx-c}b^2x^5}{2d}+\frac{e^{dx+c}b^2x^5}{2d}-\frac{5\,e^{-dx-c}b^2x^4}{2d^2}-\frac{5\,e^{dx+c}b^2x^4}{2d^2}-\frac{a^2e^c\,\mathrm{Ei}_1(-dx)}{2}-\frac{e^{-dx-c}ab\,x^2}{d}-\frac{a^2e^{-c}\,\mathrm{Ei}_1(dx)}{2}+\frac{e^{dx+c}ab}{d}$

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -32/d^6*b^2*cosh(c)*Pi^(1/2)*(-15/4/Pi^(1/2)+1/12/Pi^(1/2)*(15/8*d^4*x^4+45
/2*x^2*d^2+45)*cosh(d*x)-1/12/Pi^(1/2)*x*d*(3/8*d^4*x^4+15/2*x^2*d^2+45)*si
nh(d*x))+32*I/d^6*b^2*sinh(c)*Pi^(1/2)*(-1/28*I/Pi^(1/2)*x*d*(7/8*d^4*x^4+3
5/2*x^2*d^2+105)*cosh(d*x)+1/28*I/Pi^(1/2)*(35/8*d^4*x^4+105/2*x^2*d^2+105)
*sinh(d*x))+8*I/d^3*a*b*cosh(c)*Pi^(1/2)*(1/2*I/Pi^(1/2)*x*d*cosh(d*x)-1/6*
I/Pi^(1/2)*(3/2*x^2*d^2+3)*sinh(d*x))+8/d^3*b*a*sinh(c)*Pi^(1/2)*(-1/2/Pi^(
1/2)+1/2/Pi^(1/2)*(1/2*x^2*d^2+1)*cosh(d*x)-1/2/Pi^(1/2)*d*x*sinh(d*x))+1/2
*a^2*cosh(c)*Pi^(1/2)*((2*gamma+2*ln(x)+2*ln(I*d))/Pi^(1/2)+2/Pi^(1/2)*(Chi
(d*x)-ln(d*x)-gamma))+a^2*Shi(d*x)*sinh(c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = -\frac{2\,(5\,b^2d^4x^4+4\,abd^4x+60\,b^2d^2x^2+120\,b^2)\cosh(dx+c)-(a^2d^6\mathrm{Ei}(dx)+a^2d^6\mathrm{Ei}(-dx))\cosh(c)-2\,(b^2d}{2\,d^6}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x + 60*b^2*d^2*x^2 + 120*b^2)*cosh(d*x +
 c) - (a^2*d^6*Ei(d*x) + a^2*d^6*Ei(-d*x))*cosh(c) - 2*(b^2*d^5*x^5 + 2*a*b
*d^5*x^2 + 20*b^2*d^3*x^3 + 4*a*b*d^3 + 120*b^2*d*x)*sinh(d*x + c) - (a^2*d
^6*Ei(d*x) - a^2*d^6*Ei(-d*x))*sinh(c))/d^6
```

Sympy [A] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = a^2\sinh(c)\,\mathrm{Shi}(dx) + a^2\cosh(c)\,\mathrm{Chi}(dx) + 2ab\left(\begin{cases}\frac{x^2\sinh(c+dx)}{d} - \frac{2x\cosh(c+dx)}{d^2} + \frac{2\sinh(c+dx)}{d^3} & \text{for } d\neq 0\\ \frac{x^3\cosh(c)}{3} & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases}\frac{x^5\sinh(c+dx)}{d} - \frac{5x^4\cosh(c+dx)}{d^2} + \frac{20x^3\sinh(c+dx)}{d^3} - \frac{60x^2\cosh(c+dx)}{d^4} + \frac{120x\sinh(c+dx)}{d^5} - \frac{120\cosh(c+dx)}{d^6} & \text{for } d \\ \frac{x^6\cosh(c)}{6} & \text{other}\end{cases}\right.$$

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x**2*sinh(
c + d*x)/d - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**
3*cosh(c)/3, True)) + b**2*Piecewise((x**5*sinh(c + d*x)/d - 5*x**4*cosh(c
+ d*x)/d**2 + 20*x**3*sinh(c + d*x)/d**3 - 60*x**2*cosh(c + d*x)/d**4 + 120
*x*sinh(c + d*x)/d**5 - 120*cosh(c + d*x)/d**6, Ne(d, 0)), (x**6*cosh(c)/6,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = -\frac{1}{12}\left(4\,ab\left(\frac{(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)e^{(dx)}}{d^4} + \frac{(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)e^{(-dx-c)}}{d^4}\right) + b^2\left(\frac{(d^6x^6e\right.\right. + \frac{1}{6}\left(b^2x^6 + 4\,abx^3 + 2\,a^2\log\left(x^3\right)\right)\cosh(dx+c)$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4
 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + b^2*((d^6*x^6*e^c
- 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*
d*x*e^c + 720*e^c)*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^
3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*e^(-d*x - c)/d^7) + 4*a^2*cosh(d*x + c
)*log(x^3)/d - 6*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/6*(b^2*x^6 +
4*a*b*x^3 + 2*a^2*log(x^3))*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(160) = 320.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = \frac{b^2d^5x^5e^{(dx+c)} - b^2d^5x^5e^{(-dx-c)} - 5\,b^2d^4x^4e^{(dx+c)} - 5\,b^2d^4x^4e^{(-dx-c)} + 2\,abd^5x^2e^{(dx+c)} - 2\,abd^5x^2e^{(-dx-c)} + a}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^5*x^5*e^(d*x + c) - b^2*d^5*x^5*e^(-d*x - c) - 5*b^2*d^4*x^4*e^(
d*x + c) - 5*b^2*d^4*x^4*e^(-d*x - c) + 2*a*b*d^5*x^2*e^(d*x + c) - 2*a*b*d
^5*x^2*e^(-d*x - c) + a^2*d^6*Ei(-d*x)*e^(-c) + a^2*d^6*Ei(d*x)*e^c + 20*b^
2*d^3*x^3*e^(d*x + c) - 20*b^2*d^3*x^3*e^(-d*x - c) - 4*a*b*d^4*x*e^(d*x +
c) - 4*a*b*d^4*x*e^(-d*x - c) - 60*b^2*d^2*x^2*e^(d*x + c) - 60*b^2*d^2*x^2
*e^(-d*x - c) + 4*a*b*d^3*e^(d*x + c) - 4*a*b*d^3*e^(-d*x - c) + 120*b^2*d*
x*e^(d*x + c) - 120*b^2*d*x*e^(-d*x - c) - 120*b^2*e^(d*x + c) - 120*b^2*e^
(-d*x - c))/d^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x, x)
```

3.90 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} \, dx = -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)$$

```
[Out] -2*a*b*cosh(d*x+c)/d^2-a^2*cosh(d*x+c)/x-24*b^2*x*cosh(d*x+c)/d^4-4*b^2*x^3
*cosh(d*x+c)/d^2+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi(d*x)*sinh(c)+24*b^2*sinh(
d*x+c)/d^5+2*a*b*x*sinh(d*x+c)/d+12*b^2*x^2*sinh(d*x+c)/d^3+b^2*x^4*sinh(d*
x+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used

$= \{5395, 3378, 3384, 3379, 3382, 3377, 2718, 2717\}$

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx = a^2d\sinh(c)\text{Chi}(dx) + a^2d\cosh(c)\text{Shi}(dx) - \frac{a^2\cosh(c+dx)}{x} - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} + \frac{24b^2\sinh(c+dx)}{d^5} - \frac{24b^2x\cosh(c+dx)}{d^4} + \frac{12b^2x^2\sinh(c+dx)}{d^3} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{b^2x^4\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^2} + 2abx\cosh(c+dx) + b^2x^4\cosh(c+dx)\right) dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx + (2ab)\int x\cosh(c+dx)\,dx + b^2\int x^4\cosh(c+dx)\,dx \\
&= -\frac{a^2\cosh(c+dx)}{x} + \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d} \\
&\quad - \frac{(2ab)\int\sinh(c+dx)\,dx}{d} - \frac{(4b^2)\int x^3\sinh(c+dx)\,dx}{d} + \left(a^2d\right)\int\frac{\sinh(c+dx)}{x}\,dx \\
&= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&\quad + \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d} + \frac{(12b^2)\int x^2\cosh(c+dx)\,dx}{d^2} \\
&\quad + \left(a^2d\cosh(c)\right)\int\frac{\sinh(dx)}{x}\,dx + \left(a^2d\sinh(c)\right)\int\frac{\cosh(dx)}{x}\,dx \\
&= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&\quad + a^2d\text{Chi}(dx)\sinh(c) + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} \\
&\quad + \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx) - \frac{(24b^2)\int x\sinh(c+dx)\,dx}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&\quad + a^2d\text{Chi}(dx)\sinh(c) + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} \\
&\quad + \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx) + \frac{(24b^2)\int\cosh(c+dx)\,dx}{d^4} \\
&= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&\quad + a^2d\text{Chi}(dx)\sinh(c) + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{2abx\sinh(c+dx)}{d} \\
&\quad + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx = &-\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} \\
&- \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\
&+ a^2d\text{Chi}(dx)\sinh(c) + \frac{24b^2\sinh(c+dx)}{d^5} \\
&+ \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} \\
&+ \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.14

method	result
meijerg	$-\frac{16 i b^2 \cosh(c)\sqrt{\pi}\left(-\frac{i x d\left(\frac{5 x^2 d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8} d^4 x^4+\frac{15}{2} x^2 d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16 b^2 \sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8} d^4 x^4+\frac{9}{2} x^2 d^2+9\right) \text{co}}{6\sqrt{\pi}}\right.}{d^5}$
risch	$-\frac{e^{-dx-c} b^2 d^4 x^5 - e^{dx+c} b^2 d^4 x^5 - e^{-c} \operatorname{Ei}_1(dx) a^2 d^6 x + e^{c} \operatorname{Ei}_1(-dx) a^2 d^6 x + 4 e^{-dx-c} b^2 d^3 x^4 + 4 e^{dx+c} b^2 d^3 x^4 + 2 e^{-dx-c} a b d^4 x^2 - 2 e^{dx+}}{}$

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -16*I/d^5*b^2*cosh(c)*Pi^(1/2)*(-1/10*I/Pi^(1/2)*x*d*(5/2*x^2*d^2+15)*cosh(
d*x)+1/10*I/Pi^(1/2)*(5/8*d^4*x^4+15/2*x^2*d^2+15)*sinh(d*x))-16/d^5*b^2*si
nh(c)*Pi^(1/2)*(3/2/Pi^(1/2)-1/6/Pi^(1/2)*(3/8*d^4*x^4+9/2*x^2*d^2+9)*cosh(
d*x)+1/6/Pi^(1/2)*x*d*(3/2*x^2*d^2+9)*sinh(d*x))-4*b/d^2*a*cosh(c)*Pi^(1/2)
*(-1/2/Pi^(1/2)+1/2/Pi^(1/2)*cosh(d*x)-1/2/Pi^(1/2)*d*x*sinh(d*x))+2*b/d^2*
a*sinh(c)*(cosh(d*x)*x*d-sinh(d*x))+1/4*I*a^2*cosh(c)*Pi^(1/2)*d*(4*I/d/x*c
osh(d*x)/Pi^(1/2)-4*I/Pi^(1/2)*Shi(d*x))+1/4*a^2*sinh(c)*Pi^(1/2)*d*(2*(2*g
amma-2+2*ln(x)+2*ln(I*d))/Pi^(1/2)+4/Pi^(1/2)-4/Pi^(1/2)/x/d*sinh(d*x)+4/Pi
^(1/2)*(Chi(d*x)-ln(d*x)-gamma))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2}\,dx = -\frac{2\left(4 b^2 d^3 x^4 + a^2 d^5 + 2 a b d^3 x + 24 b^2 d x^2\right)\cosh(dx+c) - \left(a^2 d^6 x \operatorname{Ei}(dx) - a^2 d^6 x \operatorname{Ei}(-dx)\right)\cosh(c) - 2\,(b}{2\, d^5 x}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(4*b^2*d^3*x^4 + a^2*d^5 + 2*a*b*d^3*x + 24*b^2*d*x^2)*cosh(d*x + c
) - (a^2*d^6*x*Ei(d*x) - a^2*d^6*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^4*x^5 + 2*a
*b*d^4*x^2 + 12*b^2*d^2*x^3 + 24*b^2*x)*sinh(d*x + c) - (a^2*d^6*x*Ei(d*x)
+ a^2*d^6*x*Ei(-d*x))*sinh(c))/(d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx = \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$$

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**2,x)
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx = -\frac{1}{10}\left(5a^2\text{Ei}(-dx)e^{(-c)} - 5a^2\text{Ei}(dx)e^c + \frac{5(d^2x^2e^c - 2dxe^c + 2e^c)abe^{(dx)}}{d^3} + \frac{5(d^2x^2 + 2dx + 2)abe^{(-dx-c)}}{d^3}\right. + \frac{1}{5}\left(b^2x^5 + 5abx^2 - \frac{5a^2}{x}\right)\cosh(dx+c)$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
[Out] -1/10*(5*a^2*Ei(-d*x)*e^(-c) - 5*a^2*Ei(d*x)*e^c + 5*(d^2*x^2*e^c - 2*d*x*e
^c + 2*e^c)*a*b*e^(d*x)/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3
+ (d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*
e^c - 120*e^c)*b^2*e^(d*x)/d^6 + (d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2
*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6)*d + 1/5*(b^2*x^5 + 5*a*b*x^2 -
5*a^2/x)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(143) = 286$.

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.15

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx = \frac{b^2d^4x^5e^{(dx+c)} - b^2d^4x^5e^{(-dx-c)} - a^2d^6x\text{Ei}(-dx)e^{(-c)} + a^2d^6x\text{Ei}(dx)e^c - 4b^2d^3x^4e^{(dx+c)} - 4b^2d^3x^4e^{(-dx-c)}}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^5*e^(d*x + c) - b^2*d^4*x^5*e^(-d*x - c) - a^2*d^6*x*Ei(-d*x
)*e^(-c) + a^2*d^6*x*Ei(d*x)*e^c - 4*b^2*d^3*x^4*e^(d*x + c) - 4*b^2*d^3*x^
4*e^(-d*x - c) + 2*a*b*d^4*x^2*e^(d*x + c) - 2*a*b*d^4*x^2*e^(-d*x - c) - a
^2*d^5*e^(d*x + c) + 12*b^2*d^2*x^3*e^(d*x + c) - a^2*d^5*e^(-d*x - c) - 12
*b^2*d^2*x^3*e^(-d*x - c) - 2*a*b*d^3*x*e^(d*x + c) - 2*a*b*d^3*x*e^(-d*x -
 c) - 24*b^2*d*x^2*e^(d*x + c) - 24*b^2*d*x^2*e^(-d*x - c) + 24*b^2*x*e^(d*
x + c) - 24*b^2*x*e^(-d*x - c))/(d^5*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^2}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^2,x)
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^2, x)
```

3.91 $\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{2x}+\frac{6b^2x\sinh(c+dx)}{d^3}+\frac{b^2x^3\sinh(c+dx)}{d}+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)$$

```
[Out] 1/2*a^2*d^2*Chi(d*x)*cosh(c)-6*b^2*cosh(d*x+c)/d^4-1/2*a^2*cosh(d*x+c)/x^2-
3*b^2*x^2*cosh(d*x+c)/d^2+1/2*a^2*d^2*Shi(d*x)*sinh(c)+2*a*b*sinh(d*x+c)/d-
1/2*a^2*d*sinh(d*x+c)/x+6*b^2*x*sinh(d*x+c)/d^3+b^2*x^3*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5395, 2717, 3378, 3384, 3379, 3382, 3377, 2718}

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+\frac{2ab\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/(2*x^2) - (3*b^2*x^2*Cosh[
c + d*x])/d^2 + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (2*a*b*Sinh[c + d*x
])/d - (a^2*d*Sinh[c + d*x])/(2*x) + (6*b^2*x*Sinh[c + d*x])/d^3 + (b^2*x^3
*Sinh[c + d*x])/d + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^3}+b^2x^3\cosh(c+dx)\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int \cosh(c+dx)\,dx+b^2\int x^3\cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}\\
&\quad -\frac{(3b^2)\int x^2\sinh(c+dx)\,dx}{d}+\frac{1}{2}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^2}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{2x}\\
&\quad +\frac{b^2x^3\sinh(c+dx)}{d}+\frac{(6b^2)\int x\cosh(c+dx)\,dx}{d^2}+\frac{1}{2}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{2x}\\
&\quad +\frac{6b^2x\sinh(c+dx)}{d^3}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{(6b^2)\int \sinh(c+dx)\,dx}{d^3}\\
&\quad +\frac{1}{2}\left(a^2d^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx+\frac{1}{2}\left(a^2d^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx\\
&= -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}\\
&\quad +\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{2x}\\
&\quad +\frac{6b^2x\sinh(c+dx)}{d^3}+\frac{b^2x^3\sinh(c+dx)}{d}+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx = \frac{1}{2}\left(-\frac{12b^2\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{x^2} - \frac{6b^2x^2\cosh(c+dx)}{d^2} + a^2d^2\cosh(c)\text{Chi}(dx) + \frac{4ab\sinh(c+dx)}{d} - \frac{a^2d\sinh(c+dx)}{x} + \frac{12b^2x\sinh(c+dx)}{d^3} + \frac{2b^2x^3\sinh(c+dx)}{d} + a^2d^2\sinh(c)\text{Shi}(dx)\right)$$

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] ((-12*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/x^2 - (6*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*d^2*Cosh[c]*CoshIntegral[d*x] + (4*a*b*Sinh[c + d*x])/d -
 (a^2*d*Sinh[c + d*x])/x + (12*b^2*x*Sinh[c + d*x])/d^3 + (2*b^2*x^3*Sinh[c
 + d*x])/d + a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(133) = 266.

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.99

method	result
risch	$-\frac{e^c \text{Ei}_1(-dx)a^2d^6x^2+e^{-c}\text{Ei}_1(dx)a^2d^6x^2-2e^{dx+c}b^2d^3x^5+2e^{-dx-c}b^2d^3x^5+e^{dx+c}a^2d^5x+6e^{dx+c}b^2d^2x^4-e^{-dx-c}a^2d^5x+6e^{-dx-}}{}$
meijerg	$\frac{8b^2\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib^2\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i\left(\frac{15x^2d^2}{2}+15\right)}{20\sqrt{\pi}}\right.}{d^4}$

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d^4*(exp(c)*Ei(1,-d*x)*a^2*d^6*x^2+exp(-c)*Ei(1,d*x)*a^2*d^6*x^2-2*exp
(d*x+c)*b^2*d^3*x^5+2*exp(-d*x-c)*b^2*d^3*x^5+exp(d*x+c)*a^2*d^5*x+6*exp(d*
x+c)*b^2*d^2*x^4-exp(-d*x-c)*a^2*d^5*x+6*exp(-d*x-c)*b^2*d^2*x^4-4*exp(d*x+
c)*a*b*d^3*x^2+4*exp(-d*x-c)*a*b*d^3*x^2+d^4*exp(d*x+c)*a^2-12*exp(d*x+c)*b
^2*d*x^3+d^4*exp(-d*x-c)*a^2+12*exp(-d*x-c)*b^2*d*x^3+12*exp(d*x+c)*b^2*x^2
+12*exp(-d*x-c)*b^2*x^2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = -\frac{2\,(6\,b^2d^2x^4 + a^2d^4 + 12\,b^2x^2)\cosh\,(dx+c) - (a^2d^6x^2\mathrm{Ei}(dx) + a^2d^6x^2\mathrm{Ei}(-dx))\cosh\,(c) - 2\,(2\,b^2d^3x^5 - a}{4\,d^4x^2}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*b^2*d^2*x^4 + a^2*d^4 + 12*b^2*x^2)*cosh(d*x + c) - (a^2*d^6*x^2
*Ei(d*x) + a^2*d^6*x^2*Ei(-d*x))*cosh(c) - 2*(2*b^2*d^3*x^5 - a^2*d^5*x + 4
*a*b*d^3*x^2 + 12*b^2*d*x^3)*sinh(d*x + c) - (a^2*d^6*x^2*Ei(d*x) - a^2*d^6
*x^2*Ei(-d*x))*sinh(c))/(d^4*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)^2\cosh\,(c+dx)}{x^3}\,dx$$

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{8}\left(2\,a^2de^{(-c)}\Gamma(-1,dx) + 2\,a^2de^{c}\Gamma(-1,-dx) - \frac{8\,(dxe^c - e^c)abe^{(dx)}}{d^2} - \frac{8\,(dx+1)abe^{(-dx-c)}}{d^2} - \frac{(d^4x^4e^c - 4\,}{}\right. + \frac{1}{4}\left(b^2x^4 + 8\,abx - \frac{2\,a^2}{x^2}\right)\cosh\,(dx+c)$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*a^2*d*e^(-c)*gamma(-1, d*x) + 2*a^2*d*e^c*gamma(-1, -d*x) - 8*(d*x*e
^c - e^c)*a*b*e^(d*x)/d^2 - 8*(d*x + 1)*a*b*e^(-d*x - c)/d^2 - (d^4*x^4*e^c
 - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 -
(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d +
1/4*(b^2*x^4 + 8*a*b*x - 2*a^2/x^2)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(133) = 266$.

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx$$
$$= \frac{a^2d^6x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^6x^2\mathrm{Ei}(dx)\,e^c + 2\,b^2d^3x^5e^{(dx+c)} - 2\,b^2d^3x^5e^{(-dx-c)} - a^2d^5xe^{(dx+c)} - 6\,b^2d^2x^4e^{(dx-}}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^6*x^2*Ei(-d*x)*e^(-c) + a^2*d^6*x^2*Ei(d*x)*e^c + 2*b^2*d^3*x^5*
e^(d*x + c) - 2*b^2*d^3*x^5*e^(-d*x - c) - a^2*d^5*x*e^(d*x + c) - 6*b^2*d^
2*x^4*e^(d*x + c) + a^2*d^5*x*e^(-d*x - c) - 6*b^2*d^2*x^4*e^(-d*x - c) + 4
*a*b*d^3*x^2*e^(d*x + c) - 4*a*b*d^3*x^2*e^(-d*x - c) - a^2*d^4*e^(d*x + c)
 + 12*b^2*d*x^3*e^(d*x + c) - a^2*d^4*e^(-d*x - c) - 12*b^2*d*x^3*e^(-d*x -
 c) - 12*b^2*x^2*e^(d*x + c) - 12*b^2*x^2*e^(-d*x - c))/(d^4*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^3}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^3, x)
```

3.92 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx = -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{2b^2x\cosh(c+dx)}{d^2} + 2ab\cosh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3\text{Chi}(dx)\sinh(c) + \frac{2b^2\sinh(c+dx)}{d^3} - \frac{a^2d\sinh(c+dx)}{6x^2} + \frac{b^2x^2\sinh(c+dx)}{d} + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) + 2ab\sinh(c)\text{Shi}(dx)$$

```
[Out] 2*a*b*Chi(d*x)*cosh(c)-1/3*a^2*cosh(d*x+c)/x^3-1/6*a^2*d^2*cosh(d*x+c)/x-2*
b^2*x*cosh(d*x+c)/d^2+1/6*a^2*d^3*cosh(c)*Shi(d*x)+1/6*a^2*d^3*Chi(d*x)*sin
h(c)+2*a*b*Shi(d*x)*sinh(c)+2*b^2*sinh(d*x+c)/d^3-1/6*a^2*d*sinh(d*x+c)/x^2
+b^2*x^2*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2717}

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx = \frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{a^2\cosh(c+dx)}{3x^3} - \frac{a^2d\sinh(c+dx)}{6x^2} + 2ab\cosh(c)\text{Chi}(dx) + 2ab\sinh(c)\text{Shi}(dx) + \frac{2b^2\sinh(c+dx)}{d^3} - \frac{2b^2x\cosh(c+dx)}{d^2} + \frac{b^2x^2\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/(6*x) - (2*b^2*x*Cos
h[c + d*x])/d^2 + 2*a*b*Cosh[c]*CoshIntegral[d*x] + (a^2*d^3*CoshIntegral[d
*x]*Sinh[c])/6 + (2*b^2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/(6*x^2)
+ (b^2*x^2*Sinh[c + d*x])/d + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + 2*a*b
*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x}+b^2x^2\cosh(c+dx)\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x}\,dx+b^2\int x^2\cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}+\frac{b^2x^2\sinh(c+dx)}{d}\\
&\quad -\frac{(2b^2)\int x\sinh(c+dx)\,dx}{d}+\frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^3}\,dx\\
&\quad +(2ab\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(2ab\sinh(c))\int \frac{\sinh(dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)\\
&\quad -\frac{a^2d\sinh(c+dx)}{6x^2}+\frac{b^2x^2\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)\\
&\quad +\frac{(2b^2)\int\cosh(c+dx)\,dx}{d^2}+\frac{1}{6}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}\\
&\quad +2ab\cosh(c)\text{Chi}(dx)+\frac{2b^2\sinh(c+dx)}{d^3}-\frac{a^2d\sinh(c+dx)}{6x^2}\\
&\quad +\frac{b^2x^2\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)+\frac{1}{6}\left(a^2d^3\right)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)\\
&\quad +\frac{2b^2\sinh(c+dx)}{d^3}-\frac{a^2d\sinh(c+dx)}{6x^2}+\frac{b^2x^2\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(dx)\\
&\quad +\frac{1}{6}\left(a^2d^3\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx+\frac{1}{6}\left(a^2d^3\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)\\
&\quad +\frac{1}{6}a^2d^3\text{Chi}(dx)\sinh(c)+\frac{2b^2\sinh(c+dx)}{d^3}-\frac{a^2d\sinh(c+dx)}{6x^2}\\
&\quad +\frac{b^2x^2\sinh(c+dx)}{d}+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)+2ab\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\Bigg(-\frac{2a^2\cosh(c+dx)}{x^3} - \frac{a^2d^2\cosh(c+dx)}{x} - \frac{12b^2x\cosh(c+dx)}{d^2} + a\text{Chi}(dx)\left(12b\cosh(c)+ad^3\sinh(c)\right) + \frac{12b^2\sinh(c+dx)}{d^3} - \frac{a^2d\sinh(c+dx)}{x^2} + \frac{6b^2x^2\sinh(c+dx)}{d} + a\left(ad^3\cosh(c)+12b\sinh(c)\right)\text{Shi}(dx)\Bigg)$$

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/x - (12*b^2*x*Cosh[c
+ d*x])/d^2 + a*CoshIntegral[d*x]*(12*b*Cosh[c] + a*d^3*Sinh[c]) + (12*b^2*
Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/x^2 + (6*b^2*x^2*Sinh[c + d*x])/
d + a*(a*d^3*Cosh[c] + 12*b*Sinh[c])*SinhIntegral[d*x])/6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(140) = 280.

Time = 0.31 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{-e^{-c}\,\text{Ei}_1(dx)a^2d^6x^3+e^{c}\,\text{Ei}_1(-dx)a^2d^6x^3+e^{dx+c}a^2d^5x^2-6\,e^{dx+c}b^2d^2x^5+e^{-dx-c}a^2d^5x^2+6\,e^{-dx-c}b^2d^2x^5+12\,e^{-c}\,\text{Ei}_1(dx)ab\,d^3}{}$
meijerg	$\frac{4ib^2\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3}+\frac{4b^2\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3}+ab\cosh$

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/d^3*(-exp(-c)*Ei(1,d*x)*a^2*d^6*x^3+exp(c)*Ei(1,-d*x)*a^2*d^6*x^3+exp
(d*x+c)*a^2*d^5*x^2-6*exp(d*x+c)*b^2*d^2*x^5+exp(-d*x-c)*a^2*d^5*x^2+6*exp(
-d*x-c)*b^2*d^2*x^5+12*exp(-c)*Ei(1,d*x)*a*b*d^3*x^3+12*exp(c)*Ei(1,-d*x)*a
*b*d^3*x^3+d^4*exp(d*x+c)*a^2*x+12*exp(d*x+c)*b^2*d*x^4-d^4*exp(-d*x-c)*a^2
*x+12*exp(-d*x-c)*b^2*d*x^4+2*exp(d*x+c)*a^2*d^3-12*exp(d*x+c)*b^2*x^3+2*ex
p(-d*x-c)*a^2*d^3+12*exp(-d*x-c)*b^2*x^3)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx = -\frac{2\left(a^2d^5x^2+12\,b^2dx^4+2\,a^2d^3\right)\cosh\left(dx+c\right)-\left(\left(a^2d^6+12\,abd^3\right)x^3\mathrm{Ei}(dx)-\left(a^2d^6-12\,abd^3\right)x^3\mathrm{Ei}(-dx)\right)}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(a^2*d^5*x^2 + 12*b^2*d*x^4 + 2*a^2*d^3)*cosh(d*x + c) - ((a^2*d^6
 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*cosh(c) -
 2*(6*b^2*d^2*x^5 - a^2*d^4*x + 12*b^2*x^3)*sinh(d*x + c) - ((a^2*d^6 + 12*
a*b*d^3)*x^3*Ei(d*x) + (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))/(d^3*x
^3)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)^2\cosh\left(c+dx\right)}{x^4}\,dx$$

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx$$
$$=\frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma(-2,dx)-d^2e^c\Gamma(-2,-dx)\right)a^2-b^2\left(\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)e^{(dx)}}{d^4}+\frac{(d^3x^3+3\,d^2x}{}\right.\right.$$
$$+\frac{1}{3}\left(b^2x^3+2\,ab\log\left(x^3\right)-\frac{a^2}{x^3}\right)\cosh\left(dx+c\right)$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a^2 - b^2*((d^3*
x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2
*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) - 4*a*b*cosh(d*x + c)*log(x^3)/d + 6*(E
i(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + 1/3*(b^2*x^3 + 2*a*b*log(x^3) - a^
2/x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx =$$
$$-\frac{a^2d^6x^3\text{Ei}(-dx)\,e^{(-c)} - a^2d^6x^3\text{Ei}(dx)\,e^c + a^2d^5x^2e^{(dx+c)} - 6\,b^2d^2x^5e^{(dx+c)} + a^2d^5x^2e^{(-dx-c)} + 6\,b^2d^2x^5e^{(-dx-c)}}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^6*x^3*Ei(-d*x)*e^(-c) - a^2*d^6*x^3*Ei(d*x)*e^c + a^2*d^5*x^2*
e^(d*x + c) - 6*b^2*d^2*x^5*e^(d*x + c) + a^2*d^5*x^2*e^(-d*x - c) + 6*b^2*
d^2*x^5*e^(-d*x - c) - 12*a*b*d^3*x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^3*x^3*Ei(d
*x)*e^c + a^2*d^4*x*e^(d*x + c) + 12*b^2*d*x^4*e^(d*x + c) - a^2*d^4*x*e^(-
d*x - c) + 12*b^2*d*x^4*e^(-d*x - c) + 2*a^2*d^3*e^(d*x + c) - 12*b^2*x^3*e
^(d*x + c) + 2*a^2*d^3*e^(-d*x - c) + 12*b^2*x^3*e^(-d*x - c))/(d^3*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^4} \,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^4, x)
```

3.93 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} \, dx = & -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} \\ & - \frac{2ab\cosh(c+dx)}{x} + \frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx) \\ & + 2abd\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{12x^3} \\ & - \frac{a^2d^3\sinh(c+dx)}{24x} + \frac{b^2x\sinh(c+dx)}{d} \\ & + 2abd\cosh(c)\text{Shi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)\end{aligned}$$

```
[Out] 1/24*a^2*d^4*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2-1/4*a^2*cosh(d*x+c)/x^4-1
/24*a^2*d^2*cosh(d*x+c)/x^2-2*a*b*cosh(d*x+c)/x+2*a*b*d*cosh(c)*Shi(d*x)+2*
a*b*d*Chi(d*x)*sinh(c)+1/24*a^2*d^4*Shi(d*x)*sinh(c)-1/12*a^2*d*sinh(d*x+c)
/x^3-1/24*a^2*d^3*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{5395, 3378, 3384, 3379, 3382, 3377, 2718\}$

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5}\,dx = \frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx) - \frac{a^2d^3\sinh(c+dx)}{24x} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d\sinh(c+dx)}{12x^3} + 2abd\sinh(c)\mathrm{Chi}(dx) + 2abd\cosh(c)\mathrm{Shi}(dx) - \frac{2ab\cosh(c+dx)}{x} - \frac{b^2\cosh(c+dx)}{d^2} + \frac{b^2x\sinh(c+dx)}{d}$$

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(4*x^4) - (a^2*d^2*Cosh[c
+ d*x])/(24*x^2) - (2*a*b*Cosh[c + d*x])/x + (a^2*d^4*Cosh[c]*CoshIntegral[
d*x])/24 + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(12*x^
3) - (a^2*d^3*Sinh[c + d*x])/(24*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*d*Cos
h[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^2}+b^2x\cosh(c+dx)\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int x\cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2x\sinh(c+dx)}{d}-\frac{b^2\int \sinh(c+dx)\,dx}{d}\\
&\quad+\frac{1}{4}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^4}\,dx+(2abd)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{x}\\
&\quad-\frac{a^2d\sinh(c+dx)}{12x^3}+\frac{b^2x\sinh(c+dx)}{d}+\frac{1}{12}\left(a^2d^2\right)\int \frac{\cosh(c+dx)}{x^3}\,dx\\
&\quad+(2abd\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(2abd\sinh(c))\int \frac{\cosh(dx)}{x}\,dx\\
&= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d^2\cosh(c+dx)}{24x^2}\\
&\quad-\frac{2ab\cosh(c+dx)}{x}+2abd\text{Chi}(dx)\sinh(c)-\frac{a^2d\sinh(c+dx)}{12x^3}\\
&\quad+\frac{b^2x\sinh(c+dx)}{d}+2abd\cosh(c)\text{Shi}(dx)+\frac{1}{24}\left(a^2d^3\right)\int \frac{\sinh(c+dx)}{x^2}\,dx\\
&= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{2ab\cosh(c+dx)}{x}\\
&\quad+2abd\text{Chi}(dx)\sinh(c)-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2d^3\sinh(c+dx)}{24x}\\
&\quad+\frac{b^2x\sinh(c+dx)}{d}+2abd\cosh(c)\text{Shi}(dx)+\frac{1}{24}\left(a^2d^4\right)\int \frac{\cosh(c+dx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} \\
&\quad - \frac{2ab\cosh(c+dx)}{x} + 2abd\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{12x^3} \\
&\quad - \frac{a^2d^3\sinh(c+dx)}{24x} + \frac{b^2x\sinh(c+dx)}{d} + 2abd\cosh(c)\text{Shi}(dx) \\
&\quad + \frac{1}{24}\left(a^2d^4\cosh(c)\right)\int\frac{\cosh(dx)}{x}\,dx + \frac{1}{24}\left(a^2d^4\sinh(c)\right)\int\frac{\sinh(dx)}{x}\,dx \\
&= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} \\
&\quad - \frac{2ab\cosh(c+dx)}{x} + \frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx) + 2abd\text{Chi}(dx)\sinh(c) \\
&\quad - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{a^2d^3\sinh(c+dx)}{24x} + \frac{b^2x\sinh(c+dx)}{d} \\
&\quad + 2abd\cosh(c)\text{Shi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int\frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx &= \frac{1}{24}\left(-\frac{24b^2\cosh(c+dx)}{d^2} - \frac{6a^2\cosh(c+dx)}{x^4}\right. \\
&\quad - \frac{a^2d^2\cosh(c+dx)}{x^2} - \frac{48ab\cosh(c+dx)}{x} \\
&\quad + ad\text{Chi}(dx)\left(ad^3\cosh(c)+48b\sinh(c)\right) - \frac{2a^2d\sinh(c+dx)}{x^3} \\
&\quad - \frac{a^2d^3\sinh(c+dx)}{x} + \frac{24b^2x\sinh(c+dx)}{d} \\
&\quad \left. + ad\left(48b\cosh(c)+ad^3\sinh(c)\right)\text{Shi}(dx)\right)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] ((-24*b^2*Cosh[c + d*x])/d^2 - (6*a^2*Cosh[c + d*x])/x^4 - (a^2*d^2*Cosh[c
+ d*x])/x^2 - (48*a*b*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*(a*d^3*Cosh[
c] + 48*b*Sinh[c]) - (2*a^2*d*Sinh[c + d*x])/x^3 - (a^2*d^3*Sinh[c + d*x])/
x + (24*b^2*x*Sinh[c + d*x])/d + a*d*(48*b*Cosh[c] + a*d^3*Sinh[c])*SinhInt
egral[d*x])/24
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(155) = 310.

Time = 0.35 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.93

method	result
risch	$\frac{-\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)a^2d^6x^4-\mathrm{e}^{c}\,\mathrm{Ei}_1(-dx)a^2d^6x^4+\mathrm{e}^{-dx-c}a^2d^5x^3+48\,\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)ab\,d^3x^4-\mathrm{e}^{dx+c}a^2d^5x^3-48\,\mathrm{e}^{c}\,\mathrm{Ei}_1(-dx)ab\,d^3x^4-d^4\mathrm{e}^{-dx-c}}{}$
meijerg	$-\frac{2b^2\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b^2\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{idab\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}(dx)}{\sqrt{\pi}}\right)}{2}$

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d^2*(-exp(-c)*Ei(1,d*x)*a^2*d^6*x^4-exp(c)*Ei(1,-d*x)*a^2*d^6*x^4+exp(
-d*x-c)*a^2*d^5*x^3+48*exp(-c)*Ei(1,d*x)*a*b*d^3*x^4-exp(d*x+c)*a^2*d^5*x^3
-48*exp(c)*Ei(1,-d*x)*a*b*d^3*x^4-d^4*exp(-d*x-c)*a^2*x^2-24*exp(-d*x-c)*b^
2*d*x^5-d^4*exp(d*x+c)*a^2*x^2+24*exp(d*x+c)*b^2*d*x^5-48*exp(-d*x-c)*a*b*d
^2*x^3-48*exp(d*x+c)*a*b*d^2*x^3+2*exp(-d*x-c)*a^2*d^3*x-24*exp(-d*x-c)*b^2
*x^4-2*exp(d*x+c)*a^2*d^3*x-24*exp(d*x+c)*b^2*x^4-6*d^2*exp(-d*x-c)*a^2-6*d
^2*exp(d*x+c)*a^2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx =$$
$$-\frac{2\,(a^2d^4x^2+48\,abd^2x^3+24\,b^2x^4+6\,a^2d^2)\cosh{(dx+c)}-((a^2d^6+48\,abd^3)x^4\mathrm{Ei}(dx)+(a^2d^6-48\,abd^3)x}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(a^2*d^4*x^2 + 48*a*b*d^2*x^3 + 24*b^2*x^4 + 6*a^2*d^2)*cosh(d*x +
 c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) + (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-
d*x))*cosh(c) + 2*(a^2*d^5*x^3 - 24*b^2*d*x^5 + 2*a^2*d^3*x)*sinh(d*x + c)
- ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) - (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-d*x)
)*sinh(c))/(d^2*x^4)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx^3)^2\cosh{(c+dx)}}{x^5}\,dx$$

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = \frac{1}{8}\left(a^2d^3e^{(-c)}\Gamma(-3,dx) + a^2d^3e^c\Gamma(-3,-dx) - 8\,ab\mathrm{Ei}(-dx)\,e^{(-c)} + 8\,ab\mathrm{Ei}(dx)\,e^c - \frac{2\,(d^2x^2e^c - 2\,dxe^c + 2\right.$$
$$+\frac{1}{4}\left(2\,b^2x^2 - \frac{8\,abx^3 + a^2}{x^4}\right)\cosh{(dx+c)}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*d^3*e^(-c)*gamma(-3, d*x) + a^2*d^3*e^c*gamma(-3, -d*x) - 8*a*b*Ei
(-d*x)*e^(-c) + 8*a*b*Ei(d*x)*e^c - 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b^2
*e^(d*x)/d^3 - 2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3)*d + 1/4*(2*b^2
*x^2 - (8*a*b*x^3 + a^2)/x^4)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(155) = 310.

Time = 0.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = \frac{a^2d^6x^4\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^6x^4\mathrm{Ei}(dx)\,e^c - a^2d^5x^3e^{(dx+c)} + a^2d^5x^3e^{(-dx-c)} - 48\,abd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 48\,a}{}$$

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^6*x^4*Ei(-d*x)*e^(-c) + a^2*d^6*x^4*Ei(d*x)*e^c - a^2*d^5*x^3*e
^(d*x + c) + a^2*d^5*x^3*e^(-d*x - c) - 48*a*b*d^3*x^4*Ei(-d*x)*e^(-c) + 48
```

```
*a*b*d^3*x^4*Ei(d*x)*e^c - a^2*d^4*x^2*e^(d*x + c) + 24*b^2*d*x^5*e^(d*x +
c) - a^2*d^4*x^2*e^(-d*x - c) - 24*b^2*d*x^5*e^(-d*x - c) - 48*a*b*d^2*x^3*
e^(d*x + c) - 48*a*b*d^2*x^3*e^(-d*x - c) - 2*a^2*d^3*x*e^(d*x + c) - 24*b^
2*x^4*e^(d*x + c) + 2*a^2*d^3*x*e^(-d*x - c) - 24*b^2*x^4*e^(-d*x - c) - 6*
a^2*d^2*e^(d*x + c) - 6*a^2*d^2*e^(-d*x - c))/(d^2*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^5}\,dx$$

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^5,x)
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^5, x)
```

3.94 $\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 373

$$\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx = -\frac{\cosh(c+dx)}{bd^2} + \frac{(-1)^{2/3}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} + \frac{a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}} + \frac{x\sinh(c+dx)}{bd} - \frac{(-1)^{2/3}a^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} + \frac{a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}$$

```
[Out] 1/3*a^(2/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/b^(5/3)+1/
3*(-1)^(2/3)*a^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)
*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)-cosh(d*x+c)/b/
d^2+1/3*a^(2/3)*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/b^(5/3
```

```
)+1/3*(-1)^(2/3)*a^(2/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)
^(1/3)*a^(1/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Shi((-1)^(2/3)*a^(
1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)+x*sinh(d*x
+c)/b/d
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5401, 3377, 2718, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx =& \frac{(-1)^{2/3}a^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\ &-\frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &+\frac{a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &-\frac{(-1)^{2/3}a^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\ &+\frac{a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &-\frac{\sqrt[3]{-1}a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} \\ &-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}\end{aligned}$$

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + ((-1)^(2/3)*a^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)
) - ((-1)^(1/3)*a^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b^(5/3)) + (a^(2/3)*Cosh[c
- (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
 + (x*Sinh[c + d*x])/(b*d) - ((-1)^(2/3)*a^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/
3)) + (a^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{x\cosh(c+dx)}{b} - \frac{ax\cosh(c+dx)}{b(a+bx^3)} \right) dx$$

$$= \frac{\int x\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{b}$$

$$
\begin{aligned}
&= \frac{x\sinh(c+dx)}{bd} \\
&- \frac{a\int\left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b} \\
&- \frac{\int\sinh(c+dx)\,dx}{bd}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}+\frac{a^{2/3}\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b^{4/3}} \\
&- \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3b^{4/3}}+\frac{\left((-1)^{2/3}a^{2/3}\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b^{4/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}+\frac{\left(a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b^{4/3}} \\
&- \frac{\left(\sqrt[3]{-1}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3b^{4/3}} \\
&+ \frac{\left((-1)^{2/3}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b^{4/3}} \\
&+ \frac{\left(a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b^{4/3}} \\
&- \frac{\left((-1)^{5/6}a^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3b^{4/3}} \\
&- \frac{\left(\sqrt[6]{-1}a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b^{4/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cosh(c+dx)}{bd^2} + \frac{(-1)^{2/3}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\
&\quad - \frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\
&\quad + \frac{a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}} + \frac{x\sinh(c+dx)}{bd} \\
&\quad - \frac{(-1)^{2/3}a^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} \\
&\quad + \frac{a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}} \\
&\quad - \frac{\sqrt[3]{-1}a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.57

$$
\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx = -\frac{ad^2\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\text{sin}\cdots}{\#1}\right.}{\cdots}
$$

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/6*(a*d^2*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
 - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*
(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + a*d^2*RootSum
[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c
+ d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + 6*b*(Cosh[c + d*x] - d*x*Sinh[c
+ d*x]))/(b^2*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.48

method	result	size
risch	Expression too large to display	925

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d^2/b*c^4*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2/b*c^4*sum(1/(_R1^2-2*
_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^
2+a*d^3-b*c^3))+2/3/d^2/b*c^3*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*
x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2/b*c^
3*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*exp(d*x+c)*x-1/2/d/b*exp(-d*x-c)
*x-1/d^2/b*c^2*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=
RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/d^2/b*c^2*sum(_R1^2/(_R
1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))-1/2/d^2/b*exp(d*x+c)-1/2/d^2/b*exp(-d*x-c)+2/3/d^2/b^
2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*E
i(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*c+2/3/
d^2/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_
R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*
c-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(
_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3
*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1*b*c^
3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(265) = 530.

Time = 0.30 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.65

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
```

```
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(
(sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(
a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((s
qrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
 - c) + (a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x +
 c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(2
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*d*x*sinh(d*x + c
) - 12*cosh(d*x + c))/(b*d^2*cosh(d*x + c)^2 - b*d^2*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^3} \, dx = \int \frac{x^4 \cosh{(c + dx)}}{a + bx^3} \, dx$$

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a),x)
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^3} \, dx = \text{Timed out}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \cosh(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^3),x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x^3), x)
```

3.95 $\int \frac{x^3 \cosh(c+dx)}{a+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 358

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} \, dx = \frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} + \frac{\sinh(c+dx)}{bd} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}$$

```
[Out] -1/3*a^(1/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/b^(4/3)+1
/3*(-1)^(1/3)*a^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/
3)*a^(1/3)*d/b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Chi(-(-1)^(2/3)*a^(1/3
)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)-1/3*a^(1/3)*S
hi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/b^(4/3)+1/3*(-1)^(1/3)*
a^(1/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)+sinh(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5401, 2717, 5389, 3384, 3379, 3382}

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx = \frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} + \frac{\sinh(c+dx)}{bd}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((-1)^(1/3)*a^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*Cos
h[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*Cosh
Integral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) + Sinh[c + d*x]/(b*d) - ((
-1)^(1/3)*a^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^
(2/3)*a^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b(a+bx^3)}\right) dx \\
&= \frac{\int \cosh(c+dx)\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{b} \\
&= \frac{\sinh(c+dx)}{bd} \\
&\quad - \frac{a\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{b} \\
&= \frac{\sinh(c+dx)}{bd} + \frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}
\end{aligned}
$$

$$= \frac{\sinh(c+dx)}{bd} + \frac{\left(\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b}$$

$$+ \frac{\left(\sqrt[3]{a}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3b}$$

$$+ \frac{\left(i\sqrt[3]{a}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{3b}$$

$$+ \frac{\left(i\sqrt[3]{a}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3b}$$

$$= \frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}$$

$$- \frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}$$

$$- \frac{\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} + \frac{\sinh(c+dx)}{bd}$$

$$- \frac{\sqrt[3]{-1}\sqrt[3]{a}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}$$

$$- \frac{\sqrt[3]{a}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}$$

$$- \frac{(-1)^{2/3}\sqrt[3]{a}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx =$$
$$-\frac{ad\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh}{\#1^2}\right.}{}$$

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3),x]

[Out] -1/6*(a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] + a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] - 6*b*Sinh[c + d*x])/(b^2*d)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.87

method	result
risch	$\frac{c^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-b\,c^3\right)}\frac{e^{\_R1}\,\text{Ei}_1\left(-dx+\_R1-c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6db}+\frac{c^3\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d\right.}\right.}{6db}$

[In] int(x^3*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/6/d/b*c^3*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d/b*c^3*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d/b*c^2*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d/b*c^2*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*c*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*c*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d/b*exp(-d*x-c)+1/2/d/b*exp(d*x+c)-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b

```
*c^3))-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(250) = 500$.

Time = 0.27 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.73

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c
)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(
(sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(
a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c
)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((s
qrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
 - c) + (a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x +
 c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*sinh(d*x + c))/(
b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \cosh{(c+dx)}}{a+bx^3}\,dx$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a),x)
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \cosh{(dx+c)}}{bx^3+a}\,dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^3),x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^3), x)
```

3.96 $\int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\begin{aligned}\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx &= \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}\\
&+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}\\
&+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}\\
&-\frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}\\
&+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}\\
&+\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}\end{aligned}$$

```
[Out] 1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/b+1/3*Chi((-1)^(1/
3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Chi(-(
-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b+1/3
*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/b+1/3*Shi(-(-1)^(1/3)
*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Shi((-1)
^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5401, 3384, 3379, 3382}

$$\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx = \frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} + \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} - \frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b} + \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(3*b) + (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIn
tegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b) + (Cosh[c - (a^(1/3)
*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b) - (Sinh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(3*b) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b) + (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)} \right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$= \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+\frac{\left(i\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$+\frac{\left(i\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$= \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}$$

$$+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}$$

$$+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}$$

$$-\frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}$$

$$+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}$$

$$+\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.60

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx$$
$$= \frac{\text{RootSum}\left[a+b\#1^3\&, \cosh(c+d\#1)\text{Chi}(d(x-\#1)) - \text{Chi}(d(x-\#1))\sinh(c+d\#1) - \cosh(c+d\#1)\text{S}\right.}{}$$

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3),x]

[Out] (RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInteg ral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] &] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d* (x - #1)] &])/(6*b)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{c^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-b\,c^3\right)}\frac{e^{\_R1}\text{Ei}_1\left(-dx+\_R1-c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}-\frac{c^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+c\right.}\right.}{6b}$

[In] int(x^2*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/6/b*c^2*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*c^2*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3/b*c*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3/b*c*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(207) = 414$.

Time = 0.27 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.77

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\mathrm{Ei}\left(dx - \frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+c\right)+\mathrm{Ei}\left(-dx-\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)}{}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c
) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + Ei(d*x + (a*d^
3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c)
 - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))
/b
```

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \cosh(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b*d^2*x^3
*e^c + a*d^2*e^c) + 1/2*integrate((2*b*x^3*e^c - 3*a*d*x*e^c - a*e^c)*e^(d*
x)/(b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2), x) + 1/2*integrate((2*b*x^3 + 3
*a*d*x - a)*e^(-d*x)/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c), x
)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \cosh(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3), x)
```

3.97 $\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 345

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx =& -\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\&+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\&-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\\&+\frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\&-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\\&+\frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\end{aligned}$$

```
[Out] -1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1
/3*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1
/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*Shi(a^(1/
3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*
Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
```

```
/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(
c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5401, 3384, 3379, 3382}

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = -\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{(-1)^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)) - d*x])/(3*a^(1/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshInteg
ral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Sinh[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(3*a^(1/3)*b^(2/3)) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(1/3)*b^(2/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)} \right) dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$
\begin{aligned}
= & -\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
& +\frac{\left(\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
& -\frac{\left((-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
& -\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
& +\frac{\left((-1)^{5/6}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
& +\frac{\left(\sqrt[6]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}
\end{aligned}
$$

$$= -\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}$$
$$+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}$$
$$-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$
$$+\frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}$$
$$-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$
$$+\frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.05 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#}{\#1}\right.}{}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
 Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + RootSum[a + b*#1^3 & , (
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c +
 d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInteg
ral[d*(x - #1)])/#1 & ])/(6*b)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.81

method	result
risch	$\frac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-b\,c^3\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{Ei}_1\left(-dx+\_R1-c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}+\frac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a\right.}\right.}{6b}$

```
[In] int(x*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d/b*c*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d/b*c*sum(1/(_R1^2-2*_R1*c+c^2
)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3))-1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Ro
otOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d/b*sum(_R1/(_R1^2-2*_R
1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+
a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(237) = 474$.

Time = 0.28 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.94

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) - (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(2/3)*(s
qrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3)
```

```
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a),x)
[Out] Integral(x*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x\cosh(dx+c)}{bx^3+a}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x^3),x)
[Out] int((x*cosh(c + d*x))/(a + b*x^3), x)
```

3.98 $\int \frac{\cosh(c+dx)}{a+bx^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 345

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = &-\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}\\
&+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}\\
&+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\\
&+\frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}\\
&+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\\
&+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\end{aligned}$$

```
[Out] 1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/
3*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)+1/3*Shi(a^(1/3
)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*S
hi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/
```

```
a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c
-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5389, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = -\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x^3),x]
```

```
[Out] -1/3*((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cosh[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshInteg
ral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Sinh[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(2/3)*b^(1/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= -\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\left(i\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$-\frac{\left(i\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= -\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.05 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1}{\#1^2}\right.}{}$$

[In] Integrate[Cosh[c + d*x]/(a + b*x^3),x]

[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] + RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &])/(6*b)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.41

method	result
risch	$-\frac{d^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-bc^3\right)}\frac{e^{-\_R1}\text{Ei}_1\left(dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}-\frac{d^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+\right.}\right.}{6b}$

[In] int(cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(237) = 474$.

Time = 0.27 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.95

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) + (-a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(1/3)*(s
qrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{\cosh(c+dx)}{a+bx^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh(c + dx)}{a + bx^3} dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cosh(c + dx)}{a + bx^3} dx = \int \frac{\cosh(dx + c)}{bx^3 + a} dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c + dx)}{a + bx^3} dx = \int \frac{\cosh(c + d\,x)}{b\,x^3 + a} dx$$

```
[In] int(cosh(c + d*x)/(a + b*x^3),x)
[Out] int(cosh(c + d*x)/(a + b*x^3), x)
```

3.99 $\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 303

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = {} & \frac{\cosh(c)\operatorname{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\
& - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\
& - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} + \frac{\sinh(c)\operatorname{Shi}(dx)}{a} \\
& + \frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\
& - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} \\
& - \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}
\end{aligned}
$$

```
[Out] Chi(d*x)*cosh(c)/a-1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))
/a-1/3*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))/a-1/3*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a+Shi(d*x)*sinh(c)/a-1/3*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^
(1/3)*d/b^(1/3))/a-1/3*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))/a-1/3*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-
(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5401, 3384, 3379, 3382}

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = -\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} + \frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} - \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a} + \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*C
oshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a)
 - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(3*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx^2\cosh(c+dx)}{a(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b\int \frac{x^2\cosh(c+dx)}{a+bx^3} dx}{a}$$

$$= -\frac{b\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x} dx}{a} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x} dx}{a}$$

$$= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x} dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x} dx}{3a}$$

$$= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\left(\sqrt[3]{b}\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(\sqrt[3]{b}\sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(i\sqrt[3]{b}\sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$- \frac{\left(i\sqrt[3]{b}\sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3a}$$

$$- \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3a}$$

$$- \frac{\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{3a} + \frac{\sinh(c)\text{Shi}(dx)}{a}$$

$$+ \frac{\sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3a}$$

$$- \frac{\sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{3a}$$

$$- \frac{\sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{3a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.61

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = -\frac{-6\cosh(c)\text{Chi}(dx)+\text{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+$$

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)),x]

[Out] -1/6*(-6*Cosh[c]*CoshIntegral[d*x] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] &] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] &] - 6*Sinh[c]*SinhIntegral[d*x])/a

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{e^{-c}\,\text{Ei}_1(dx)}{2a}+\frac{\sum_{\_R2=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-b\,c^3\right)} e^{-\_R2}\,\text{Ei}_1\left(dx-\_R2+c\right)}{6a}-\frac{e^{c}\,\text{Ei}_1(-dx)}{2a}+\frac{\sum_{\_R2=\text{RootOf}\left(\right.}$

[In] int(cosh(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/6/a*sum(exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a*exp(c)*Ei(1,-d*x)+1/6/a*sum(exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(227) = 454.

Time = 0.26 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.75

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = -\frac{\text{Ei}\left(dx-\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+c\right)+\text{Ei}\left(-dx-\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)}{}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)
*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(
-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) +
c) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) +
Ei(-d*x))*cosh(c) + Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) +
Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
 - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - E
i(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) - Ei(-d*
x))*sinh(c) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/a
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^3+a)x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^3+a)x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x^3)),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^3)), x)
```

3.100 $\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 381

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = -\frac{\cosh(c+dx)}{ax} + \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} + \frac{d\,\mathrm{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a} - \frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}$$

```
[Out] 1/3*b^(1/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)+1/
3*(-1)^(2/3)*b^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)
```

```
*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-cosh(d*x+c)/a/
x+d*cosh(c)*Shi(d*x)/a+d*Chi(d*x)*sinh(c)/a+1/3*b^(1/3)*Shi(a^(1/3)*d/b^(1/
3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*Shi(-(-1)^
(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-1
/3*(-1)^(1/3)*b^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/
3)*a^(1/3)*d/b^(1/3))/a^(4/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5401, 3378, 3384, 3379, 3382}

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = {} & \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\
& - \frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
& + \frac{\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
& - \frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\
& + \frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
& - \frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} \\
& + \frac{d\sinh(c)\text{Chi}(dx)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}
\end{aligned}
$$

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) + ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3))
- ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral
[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) +
 (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - ((-1)^
(2/3)*b^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Sinh[c - (a^(1/3)*d)
```

```
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3
)*b^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a\,(a+bx^3)}\right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{a}
\end{aligned}
$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int\left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{d\int\frac{\sinh(c+dx)}{x}dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{b^{2/3}\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{(d\cosh(c))\int\frac{\sinh(dx)}{x}dx}{a} + \frac{(d\sinh(c))\int\frac{\cosh(dx)}{x}dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} + \frac{\left(b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3a^{4/3}} + \frac{\left(b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}} - \frac{\left((-1)^{5/6}b^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{3a^{4/3}} - \frac{\left(\sqrt[6]{-1}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{3a^{4/3}}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}$$
$$-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}$$
$$+\frac{\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} + \frac{d\text{Chi}(dx)\sinh(c)}{a}$$
$$+\frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}$$
$$+\frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}$$
$$-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.56

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx =$$
$$-\frac{6\cosh(c+dx) + x\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Sh}}{\#1}\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] -1/6*(6*Cosh[c + d*x] + x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegr
al[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*S
inhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] +
 x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIn
tegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
 + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] - 6*d*x*CoshIntegral[d*x
]*Sinh[c] - 6*d*x*Cosh[c]*SinhIntegral[d*x])/(a*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{3\,\mathrm{e}^{c}\,\mathrm{Ei}_{1}(-dx)xd-3\,\mathrm{e}^{-c}\,\mathrm{Ei}_{1}(dx)xd-\left(\sum_{\_R2=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3c^{2}b\_Z+d^{3}a-b\,c^{3}\right)}\frac{\mathrm{e}^{\_R2}\,\mathrm{Ei}_{1}\left(-dx+\_R2-c\right)}{\_R2-c}\right)xd-\left(\sum_{\_R2=\mathrm{I}}\right.}{6ax}$

```
[In] int(cosh(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(3*exp(c)*Ei(1,-d*x)*x*d-3*exp(-c)*Ei(1,d*x)*x*d-sum(1/(_R2-c)*exp(_R2
)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*x*
d-sum(1/(_R2-c)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))*x*d+3*exp(-d*x-c)+3*exp(d*x+c))/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(273) = 546$.

Time = 0.28 (sec) , antiderivative size = 1154, normalized size of antiderivative = 3.03

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*d^2*cosh(d*x + c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(
d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3
/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh
(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^
3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^
3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sin
h(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d
^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c
)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)
) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(2/3)*Ei(d*x +
(a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*
x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1)
+ c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b
*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*s
```

```
inh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b
*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x +
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
 - c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*
b*x + b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*
sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x*cosh(d*x + c)^2 - b*
x*sinh(d*x + c)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-
a*d^3/b)^(1/3)) + 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(
2/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 6*(a*d^3*x*Ei(d
*x) - a*d^3*x*Ei(-d*x))*cosh(c) - 6*(a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*si
nh(c))/(a^2*d^2*x*cosh(d*x + c)^2 - a^2*d^2*x*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \frac{\cosh\left(c+dx\right)}{x^2\left(a+bx^3\right)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**3+a),x)
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**3)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^3+a)x^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx=\int \frac{\cosh(c+d\,x)}{x^2\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^3)),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^3)), x)
```

3.101 $\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 410

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = &-\frac{\cosh(c+dx)}{2ax^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a} \\
&+\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\
&-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\
&-\frac{b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} \\
&-\frac{d\sinh(c+dx)}{2ax}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a} \\
&-\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\
&-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} \\
&-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}
\end{aligned}$$

```
[Out] 1/2*d^2*Chi(d*x)*cosh(c)/a-1/3*b^(2/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^
(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/3*(-1)^(2/3)*b^(2
```

```
/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))/a^(5/3)-1/2*cosh(d*x+c)/a/x^2+1/2*d^2*Shi(d*x)*sinh(c)/a-1/3*b^(2/3)*S
hi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*
b^(2/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/2*d*sinh(d*x+c)/a/x
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = &\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}\\&-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\&-\frac{b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\&-\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}\\&-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\&-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}\\&+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}\\&-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}\end{aligned}$$

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a*x^2) + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) + ((-1)^
(1/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - (d*Sinh[c + d*x])/(2*a*x) + (d
```

```
^2*Sinh[c]*SinhIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Sinh[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])
/(3*a^(5/3)) - (b^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)
*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Sinh[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*
a^(5/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{d^2\int \frac{\cosh(c+dx)}{x}\,dx}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax} + \frac{(d^2\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{2a}$$

$$+ \frac{\left(b\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$+ \frac{\left(b\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$+ \frac{\left(b\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$+ \frac{(d^2\sinh(c))\int \frac{\sinh(dx)}{x}\,dx}{2a} + \frac{\left(b\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$+ \frac{\left(ib\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$+ \frac{\left(ib\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} + \frac{\sqrt[3]{-1}b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} - \frac{b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} - \frac{d\sinh(c+dx)}{2ax} + \frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a} - \frac{\sqrt[3]{-1}b^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} - \frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.58

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = -\frac{3\cosh(c+dx) - 3d^2x^2\cosh(c)\mathrm{Chi}(dx) + x^2\mathrm{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1)) - \mathrm{Chi}(d(x-\#1)\dots}{\dots}\right.}{\dots}$$

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/6*(3*Cosh[c + d*x] - 3*d^2*x^2*Cosh[c]*CoshIntegral[d*x] + x^2*RootSum[a
 + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c +
d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + x^2*RootSum[a + b*#1^3 & , (Cosh
[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#
1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[
d*(x - #1)])/#1^2 & ] + 3*d*x*Sinh[c + d*x] - 3*d^2*x^2*Sinh[c]*SinhIntegra
l[d*x])/(a*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{3\,\mathrm{e}^{c}\,\mathrm{Ei}_1(-dx)x^2d^2+3\,\mathrm{e}^{-c}\,\mathrm{Ei}_1(dx)x^2d^2-2\left(\sum_{\_R2=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-b\,c^3\right)}\frac{\mathrm{e}^{\_R2}\,\mathrm{Ei}_1\left(-dx+\_R2-c\right)}{\_R2^2-2\_R2c+c^2}\right)x^2d^2-2\Big(}{12a}$

```
[In] int(cosh(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(3*exp(c)*Ei(1,-d*x)*x^2*d^2+3*exp(-c)*Ei(1,d*x)*x^2*d^2-2*sum(1/(_R2
^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))*x^2*d^2-2*sum(1/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x
-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*x^2*d^2-3*exp
(-d*x-c)*d*x+3*d*x*exp(d*x+c)+3*exp(-d*x-c)+3*exp(d*x+c))/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. 2(294) = 588.

Time = 0.29 (sec) , antiderivative size = 1251, normalized size of antiderivative = 3.05

$$\int \frac{\cosh(c+dx)}{x^3\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*d^2*x*sinh(d*x + c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*
cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*
x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(1/3)*((sqrt(-3
)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2
)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)
^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x^2
*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^
3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*s
inh(d*x + c)^2)*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/
b)^(1/3)) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sq
rt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(1/3)*((
```

```
sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x
 + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-
a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2
- b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*si
nh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x^2*cosh(d*x + c)^2 - b*
x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c +
(-a*d^3/b)^(1/3)) - 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(a*d^
3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 6*a*d*cos
h(d*x + c) - 3*(a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c) - 3*(a*d^3*
x^2*Ei(d*x) - a*d^3*x^2*Ei(-d*x))*sinh(c))/(a^2*d*x^2*cosh(d*x + c)^2 - a^2
*d*x^2*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx=\int \frac{\cosh\left(c+dx\right)}{x^3\left(a+bx^3\right)}\,dx$$

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**3+a),x)
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x**3)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx=\text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx=\int \frac{\cosh\left(dx+c\right)}{(bx^3+a)x^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3\,(a+bx^3)}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x^3)),x)
[Out] int(cosh(c + d*x)/(x^3*(a + b*x^3)), x)
```

3.102 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 718

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} + \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}} - \frac{(-1)^{2/3}d\,\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\sqrt[3]{-1}d\,\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{(-1)^{2/3}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} + \frac{\sqrt[3]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

```
[Out] 1/9*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/
9*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/3*x*cosh(d*x
+c)/b/(b*x^3+a)-1/9*(-1)^(2/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-
(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)-1/9*d*cosh(c-a^(1/3)*d/b^
(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*(-1)^(1/3)*d*cosh(c-(
-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/
b^(5/3)-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/
b^(5/3)+1/9*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^
(4/3)-1/9*(-1)^(2/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)*Shi(-(-1)^(1/3)*a^(1/
3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*
(-1)^(1/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{5399, 5389, 3384, 3379, 3382, 5400\}$

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$-\frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{(-1)^{2/3}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{(-1)^{2/3}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{x\cosh(c+dx)}{3b\,(a+bx^3)}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(x*Cosh[c + d*x])/(b*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a
^(2/3)*b^(4/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Cosh
Integral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(2/3)*b^(4/3)) + (C
osh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^
(2/3)*b^(4/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/
3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) - ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3
)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^
(1/3)*b^(5/3)) + ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + (
(-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*Sinh[c + (
(-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(9*a^(2/3)*b^(4/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (Sinh[c - (a^(1/3)*d)/b^
(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)
^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Sinh[c - ((-1)
^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*
x])/(9*a^(2/3)*b^(4/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{x\cosh(c+dx)}{3b(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{3b} + \frac{d\int \frac{x\sinh(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b} + \frac{d\int \left(-\frac{\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}} + \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\sinh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}} - \frac{\left((-1)^{2/3}d\right)\int \frac{\sinh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$= -\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$-\frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$+\frac{\left((-1)^{5/6}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$+\frac{\left(\sqrt[6]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$-\frac{\left(d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\left(i\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$+\frac{\left(\sqrt[3]{-1}d\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$-\frac{\left(i\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$-\frac{\left((-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} - \frac{d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$-\frac{(-1)^{2/3}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{(-1)^{2/3}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$-\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$
$$+\frac{\sqrt[3]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}}$$
$$+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.51

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6bx\cosh(c+dx)}{a+bx^3} - \text{RootSum}\left[a+b\#1^3\&, \frac{-\cosh(c+d\#1)\text{Chi}(d(x-\#1))+\text{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\text{Shi}(d}{}\right.}{}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*x*Cosh[c + d*x])/(a + b*x^3) - RootSum[a + b*#1^3 & , (-(Cosh[c + d*
#1]*CoshIntegral[d*(x - #1)]) + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d*#1]*SinhIntegral[d*(x -
 #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d
*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3
& , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sin
h[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ])/(18*b
^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.22

method	result	size
risch	Expression too large to display	877

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)*x-1/18/d/a/b^2*sum((3*_R2^2*b*c^2-
_R2*a*d^3-5*_R2*b*c^3-2*a*c*d^3+2*b*c^4+3*_R2*b*c^2+a*d^3-b*c^3)/(_R2^2-2*_
R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))+1/18/d*c^3/a/b*sum((_R2-c+2)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*E
i(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*
c^2/a/b*sum((_R2^2-_R2*c+_R2+c)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+
c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c/a/b^2*sum(
(2*_R2^2*b*c-3*_R2*b*c^2-a*d^3+b*c^3+2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)*exp(-_R
2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/
6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)*x+1/18/d/a/b^2*sum((3*_R2^2*b*c^2-_R2*
a*d^3-5*_R2*b*c^3-2*a*c*d^3+2*b*c^4-3*_R2*b*c^2-a*d^3+b*c^3)/(_R2^2-2*_R2*c
+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))-1/18/d*c^3/a/b*sum((_R2-c-2)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-
d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^2/
a/b*sum((_R2^2-_R2*c-_R2-c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_
R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*c/a/b^2*sum((2*_
R2^2*b*c-3*_R2*b*c^2-a*d^3+b*c^3-2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei
(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2050 vs. $2(500) = 1000$.

Time = 0.30 (sec) , antiderivative size = 2050, normalized size of antiderivative = 2.86

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^3)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
 a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
 a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3
))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
 a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
```

```
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
 a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3
))*sinh(-c + (a*d^3/b)^(1/3)))/((a*b^2*d*x^3 + a^2*b*d)*cosh(d*x + c)^2 - (
a*b^2*d*x^3 + a^2*b*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3
- 3*d*x^2 + 12*x)*e^(-d*x))/(b^2*d^3*x^6*e^c + 2*a*b*d^3*x^3*e^c + a^2*d^3*
e^c) + 1/2*integrate(-6*(a*d^2*x^2*e^c - 10*b*x^3*e^c + 3*a*d*x*e^c + 2*a*e
^c)*e^(d*x)/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3), x)
 - 1/2*integrate(-6*(a*d^2*x^2 - 10*b*x^3 - 3*a*d*x + 2*a)*e^(-d*x)/(b^3*d^
3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*b*d^3*x^3*e^c + a^3*d^3*e^c), x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^2} \, dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^3+a)^2} \, dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^3)^2, x)
```

3.103 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 373

$$\begin{aligned}\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} \, dx = & -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & - \frac{\sqrt[3]{-1}d\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & + \frac{(-1)^{2/3}d\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} \\ & + \frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} \\ & + \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} \\ & + \frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}\end{aligned}$$

```
[Out] -1/3*cosh(d*x+c)/b/(b*x^3+a)-1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b
^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*d*cosh(c
-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*(-1)^(2/
3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+
d*x)/a^(2/3)/b^(4/3)+1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1
/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)
*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*d*Chi(
```

```
-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(
2/3)/b^(4/3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5397, 5388, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx =& \frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\ &-\frac{\sqrt[3]{-1}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}\\ &+\frac{(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\ &+\frac{\sqrt[3]{-1}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}\\ &+\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\ &+\frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\ &-\frac{\cosh(c+dx)}{3b\,(a+bx^3)}\end{aligned}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Cosh[c + d*x]/(b*(a + b*x^3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/3)*d*Cos
hIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*
a^(2/3)*b^(4/3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*S
inhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + (d
*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*
a^(2/3)*b^(4/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)}+\frac{d\int\frac{\sinh(c+dx)}{a+bx^3}\,dx}{3b}\\
&= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)}\\
&\quad+\frac{d\int\left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b}\\
&= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)}-\frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}-\frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}-\frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}
\end{aligned}$$

$$= -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$
$$- \frac{\left(id\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$
$$- \frac{\left(id\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$
$$- \frac{\left(d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$
$$- \frac{\left(d\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$
$$- \frac{\left(d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$= -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} + \frac{d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$- \frac{\sqrt[3]{-1}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+ \frac{(-1)^{2/3}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$
$$+ \frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}$$
$$+ \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$
$$+ \frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.54

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx$$
$$= \frac{-\frac{6b\cosh(c+dx)}{a+bx^3} - d\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d}{\#1^2}\right.}{}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*Cosh[c + d*x])/(a + b*x^3) - d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh
[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1
)])/#1^2 & ] + d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x -
 #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegr
al[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/(18*b^2
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{d^3 e^{-dx-c}}{6b(b d^3 x^3+d^3 a)} - \frac{\sum_{\_R2=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3c^2b\_Z+d^3a-bc^3\right)} \frac{\left(2\_R2^2bc-3\_R2bc^2-d^3a+bc^3+2\_R2bc\right)e^{-\_R2}\text{Ei}_1\left(\right.}{\_R2^2-2\_R2c+c^2}}{18ab^2}$

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)-1/18/a/b^2*sum((2*_R2^2*b*c-3*_R2*
b*c^2-a*d^3+b*c^3+2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),
_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*c^2/a/b*sum((_R2
-c+2)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*c/a/b*sum((_R2^2-_R2*c+_R2+c)/(_R2^2-2*_
R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)+1/18/a/b^2*sum((2*_R2
^2*b*c-3*_R2*b*c^2-a*d^3+b*c^3-2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1
,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*c^2
/a/b*sum((_R2-c-2)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c/a/b*sum((_R2^2-_R2*c-_R2-
```

```
c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. $2(263) = 526$.

Time = 0.29 (sec) , antiderivative size = 1276, normalized size of antiderivative = 3.42

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*((a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*
d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 +
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(
1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
- 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(1/3)*((b*x
^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 +
 a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1/3)*((b*x^3 +
a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3
))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c
)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d
^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x
 + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 -
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*
d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 -
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/
3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*
(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3)
- 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*((
b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3
/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*((b*x^3 + a)*cosh
(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-
c + (a*d^3/b)^(1/3)) + 12*a*cosh(d*x + c))/((a*b^2*x^3 + a^2*b)*cosh(d*x +
c)^2 - (a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^2} dx = \text{Timed out}$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^2} dx = \int \frac{x^2 \cosh(dx + c)}{(bx^3 + a)^2} dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 4*x*e^(2*c))*e^(d*x) - (d*x^2 - 4*x)*e^(-d*x))/(b^2*d
^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c) + 1/2*integrate(2*(10*b*x^3*e
^c - 3*a*d*x*e^c - 2*a*e^c)*e^(d*x)/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^2*
b*d^2*x^3 + a^3*d^2), x) + 1/2*integrate(2*(10*b*x^3 + 3*a*d*x - 2*a)*e^(-d
*x)/(b^3*d^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*
e^c), x)
```

Giac [F]

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^2} dx = \int \frac{x^2 \cosh(dx + c)}{(bx^3 + a)^2} dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^2} dx = \int \frac{x^2 \cosh(c + d\,x)}{(b\,x^3 + a)^2} dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3)^2,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3)^2, x)
```

3.104 $\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 695

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} + \frac{d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} + \frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} + \frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}$$

```
[Out] -1/9*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-1
/9*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/
```

```
3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1
/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/3*cosh(d*x+
c)/a/b/x-1/3*cosh(d*x+c)/b/x/(b*x^3+a)-1/9*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*d*cosh(c-a^(1/3)*d/b^
(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(
1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*d*Chi(a^(1/3)*d/b^(1/3)
+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a/b-1/9*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a
^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-1/9*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*
x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b-1/9*(-1)^(2/3)*Shi(-(-1)^(1/3)*
a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)
-1/9*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))/a/b+1/9*(-1)^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1
)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used

$= \{5399, 5401, 3378, 3384, 3379, 3382, 5400\}$

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{(-1)^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} - \frac{d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} + \frac{d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} + \frac{\cosh(c+dx)}{3abx}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x) - Cosh[c + d*x]/(3*b*x*(a + b*x^3)) - ((-1)^(2/3)*C
osh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*
a^(4/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a*b) - (d*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b)
 - (d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b) + (d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a*b) + ((-1)^(
2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3
)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d
*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/
3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\sinh(c+dx)}{x(a+bx^3)}\,dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax} - \frac{bx^2\sinh(c+dx)}{a(a+bx^3)}\right)dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{3a} - \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{3ab} - \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{3ab} \\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} \\
&\quad + \frac{\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} \\
&\quad - \frac{d\int \left(\frac{\sinh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{3a} \\
&\quad - \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{3ab} + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{3ab} + \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{3ab}
\end{aligned}
$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} + \frac{d\text{Chi}(dx)\sinh(c)}{3ab} + \frac{d\cosh(c)\text{Shi}(dx)}{3ab} - \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}} - \frac{d\int \frac{\sinh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}} - \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{3ab} - \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{3ab}$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\,(a+bx^3)} - \frac{\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(d\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$+ \frac{\left(\sqrt[3]{-1}\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(id\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$- \frac{\left((-1)^{2/3}\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(id\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$- \frac{\sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(d\sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$+ \frac{\left((-1)^{5/6}\sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(d\sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$+ \frac{\left(\sqrt[6]{-1}\sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\left(d\sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\,(a+bx^3)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} + \frac{d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} + \frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} + \frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.56

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx$$

$$= \frac{6bx^2\cosh(c+dx) + (a+bx^3)\,\mathrm{RootSum}\Big[a+b\#1^3\&,\ \frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(}{}}{}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (6*b*x^2*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[
c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)
] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sin
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ] - (a + b*x^3)*RootSum[a + b
*#1^3 & , (-(Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) - CoshIntegral[d*(x -
 #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d
*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#
1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ]
)/(18*a*b*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.57

method	result
risch	$\frac{d^3 e^{-dx-c} x^2}{6a(b d^3 x^3 + d^3 a)} - \frac{d\left(\sum_{\_R2=\mathrm{RootOf}\left(b\_Z^3 - 3cb\_Z^2 + 3c^2 b\_Z + d^3 a - b c^3\right)} \frac{\left(\_R2^2 - \_R2c + \_R2 + c\right) e^{-\_R2} \mathrm{Ei}_1\left(dx - \_R2 + c\right)}{\_R2^2 - 2\_R2c + c^2}\right)}{18ab}$

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x^2/a/(b*d^3*x^3+a*d^3)-1/18*d/a/b*sum((_R2^2-_R2*c+_R2
+c)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d*c/a/b*sum((_R2-c+2)/(_R2^2-2*_R2*c+c^2)
*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/6*d^3*exp(d*x+c)*x^2/a/(b*d^3*x^3+a*d^3)+1/18*d/a/b*sum((_R2^2-_R2*
c-_R2-c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*d*c/a/b*sum((_R2-c-2)/(_R2^2-2*_R2*c
+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. $2(507) = 1014$.

Time = 0.30 (sec) , antiderivative size = 2135, normalized size of antiderivative = 3.07

$$\int \frac{x \cosh(c + dx)}{(a + bx^3)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*cosh(d*x + c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x +
c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^
3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt
(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (2*(a*b*d^3*x^3 +
a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*
d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 -
(b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) -
 c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3
)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a
*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x
+ c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1
/3)*(sqrt(-3) - 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a
*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b +
sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*
x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((a*b*d^3*x^3 + a^2*d^3
)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(2
/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei
(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*b*d^3*x^3 + a^
2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (a*d^3/b
)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2)
)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (2*(a*b*d^3*x^3 +
a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d
^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (
b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*si
nh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b)
)*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c
)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1
/3)*(sqrt(-3) + 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a
*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + s
qrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x
```

```
^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*s
inh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*co
sh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3
)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 +
a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^
(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((a
*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x +
c)^2 - (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*
sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) +
2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d
*x + c)^2 + (a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a
*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))
/((a^2*b^2*d^2*x^3 + a^3*b*d^2)*cosh(d*x + c)^2 - (a^2*b^2*d^2*x^3 + a^3*b*
d^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\cosh(dx+c)}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^6*e^c + 2*a*b*d*x^3*e^c + a^2*d
*e^c) + 1/2*integrate((5*b*x^3*e^c - a*e^c)*e^(d*x)/(b^3*d*x^9 + 3*a*b^2*d*
x^6 + 3*a^2*b*d*x^3 + a^3*d), x) - 1/2*integrate((5*b*x^3 - a)*e^(-d*x)/(b^
3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a^2*b*d*x^3*e^c + a^3*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx^3+a\right)^2}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\cosh(c+d\,x)}{\left(b\,x^3+a\right)^2}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x*cosh(c + d*x))/(a + b*x^3)^2, x)
```

3.105 $\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 739

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{(-1)^{2/3}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{(-1)^{2/3}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{2\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} + \frac{2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} + \frac{2(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}$$

```
[Out] 2/9*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-2/
9*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(5/3)/b^(1/3)+2/9*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/3*cosh(d*x+c
)/a/b/x^2-1/3*cosh(d*x+c)/b/x^2/(b*x^3+a)+1/9*(-1)^(2/3)*d*cosh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)
+1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)
-1/9*(-1)^(1/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a
^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+2/9*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(
1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/9*(-1)^(2/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b
^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-2/9*(-1)^(
1/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(
1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(1/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*
x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+2/9*(-1)^(2/3)*Shi(
(-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5
/3)/b^(1/3)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{5387, 5401, 3378, 3384, 3379, 3382, 5389, 5400\}$

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{(-1)^{2/3}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{(-1)^{2/3}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{2\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)}$$

```
[In] Int[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x^2) - Cosh[c + d*x]/(3*b*x^2*(a + b*x^3)) - (2*(-1)^(
1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*Cosh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d
*x])/(9*a^(5/3)*b^(1/3)) + (2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (d*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^
(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*d*CoshIntegral[
-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(9*a^(5/3)*b^(1/3)) - ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5387

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)}-\frac{2\int\frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx}{3b}+\frac{d\int\frac{\sinh(c+dx)}{x^2(a+bx^3)}\,dx}{3b}\\
&=-\frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)}-\frac{2\int\left(\frac{\cosh(c+dx)}{ax^3}-\frac{b\cosh(c+dx)}{a(a+bx^3)}\right)dx}{3b}+\frac{d\int\left(\frac{\sinh(c+dx)}{ax^2}-\frac{bx\sinh(c+dx)}{a(a+bx^3)}\right)dx}{3b}\\
&=-\frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)}+\frac{2\int\frac{\cosh(c+dx)}{a+bx^3}\,dx}{3a}-\frac{2\int\frac{\cosh(c+dx)}{x^3}\,dx}{3ab}-\frac{d\int\frac{x\sinh(c+dx)}{a+bx^3}\,dx}{3a}+\frac{d\int\frac{\sinh(c+dx)}{x^2}\,dx}{3ab}
\end{aligned}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{d\sinh(c+dx)}{3abx} + \frac{2\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} - \frac{d\int \left(-\frac{\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a} - \frac{d\int \frac{\sinh(c+dx)}{x^2}dx}{3ab} + \frac{d^2\int \frac{\cosh(c+dx)}{x}dx}{3ab}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{5/3}} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{5/3}} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{5/3}} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{-1}d\right)\int \frac{\sinh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\left((-1)^{2/3}d\right)\int \frac{\sinh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{4/3}\sqrt[3]{b}} - \frac{d^2\int \frac{\cosh(c+dx)}{x}dx}{3ab} + \frac{(d^2\cosh(c))\int \frac{\cosh(dx)}{x}dx}{3ab} + \frac{(d^2\sinh(c))\int \frac{\sinh(dx)}{x}dx}{3ab}$$

$$= \text{Too large to display}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.52

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}dx = \frac{6bx\cosh(c+dx) + (a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&, \frac{2\cosh(c+d\#1)\text{Chi}(d(x-\#1))-2\text{Chi}(d(x-\#1))\sinh(c+d\#1)-2}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] (6*b*x*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x
 - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
```

```
d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (a + b*x^3)*RootS
um[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshInteg
ral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
- 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral
[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c +
d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #
1)]*#1)/#1^2 & ])/(18*a*b*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.31

method	result
risch	$\frac{d^{3} \mathrm{e}^{-d x -c} x}{6 a \left(b \,d^{3} x^{3}+d^{3} a \right)}-\frac{d^{2} \left(\sum_{\_R2 =\operatorname{RootOf}\left(b \,\_Z^{3}-3 c b \,\_Z^{2}+3 c^{2} b \,\_Z +d^{3} a -b \,c^{3}\right)}\frac{\left(\_R2 -c +2\right) \mathrm{e}^{-\_R2} \operatorname{Ei}_{1}\left(d x -\_R2 +c \right)}{\_R2^{2}-2 \_R2 c +c^{2}}\right)}{18 a b}+\frac{d^{3} \mathrm{e}^{d x +c}}{6 a \left(b \,d^{3} x^{3}+\right.}$

```
[In] int(cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x/a/(b*d^3*x^3+a*d^3)-1/18*d^2/a/b*sum((_R2-c+2)/(_R2^2
-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))+1/6*d^3*exp(d*x+c)*x/a/(b*d^3*x^3+a*d^3)+1/18*d^2/a/b*su
m((_R2-c-2)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. 2(519) = 1038.

Time = 0.29 (sec) , antiderivative size = 2048, normalized size of antiderivative = 2.77

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 +
a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)
^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((
-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^
3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b*x^3
+ sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a
```

```
) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cos
h(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqr
t(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)
*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*c
osh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*
x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
- 1) - c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x
+ c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)
^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(
-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt
(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2
/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(-a*d^3
/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-
d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3
)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*
d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(
-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*s
inh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cos
h(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x
- 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1) + c) + ((-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x +
c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*(
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x
^3 + a) + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3
 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x +
c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3
)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) +
a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) +
2*(-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b
*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((
-a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)
 + 2*(-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x +
 c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/
b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(a
*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*
Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/((a^2*b*d*x^3 + a^3*d
)*cosh(d*x + c)^2 - (a^2*b*d*x^3 + a^3*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{\cosh(dx+c)}{(bx^3+a)^2} \, dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{\cosh(dx+c)}{(bx^3+a)^2} \, dx$$

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} \, dx = \int \frac{\cosh(c+d\,x)}{(b\,x^3+a)^2} \, dx$$

```
[In] int(cosh(c + d*x)/(a + b*x^3)^2,x)
[Out] int(cosh(c + d*x)/(a + b*x^3)^2, x)
```

3.106 $\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 697

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx &= \frac{\cosh(c+dx)}{3abx^3}-\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}\\
&-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}\\
&-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}\\
&-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}\\
&-\frac{d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}\\
&+\frac{\sqrt[3]{-1}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}\\
&-\frac{(-1)^{2/3}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}\\
&+\frac{\sinh(c)\text{Shi}(dx)}{a^2}\\
&-\frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}\\
&+\frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}\\
&-\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}\\
&-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}\\
&-\frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}\\
&-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}\end{aligned}$$

```
[Out] Chi(d*x)*cosh(c)/a^2-1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3
))/a^2-1/3*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a^2-1/3*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)
*a^(1/3)*d/b^(1/3))/a^2+1/3*cosh(d*x+c)/a/b/x^3-1/3*cosh(d*x+c)/b/x^3/(b*x^
3+a)+1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*
a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(
a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a
^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+Shi
(d*x)*sinh(c)/a^2-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3)
)/a^(5/3)/b^(1/3)-1/3*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/
a^2+1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+
d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-1/9*(-1)^(2/3)*d*Chi(-(-1)^(2
/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(
1/3)-1/3*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/
b^(1/3))/a^2
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used

$= \{5399, 5401, 3378, 3384, 3379, 3382, 5400, 5388\}$

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx = -\frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} + \frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} - \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c+dx)}{3abx^3}$$

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*
CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])
/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a
^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) +
(Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3
)*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a
^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) - ((-1)
^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^4(a+bx^3)}\,dx}{b} + \frac{d\int \frac{\sinh(c+dx)}{x^3(a+bx^3)}\,dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^4} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2x^2\cosh(c+dx)}{a^2(a+bx^3)}\right)dx}{b} \\
&\quad + \frac{d\int \left(\frac{\sinh(c+dx)}{ax^3} - \frac{b\sinh(c+dx)}{a(a+bx^3)}\right)dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{\int \frac{\cosh(c+dx)}{x^4}\,dx}{ab} \\
&\quad - \frac{b\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\sinh(c+dx)}{x^3}\,dx}{3ab}
\end{aligned}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{6abx^2}$$
$$- \frac{b\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{a^2}$$
$$- \frac{d\int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3a}$$
$$- \frac{d\int \frac{\sinh(c+dx)}{x^3}dx}{3ab} + \frac{d^2\int \frac{\cosh(c+dx)}{x^2}dx}{6ab} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}dx}{a^2} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}dx}{a^2}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{d^2\cosh(c+dx)}{6abx} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2}$$
$$+ \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^2} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^2}$$
$$- \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^2} + \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{9a^{5/3}} + \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{9a^{5/3}}$$
$$+ \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{9a^{5/3}} - \frac{d^2\int \frac{\cosh(c+dx)}{x^2}dx}{6ab} + \frac{d^3\int \frac{\sinh(c+dx)}{x}dx}{6ab}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2}$$

$$+ \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d^3 \int \frac{\sinh(c+dx)}{x}\,dx}{6ab} + \frac{\left(d^3\cosh(c)\right)\int \frac{\sinh(dx)}{x}\,dx}{6ab}$$

$$- \frac{\left(\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$+ \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$+ \frac{\left(id\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$+ \frac{\left(id\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$+ \frac{\left(d^3\sinh(c)\right)\int \frac{\cosh(dx)}{x}\,dx}{6ab} - \frac{\left(\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$+ \frac{\left(d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(i\sqrt[3]{b}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$+ \frac{\left(d\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$- \frac{\left(i\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} + \frac{d^3\text{Chi}(dx)\sinh(c)}{6ab} - \frac{d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{d^3\cosh(c)\text{Shi}(dx)}{6ab} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} - \frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2} - \frac{(d^3\cosh(c))\int\frac{\sinh(dx)}{x}\,dx}{6ab} - \frac{(d^3\sinh(c))\int\frac{\cosh(dx)}{x}\,dx}{6ab}$$

$$=\frac{\cosh(c+dx)}{3abx^3}-\frac{\cosh(c+dx)}{3bx^3(a+bx^3)}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}$$
$$-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}$$
$$-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}$$
$$-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}-\frac{d\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{\sqrt[3]{-1}d\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$-\frac{(-1)^{2/3}d\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}-\frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$+\frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}$$
$$-\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}$$
$$-\frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}$$
$$-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.59

$$\int\frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx$$

$$=\frac{\frac{6a\cosh(c)\cosh(dx)}{a+bx^3}+18\cosh(c)\mathrm{Chi}(dx)-3\mathrm{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*Cosh[c]*Cosh[d*x])/(a + b*x^3) + 18*Cosh[c]*CoshIntegral[d*x] - 3*Roo
tSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[
d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)] & ] - 3*RootSum[a + b*#1^3 & , Cosh[c +
 d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] +
 Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x
 - #1)] & ] + (a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x
 - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhInte
gral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/b - (
a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshI
ntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/b + (6*a*Sinh[c]*Sin
h[d*x])/(a + b*x^3) + 18*Sinh[c]*SinhIntegral[d*x])/(18*a^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.48

method	result
risch	$\frac{e^{-dx-c} d^{3}}{6a\left(b(dx+c)^{3}-3(dx+c)^{2}bc+3(dx+c)b\,c^{2}+d^{3}a-b\,c^{3}\right)}-\frac{e^{-c}\,\mathrm{Ei}_{1}(dx)}{2a^{2}}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3cb\_Z^{2}+3c^{2}b\_Z+d^{3}a-b\,c^{3}\right)}\frac{\left(-d^{3}a+3\right.}{}}{18a^{2}}$

```
[In] int(cosh(d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*exp(-d*x-c)*d^3/a/(b*(d*x+c)^3-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+d^3*a-b*
c^3)-1/2/a^2*exp(-c)*Ei(1,d*x)+1/18/a^2/b*sum((-a*d^3+3*_R1^2*b-6*_R1*b*c+3
*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*exp(d*x+c)*d^3/a/(b*(d*x+c)^3-3*(d*x+c
)^2*b*c+3*(d*x+c)*b*c^2+d^3*a-b*c^3)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/18/a^2/b*s
um((a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d
*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(509) = 1018$.

Time = 0.28 (sec) , antiderivative size = 1773, normalized size of antiderivative = 2.54

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*((6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*
d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 +
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (6*(b*x^3 + a
)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^
3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 +
a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^
2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x
^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x
+ c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1
/3)*(sqrt(-3) - 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*si
nh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x +
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1) + c) + 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2
+ (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)
^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(3*(b*x^3 +
 a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b*x
^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^
(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x
^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) +
a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*E
i(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) + 1) + c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c
)^2 - (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^
3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) -
(6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^
(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-
3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (6*(b*x^3 + a)*cosh(
d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqr
t(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)
*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*
```

```
(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3
*(b*x^3 + a)*sinh(d*x + c)^2 + (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^
2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*
d^3/b)^(1/3)) - 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x +
 c)^2 + (a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x
 + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*a*cosh(
d*x + c) - 18*((b*x^3 + a)*Ei(d*x) + (b*x^3 + a)*Ei(-d*x))*cosh(c) - 18*((b
*x^3 + a)*Ei(d*x) - (b*x^3 + a)*Ei(-d*x))*sinh(c))/((a^2*b*x^3 + a^3)*cosh(
d*x + c)^2 - (a^2*b*x^3 + a^3)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^3+a)^2 x}\,dx$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)^2}\,dx = \int \frac{\cosh(c+d\,x)}{x\,(b\,x^3+a)^2}\,dx$$

```
[In] int(cosh(c + d*x)/(x*(a + b*x^3)^2),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^3)^2), x)
```

3.107 $\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 784

$$\int \frac{x^5\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{x^3\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)}$$
$$- \frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}b^{8/3}}$$
$$+ \frac{\sqrt[3]{-1}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}b^{8/3}}$$
$$- \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54\sqrt[3]{a}b^{8/3}}$$
$$+ \frac{2d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}}$$
$$- \frac{2\sqrt[3]{-1}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}}$$
$$+ \frac{2(-1)^{2/3}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}}$$
$$- \frac{dx\sinh(c+dx)}{18b^2(a+bx^3)}$$
$$+ \frac{2\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{2/3}b^{7/3}}$$
$$+ \frac{(-1)^{2/3}d^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}b^{8/3}}$$
$$+ \frac{2d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{2/3}b^{7/3}}$$
$$- \frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54\sqrt[3]{a}b^{8/3}}$$
$$+ \frac{2(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{2/3}b^{7/3}}$$
$$+ \frac{\sqrt[3]{-1}d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54\sqrt[3]{a}b^{8/3}}$$

```
[Out] -1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8
/3)-1/54*(-1)^(2/3)*d^2*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)+1/54*(-1)^(1/3)*d^2*Chi(-(-1)^(2/3)
*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3
)-1/6*x^3*cosh(d*x+c)/b/(b*x^3+a)^2-1/6*cosh(d*x+c)/b^2/(b*x^3+a)-2/27*(-1)
^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(
1/3)+d*x)/a^(2/3)/b^(7/3)+2/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^
(1/3)+d*x)/a^(2/3)/b^(7/3)+2/27*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^
(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(7/3)+2/27*d*Chi(a^(
1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)-1/54*d^2*Shi(
a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)-2/27*(-1)^
(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b
^(1/3))/a^(2/3)/b^(7/3)-1/54*(-1)^(2/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/
3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)+2/27*(-1)^(2/3
)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))/a^(2/3)/b^(7/3)+1/54*(-1)^(1/3)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d
*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)-1/18*d*x*sinh(d*x+
c)/b^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used

$= \{5399, 5397, 5388, 3384, 3379, 3382, 5398, 5401\}$

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{2d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} - \frac{2\sqrt[3]{-1}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{2/3}b^{7/3}} + \frac{2(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} + \frac{2\sqrt[3]{-1}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{2/3}b^{7/3}} + \frac{2d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} + \frac{2(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} - \frac{(-1)^{2/3}d^2\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}b^{8/3}} + \frac{\sqrt[3]{-1}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54\sqrt[3]{a}b^{8/3}} - \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54\sqrt[3]{a}b^{8/3}} + \frac{(-1)^{2/3}d^2\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}b^{8/3}} - \frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54\sqrt[3]{a}b^{8/3}} + \frac{\sqrt[3]{-1}d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54\sqrt[3]{a}b^{8/3}} - \frac{dx\sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{x^3\cosh(c+dx)}{6b(a+bx^3)^2}$$

```
[In] Int[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x^3*Cosh[c + d*x])/(b*(a + b*x^3)^2) - Cosh[c + d*x]/(6*b^2*(a + b*x^
3)) - ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + ((-1)^(1/3)*
d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b
^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*
d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(2
7*a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^
(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3))
 + (2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c
+ d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(2/3
)*b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhI
ntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*
Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)
*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8
/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

```
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3\cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx}{2b} + \frac{d\int \frac{x^3\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b} \\
&= -\frac{x^3\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx\sinh(c+dx)}{18b^2(a+bx^3)} \\
&\quad + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{18b^2} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{6b^2} + \frac{d^2\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{18b^2}
\end{aligned}
$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{dx\sinh(c+dx)}{18b^2\left(a+bx^3\right)} + \frac{d\int\left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18b^2} + \frac{d\int\left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{6b^2} + \frac{d^2\int\left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18b^2}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{dx\sinh(c+dx)}{18b^2\left(a+bx^3\right)} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{54a^{2/3}b^2} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54a^{2/3}b^2} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{54a^{2/3}b^2} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{18a^{2/3}b^2} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{18a^{2/3}b^2} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{18a^{2/3}b^2} - \frac{d^2\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54\sqrt[3]{a}b^{7/3}} + \frac{\left(\sqrt[3]{-1}d^2\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{54\sqrt[3]{a}b^{7/3}} - \frac{\left((-1)^{2/3}d^2\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54\sqrt[3]{a}b^{7/3}}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.51

$$\int\frac{x^5\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx = \frac{d\text{RootSum}\left[a+b\#1^3\&,\frac{-4\cosh(c+d\#1)\text{Chi}(d(x-\#1))+4\text{Chi}(d(x-\#1))\sinh(c+d\#1)+4\cosh(c+d\#1)\text{Shi}(d(x-\#1))-4\text{si}}{}\right.}{}$$

```
[In] Integrate[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*RootSum[a + b*#1^3 & , (-4*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 4*C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c + d*#1]*SinhIntegral[d*(x
 - #1)] - 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*Cosh
```

```
Integral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[
d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (4*Cosh[c + d*#1]*CoshI
ntegral[d*(x - #1)] + 4*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c
+ d*#1]*SinhIntegral[d*(x - #1)] + 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)
] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sin
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*(3*(a + 2*b*x^3)*C
osh[c + d*x] + d*x*(a + b*x^3)*Sinh[c + d*x]))/(a + b*x^3)^2)/(108*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 6243, normalized size of antiderivative = 7.96

method	result	size
risch	Expression too large to display	6243

```
[In] int(x^5*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. 2(562) = 1124.

Time = 0.31 (sec) , antiderivative size = 2980, normalized size of antiderivative = 3.80

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2
*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 +
 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
(b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
 + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^
3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
```

```
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - s
qrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((-
a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6
 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2)
)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh
(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3
)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2
*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*
a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^
2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d
*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt
(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 +
2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (
b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
+ c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 +
 sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b^2*x^
6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2
 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(
d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^
2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sq
rt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3
```

```
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2
*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6
+ 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*
Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)
*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x
 + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^
(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 36*(2*a*b*x^3 + a^2)*cosh(d*x + c) - 12
*(a*b*d*x^4 + a^2*d*x)*sinh(d*x + c))/((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2
)*cosh(d*x + c)^2 - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^5 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b*d^4*x^5*e^(2*c) + 4*b*d^3*x^4*e^(2*c) + 20*b*d^2*x^3*e^(2*c) + 120*
b*d*x^2*e^(2*c) - 3*(3*a*d^3*e^(2*c) - 280*b*e^(2*c))*x)*e^(d*x) - (b*d^4*x
^5 - 4*b*d^3*x^4 + 20*b*d^2*x^3 - 120*b*d*x^2 + 3*(3*a*d^3 + 280*b)*x)*e^(-
d*x))/(b^4*d^5*x^9*e^c + 3*a*b^3*d^5*x^6*e^c + 3*a^2*b^2*d^5*x^3*e^c + a^3*
b*d^5*e^c) - 1/2*integrate(3*(60*a*b*d^2*x^2*e^c - 3*a^2*d^3*e^c + 4*(9*a*b
*d^3*e^c - 560*b^2*e^c)*x^3 + 280*a*b*e^c - 3*(a^2*d^4*e^c - 120*a*b*d*e^c)
*x)*e^(d*x)/(b^5*d^5*x^12 + 4*a*b^4*d^5*x^9 + 6*a^2*b^3*d^5*x^6 + 4*a^3*b^2
*d^5*x^3 + a^4*b*d^5), x) - 1/2*integrate(-3*(60*a*b*d^2*x^2 + 3*a^2*d^3 -
4*(9*a*b*d^3 + 560*b^2)*x^3 + 280*a*b - 3*(a^2*d^4 + 120*a*b*d)*x)*e^(-d*x)
/(b^5*d^5*x^12*e^c + 4*a*b^4*d^5*x^9*e^c + 6*a^2*b^3*d^5*x^6*e^c + 4*a^3*b^
2*d^5*x^3*e^c + a^4*b*d^5*e^c), x)
```

Giac [F]

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^5 \cosh(dx+c)}{(bx^3+a)^3} \, dx$$

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^5*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^5 \cosh(c+d\,x)}{(b\,x^3+a)^3} \, dx$$

```
[In] int((x^5*cosh(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x^5*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.108 $\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1105

$$\int \frac{x^4\cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x\,(a+bx^3)}$$

$$- \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{4/3}b^{5/3}}$$

$$- \frac{\sqrt[3]{-1}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{2/3}b^{7/3}}$$

$$+ \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{4/3}b^{5/3}}$$

$$+ \frac{(-1)^{2/3}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{2/3}b^{7/3}}$$

$$- \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{4/3}b^{5/3}}$$

$$+ \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{2/3}b^{7/3}}$$

$$- \frac{d\,\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27ab^2}$$

$$- \frac{d\,\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27ab^2}$$

$$- \frac{d\,\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27ab^2}$$

$$- \frac{d\sinh(c+dx)}{18b^2\,(a+bx^3)} + \frac{d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27ab^2}$$

$$+ \frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{4/3}b^{5/3}}$$

$$+ \frac{\sqrt[3]{-1}d^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{2/3}b^{7/3}}$$

$$- \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27ab^2}$$

$$- \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{4/3}b^{5/3}}$$

$$d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)$$

```
[Out] -1/27*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)+
1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/
3)-1/27*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*
a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(1/3)*d^2*Chi((-1)^(1/3)*a^(1/
3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)+1/27
*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3
)*d/b^(1/3))/a^(4/3)/b^(5/3)+1/54*(-1)^(2/3)*d^2*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)+1/9*cosh(
d*x+c)/a/b^2/x-1/6*x^2*cosh(d*x+c)/b/(b*x^3+a)^2-1/9*cosh(d*x+c)/b^2/x/(b*x
^3+a)-1/27*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d
/b^(1/3)+d*x)/a/b^2-1/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+
d*x)/a/b^2-1/27*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)/a/b^2-1/27*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/
b^(1/3))/a/b^2-1/27*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^
(4/3)/b^(5/3)+1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))
/a^(2/3)/b^(7/3)-1/27*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(2/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(
1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(1
/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/a^(2/3)/b^(7/3)-1/27*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/27*(-1)^(1/3)*Shi((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)+1/54
*(-1)^(2/3)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(
1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)-1/18*d*sinh(d*x+c)/b^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 1105, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{5399, 5401, 3378, 3384, 3379, 3382, 5400, 5396, 5389\}$

$$\int \frac{x^4\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^{2/3}b^{7/3}}$$

$$+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{2/3}b^{7/3}}$$

$$+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{2/3}b^{7/3}}$$

$$+\frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^{2/3}b^{7/3}}$$

$$+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{2/3}b^{7/3}}$$

$$+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{2/3}b^{7/3}}$$

$$-\frac{\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27ab^2}$$

$$-\frac{\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27ab^2}$$

$$-\frac{\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27ab^2}$$

$$-\frac{\sinh(c+dx)d}{18b^2\left(bx^3+a\right)}+\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{27ab^2}$$

$$-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27ab^2}$$

$$-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27ab^2}$$

$$+\frac{\cosh(c+dx)}{9ab^2x}-\frac{\cosh(c+dx)}{9b^2x\left(bx^3+a\right)}-\frac{x^2\cosh(c+dx)}{6b\left(bx^3+a\right)^2}$$

$$-\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{4/3}b^{5/3}}$$

$$+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{4/3}b^{5/3}}$$

$$\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)$$

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] Cosh[c + d*x]/(9*a*b^2*x) - (x^2*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[
c + d*x]/(9*b^2*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*
b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) + ((-1)^(
1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(54*a^(2/3)*b^(7/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + (d^2*Cosh[c - (a^(1/3
)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(2/3)*b^(7/3))
 - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]
)/(27*a*b^2) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a*b^2) - (d*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(2
7*a*b^2) - (d*Sinh[c + d*x])/(18*b^2*(a + b*x^3)) + (d*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(2
7*a*b^2) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(1/3)
*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) - (Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3))
 + (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) + ((-1)^(1/3)*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Sinh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(5
4*a^(2/3)*b^(7/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5396

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
```

```
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{\int\frac{x\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{3b}+\frac{d\int\frac{x^2\sinh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$=-\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)}-\frac{\int\frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{9b^2}+\frac{d\int\frac{\sinh(c+dx)}{x\left(a+bx^3\right)}\,dx}{9b^2}+\frac{d^2\int\frac{\cosh(c+dx)}{a+bx^3}\,dx}{18b^2}$$

$$=-\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)}-\frac{\int\left(\frac{\cosh(c+dx)}{ax^2}-\frac{bx\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2}+\frac{d\int\left(\frac{\sinh(c+dx)}{ax}-\frac{bx^2\sinh(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2}+\frac{d^2\int\left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18b^2}$$

$$=-\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)}-\frac{\int\frac{\cosh(c+dx)}{x^2}\,dx}{9ab^2}+\frac{\int\frac{x\cosh(c+dx)}{a+bx^3}\,dx}{9ab}+\frac{d\int\frac{\sinh(c+dx)}{x}\,dx}{9ab^2}-\frac{d\int\frac{x^2\sinh(c+dx)}{a+bx^3}\,dx}{9ab}-\frac{d^2\int\frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}-\frac{d^2\int\frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}-\frac{d^2\int\frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2(a+bx^3)}$$

$$+ \frac{\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{9ab}$$

$$- \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{9ab^2}$$

$$- \frac{d\int \left(\frac{\sinh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{9ab}$$

$$+ \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{9ab^2} - \frac{\left(d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$- \frac{\left(d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$- \frac{\left(d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$+ \frac{(d\sinh(c))\int \frac{\cosh(dx)}{x}\,dx}{9ab^2} - \frac{\left(d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$- \frac{\left(id^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$- \frac{\left(id^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[6]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{\sqrt[3]{-1}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{2/3}b^{7/3}} + \frac{(-1)^{2/3}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{2/3}b^{7/3}} + \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{2/3}b^{7/3}} + \frac{d\text{Chi}(dx)\sinh(c)}{9ab^2} - \frac{d\sinh(c+dx)}{18b^2(a+bx^3)} + \frac{d\cosh(c)\text{Shi}(dx)}{9ab^2} + \frac{\sqrt[3]{-1}d^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{2/3}b^{7/3}} + \frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{2/3}b^{7/3}} + \frac{(-1)^{2/3}d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{2/3}b^{7/3}} - \frac{\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{4/3}b^{4/3}} + \frac{\sqrt[3]{-1}\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{27a^{4/3}b^{4/3}} - \frac{(-1)^{2/3}\int\frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{27a^{4/3}b^{4/3}} - \frac{d\int\frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27ab^{5/3}} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27ab^{5/3}} - \frac{d\int\frac{\sinh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27ab^{5/3}} - \frac{(d\cosh(c))\int\frac{\sinh(dx)}{x}dx}{9ab^2} - \frac{(d\sinh(c))\int\frac{\cosh(dx)}{x}dx}{9ab^2}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.61

$$\int \frac{x^4\cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{\text{RootSum}\left[a+b\#1^3\&, \frac{ad^2\cosh(c+d\#1)\text{Chi}(d(x-\#1))-ad^2\text{Chi}(d(x-\#1))\sinh(c+d\#1)-ad^2\cosh(c+d\#1)\text{Shi}(d(x-\#1))+a\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - a*
d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - a*d^2*Cosh[c + d*#1]*SinhInte
gral[d*(x - #1)] + a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*b*Cosh
[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[
c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*Sinh[c
+ d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(
x - #1)]*#1^2 - 2*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 2*b*d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c +
 d*#1]*CoshIntegral[d*(x - #1)]) - a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c +
d*#1] - a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)] - 2*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1
 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1 - 2*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 2*b*d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1^2 + 2*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*
b*Cosh[d*x]*(b*x^2*(-a + 2*b*x^3)*Cosh[c] - a*d*(a + b*x^3)*Sinh[c]))/(a +
b*x^3)^2 + (6*b*(-(a*d*(a + b*x^3)*Cosh[c]) + b*x^2*(-a + 2*b*x^3)*Sinh[c])
*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 4708, normalized size of antiderivative = 4.26

method	result	size
risch	Expression too large to display	4708

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108/d^2*(-3*d^3*exp(d*x+c)*a^3*b-3*exp(d*x+c)*a^2*b^2*d^3*x^3+sum((4*_R1^
2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3
*d^3-b^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^
4-10*a*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*
c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*x^6+sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-
2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*d
^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^
5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*e
xp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))*b^2*x^6+sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^
2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*
a*b*c*d^3-26*_R1*b^2*c^4-10*a*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2
*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_
```

R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2+sum((4*_R1^2*a*b*c
*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^
2*c^6-2*_R1^2*a*b*d^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*
b*c^2*d^3-10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_
R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))*a^2+3*exp(-d*x-c)*a^2*b^2*d^3*x^3-sum((_R1^2-2*_R1*c
+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^4*c^4*x^6-2*sum((_R1^2-2*_R1*
c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^3*c^4*x^3+6*exp(d*x+c)*a*b
^3*d^2*x^5+8*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b
*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^3*x^3-12*sum((_R1^2*b*
c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*
b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=
RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^2*x^3-3*exp(d*x+c
)*a^2*b^2*d^2*x^2-8*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*
c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*
a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c*x^3+3*d^3*exp(-d*x-c)*a^3*b
-3*exp(-d*x-c)*a^2*b^2*d^2*x^2-12*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*
c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R
1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^2*x^3-8*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*
d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c
^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1
+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c*x^3+8*sum
((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b
*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^3*x^3+4*sum((_R1^2*b*c-2*_R1*b*c^2-a*d
^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp
(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)*b^3*c^3*x^6-6*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^
2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*e
xp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))*b^3*c^2*x^6-4*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^
2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*
d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c*x^6-sum((_R1^2-2*_R1*c+c^2-6*_R
1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b^2*c^4+4*sum((_R1^2*b*c-2*_R1*b*c^2
-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)
*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))*a^2*b*c^3+2*sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*
_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-

4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_
R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R
1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*x^3-6*sum((_
R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a
*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-
c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c^2-4*sum((_
R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c
^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R
1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+
a*d^3-b*c^3))*a^2*b*c+6*exp(-d*x-c)*a*b^3*d^2*x^5-2*sum((_R1^2-2*_R1*c+c^2+
6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^3*c^4*x^3-sum((_R1^2-2*_R1*c+c^2
+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^4*c^4*x^6+4*sum((_R1^2*b*c-2*_R1*
b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c
+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))*b^3*c^3*x^6-6*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c
^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1
*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a
*d^3-b*c^3))*b^3*c^2*x^6-4*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b
*c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b
*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=Roo
tOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c*x^6-sum((_R1^2-2*_R1*c
+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b^2*c^4+4*sum((_R1^2*b*c-2*
_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))*a^2*b*c^3+2*sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c
^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*d^3-16*_R1^2
*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^5-2*_R1*a*b
*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei
(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*x^3
-6*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*
b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1
,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c^2
-4*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12*
_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_
R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c)/a^2/b^3/(b^2*x^6+2*a*b*x^3+a^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4691 vs. $2(805) = 1610$.

Time = 0.34 (sec) , antiderivative size = 4691, normalized size of antiderivative = 4.25

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] -1/216*((4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(
2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^
2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6
+ 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqr
t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*Ei(d*x
 - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(-a*d^3/
b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*((a*b^2*d^3
*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3
+ sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*E
i(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sq
rt(-3) + 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x
+ c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*
(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b
^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3
)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*
d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^
3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)
^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d
*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 +
 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2
*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*
((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x
```

```
 + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) - 1) + c) - 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*(
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^
6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3)
)*cosh(c + (-a*d^3/b)^(1/3)) + 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d
*x + c)^2 + 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((
a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*c
osh(-c + (a*d^3/b)^(1/3)) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x +
c)^2 - 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^
6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
 - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1
/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 +
 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh
(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x
 + c)^2 + 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^
3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/
b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*
x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*
b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))
*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
sinh(d*x + c)^2 - 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (
a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^
2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 +
2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + s
qrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b
^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2)
```

```
 + (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x
^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(2*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
- (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x +
c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*
x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(2*(a*b^2*d^3*x^6 + 2*
a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
- (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c
)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x
+ (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(2*a*b^2*d^2*x^5 - a^2*b
*d^2*x^2)*cosh(d*x + c) + 12*(a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c))/((a^2
*b^4*d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*cosh(d*x + c)^2 - (a^2*b^4*
d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^4 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/2*((d^3*x^4*e^(2*c) + 5*d^2*x^3*e^(2*c) + 30*d*x^2*e^(2*c) + 210*x*e^(2*c
))*e^(d*x) - (d^3*x^4 - 5*d^2*x^3 + 30*d*x^2 - 210*x)*e^(-d*x))/(b^3*d^4*x^
9*e^c + 3*a*b^2*d^4*x^6*e^c + 3*a^2*b*d^4*x^3*e^c + a^3*d^4*e^c) - 1/2*inte
grate(3*(15*a*d^2*x^2*e^c + (3*a*d^3*e^c - 560*b*e^c)*x^3 + 90*a*d*x*e^c +
70*a*e^c)*e^(d*x)/(b^4*d^4*x^12 + 4*a*b^3*d^4*x^9 + 6*a^2*b^2*d^4*x^6 + 4*a
```

```
^3*b*d^4*x^3 + a^4*d^4), x) + 1/2*integrate(-3*(15*a*d^2*x^2 - (3*a*d^3 + 5
60*b)*x^3 - 90*a*d*x + 70*a)*e^(-d*x)/(b^4*d^4*x^12*e^c + 4*a*b^3*d^4*x^9*e
^c + 6*a^2*b^2*d^4*x^6*e^c + 4*a^3*b*d^4*x^3*e^c + a^4*d^4*e^c), x)
```

Giac [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^4 \cosh\left(dx+c\right)}{\left(bx^3+a\right)^3}\,dx$$

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{\left(b\,x^3+a\right)^3}\,dx$$

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.109 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 776

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}$$
$$- \frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$- \frac{d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$+ \frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$- \frac{d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$+ \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}$$
$$- \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2}$$
$$+ \frac{d\sinh(c+dx)}{18ab^2x} - \frac{d\sinh(c+dx)}{18b^2x\,(a+bx^3)}$$
$$+ \frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$+ \frac{d^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$+ \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}$$
$$- \frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2}$$
$$+ \frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}$$
$$- \frac{d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54ab^2}$$

```
[Out] 1/27*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1
/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1
)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b
^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh
(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/27*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1
/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/5
4*d^2*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^
(1/3))/a/b^2+1/18*cosh(d*x+c)/a/b^2/x^2-1/6*x*cosh(d*x+c)/b/(b*x^3+a)^2-1/1
8*cosh(d*x+c)/b^2/x^2/(b*x^3+a)+1/27*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1
/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a
^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(1/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d
*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Shi(-(-1)
^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/
27*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x
)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/18*d*sinh(d*x+c)/a/b^2/x-1/1
8*d*sinh(d*x+c)/b^2/x/(b*x^3+a)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules

used = {5399, 5387, 5401, 3378, 3384, 3379, 3382, 5389, 5400, 5398}

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$
$$-\frac{d^2\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$-\frac{d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$-\frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$+\frac{d^2\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54ab^2}$$
$$-\frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$-\frac{d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54ab^2}$$
$$-\frac{d\sinh(c+dx)}{18b^2x(a+bx^3)}+\frac{\cosh(c+dx)}{18ab^2x^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}$$
$$+\frac{d\sinh(c+dx)}{18ab^2x}-\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}$$

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] Cosh[c + d*x]/(18*a*b^2*x^2) - (x*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh
[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5
/3)*b^(4/3)) - (d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + ((-1)^(2/3)*Cosh[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
 - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a*b^2) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(
5/3)*b^(4/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(54*a*b^2) + (d*Sinh[c + d*x])/(18*a*b^2*x) - (d*Sinh[c +
d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^
(4/3)) + (d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]
*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[
c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2
) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(54*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5387

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}+\frac{d\int \frac{x\sinh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$=-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2}+\frac{d^2\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx}{18b^2}$$

$$=-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{\int \left(\frac{\cosh(c+dx)}{ax^3}-\frac{b\cosh(c+dx)}{a(a+bx^3)}\right)dx}{9b^2}+\frac{d^2\int \left(\frac{\cosh(c+dx)}{ax}-\frac{bx^2\cosh(c+dx)}{a(a+bx^3)}\right)dx}{18b^2}$$

$$=-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{9ab^2}+\frac{\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{9ab}+\frac{d^2\int \frac{\cosh(c+dx)}{x}\,dx}{18ab^2}-\frac{d^2\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx}{18ab}$$

$$=\frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\cosh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\sinh(c+dx)}{18b^2x\left(a+bx^3\right)}+\frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9ab}-\frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{18ab^2}-\frac{d^2\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{18ab}+\frac{\left(d^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx}{18ab^2}+\frac{\left(d^2\sinh(c)\right)\int \frac{\sinh(dx)}{x}\,dx}{18ab^2}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} + \frac{d^2\cosh(c)\text{Chi}(dx)}{18ab^2} + \frac{d\sinh(c+dx)}{18ab^2x} - \frac{d\sinh(c+dx)}{18b^2x(a+bx^3)} + \frac{d^2\sinh(c)\text{Shi}(dx)}{18ab^2} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{d^2\int \frac{\cosh(c+dx)}{x}\,dx}{18ab^2} - \frac{d^2\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}} - \frac{d^2\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}} - \frac{d^2\int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54ab^{5/3}}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.55

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\text{RootSum}\left[a+b\#1^3\&, \frac{-2\cosh(c+d\#1)\text{Chi}(d(x-\#1))+2\text{Chi}(d(x-\#1))\sinh(c+d\#1)+2\cosh(c+d\#1)\text{Shi}(d(x-\#1))-2}{}\right.}{}$$

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
+ 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 2*Cosh[c + d*#1]*SinhIntegral
[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d^2*Cosh[c + d*#
1]*CoshIntegral[d*(x - #1)]*#1^2 - d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*
#1]*#1^2 - d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c +
d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (-2*
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c
 + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*Sin
hIntegral[d*(x - #1)] + d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 +
d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + d^2*Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2
)/#1^2 & ] - (6*b*x*((-2*a + b*x^3)*Cosh[c + d*x] + d*x*(a + b*x^3)*Sinh[c
+ d*x]))/(a + b*x^3)^2)/(a*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 3281, normalized size of antiderivative = 4.23

method	result	size
risch	Expression too large to display	3281

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108/d*(2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b
*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b
*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*x^3+3*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b
*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c
^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*c+sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3
+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-
12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c)
,_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*x^6-3*sum((_R1^2
*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R
1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))*a^2*c^2+3*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*
d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^
2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*
b*c^2+a*d^3-b*c^3))*a^2*c+sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*
c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*
c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*x^6-3*sum((_R1^2*b*c-2*_R
1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1
*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a
*d^3-b*c^3))*a^2*c^2+sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a
*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2
*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2+sum((_R1^2*a*d^3-_R1^2*b*c^3+_
R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^
3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,
d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2-3*exp(
-d*x-c)*a^2*b*d^2*x^2-6*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*
_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c
),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*c^2*x^3+6*sum((
_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*
a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1
+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*c*x^3-6*exp(-
d*x-c)*a^2*b*d*x-3*exp(-d*x-c)*a*b^2*d^2*x^5+2*sum((_R1^2-2*_R1*c+c^2+6*_R1
```

```
-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^3*x^3+3*exp(-d*x-c)*a*b^2*d*x^4+3
*exp(d*x+c)*a^2*b*d^2*x^2+6*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+
b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*
_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^
2+a*d^3-b*c^3))*a*b*c*x^3-6*exp(d*x+c)*a^2*b*d*x+3*exp(d*x+c)*a*b^2*d^2*x^5
+2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-
d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^3*
x^3+3*exp(d*x+c)*a*b^2*d*x^4-6*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1
^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*
x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*c^2*x^3-
3*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*
b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*c^2*x^6+3*sum((_R1^2*b*c^2-_R1*a*d^3-
2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*
c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*c*x^6+sum((_R1^2-2*_R1*c+c^2+6*_R1-
6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c^3+2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_
R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^
3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,
d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*x^3+su
m((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_
R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c^3*x^6+sum
((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R
1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c^3*x^6-3*su
m((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*
b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*c^2*x^6+3*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R
1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*
b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*c*x^6+sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+
10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c^3)/a^2/b^2/(b^2*x^6+2*a*b*x^3+a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2962 vs. $2(582) = 1164$.

Time = 0.33 (sec) , antiderivative size = 2962, normalized size of antiderivative = 3.82

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*(
(b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*
cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*
b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
+ 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +
 a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*
b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 +
2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x +
c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(
1/3)*(sqrt(-3) + 1) - c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cos
h(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2
+ (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 +
 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x +
 c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*co
sh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(-d*x +
 (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + ((a*b^2*d^3*x^6 + 2*a^2*b*d
^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*c
osh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(d*x +
 (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^
3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*
Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)
^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/
b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3
)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*b^2*d
^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^
2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 -
```

```
 (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))
*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2
*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(-d*x +
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1) + c) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(1
/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/
b)^(1/3)) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(
1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b
)^(1/3)) - 6*(a*b^2*d*x^4 - 2*a^2*b*d*x)*cosh(d*x + c) - 6*(a*b^2*d^2*x^5 +
 a^2*b*d^2*x^2)*sinh(d*x + c))/((a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*
d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*d)*sinh(d*x
 + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^3} \, dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + 6*d*x^2*e^(2*c) + 42*x*e^(2*c))*e^(d*x) - (d^2*x^3
- 6*d*x^2 + 42*x)*e^(-d*x))/(b^3*d^3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*
b*d^3*x^3*e^c + a^3*d^3*e^c) + 1/2*integrate(-3*(3*a*d^2*x^2*e^c - 112*b*x^
3*e^c + 18*a*d*x*e^c + 14*a*e^c)*e^(d*x)/(b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 +
6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3), x) - 1/2*integrate(-3*(3*a*
```

```
d^2*x^2 - 112*b*x^3 - 18*a*d*x + 14*a)*e^(-d*x)/(b^4*d^3*x^12*e^c + 4*a*b^3
*d^3*x^9*e^c + 6*a^2*b^2*d^3*x^6*e^c + 4*a^3*b*d^3*x^3*e^c + a^4*d^3*e^c),
x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^3} \, dx$$

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^3+a)^3} \, dx$$

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.110 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 781

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}} - \frac{\sqrt[3]{-1}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}} + \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{4/3}b^{5/3}} + \frac{d\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} - \frac{\sqrt[3]{-1}d\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3}d\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} + \frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} - \frac{(-1)^{2/3}d^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}} + \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} + \frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{4/3}b^{5/3}} + \frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}} - \frac{\sqrt[3]{-1}d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{4/3}b^{5/3}}$$

```
[Out] 1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/
3)+1/54*(-1)^(2/3)*d^2*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(1/3)*d^2*Chi(-(-1)^(2/3)*
a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)
-1/6*cosh(d*x+c)/b/(b*x^3+a)^2-1/27*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/27*d*co
sh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/27*(-1
)^(2/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(
1/3)+d*x)/a^(5/3)/b^(4/3)+1/27*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*
d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/
3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/27*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*(-1)^(
2/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d
/b^(1/3))/a^(4/3)/b^(5/3)+1/27*(-1)^(2/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*(-1)^(1/3
)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(
1/3))/a^(4/3)/b^(5/3)+1/18*d*sinh(d*x+c)/a/b^2/x^2-1/18*d*sinh(d*x+c)/b^2/x
^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{5397, 5386, 5400, 3378, 3384, 3379, 3382, 5388, 5401\}$

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{(-1)^{2/3} d^2 \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}} - \frac{\sqrt[3]{-1}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} + \frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} + \frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} - \frac{\sqrt[3]{-1}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} - \frac{(-1)^{2/3}d^2\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}} + \frac{d^2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} - \frac{\sqrt[3]{-1}d^2\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} + \frac{\sqrt[3]{-1}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} + \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2(a+bx^3)} - \frac{\cosh(c+dx)}{6b(a+bx^3)^2}$$

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/6*Cosh[c + d*x]/(b*(a + b*x^3)^2) + ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(5
4*a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(4/3)*b^(5/
3)) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x])/(54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Si
nh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshInteg
ral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)])/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5
/3)*b^(4/3)) + (d*Sinh[c + d*x])/(18*a*b^2*x^2) - (d*Sinh[c + d*x])/(18*b^2
*x^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*
SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) -
((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/
3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)
) + (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d
*x])/(54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b
^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(
4/3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1))), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sinh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\int \frac{\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d\int \frac{\sinh(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2} + \frac{d^2\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{d\int\left(\frac{\sinh(c+dx)}{ax^3} - \frac{b\sinh(c+dx)}{a(a+bx^3)}\right)dx}{9b^2} + \frac{d^2\int\left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a(a+bx^3)}\right)dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{d\int\frac{\sinh(c+dx)}{x^3}dx}{9ab^2} + \frac{d\int\frac{\sinh(c+dx)}{a+bx^3}dx}{9ab} + \frac{d^2\int\frac{\cosh(c+dx)}{x^2}dx}{18ab^2} - \frac{d^2\int\frac{x\cosh(c+dx)}{a+bx^3}dx}{18ab}$$

$$= -\frac{d^2\cosh(c+dx)}{18ab^2x} - \frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)} + \frac{d\int\left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9ab} - \frac{d^2\int\frac{\cosh(c+dx)}{x^2}dx}{18ab^2} - \frac{d^2\int\left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18ab} + \frac{d^3\int\frac{\sinh(c+dx)}{x}dx}{18ab^2}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}dx}{27a^{5/3}b} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}dx}{27a^{5/3}b} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}dx}{27a^{5/3}b} + \frac{d^2\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{54a^{4/3}b^{4/3}} - \frac{\left(\sqrt[3]{-1}d^2\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}dx}{54a^{4/3}b^{4/3}} + \frac{\left((-1)^{2/3}d^2\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}dx}{54a^{4/3}b^{4/3}} - \frac{d^3\int\frac{\sinh(c+dx)}{x}dx}{18ab^2} + \frac{\left(d^3\cosh(c)\right)\int\frac{\sinh(dx)}{x}dx}{18ab^2} + \frac{\left(d^3\sinh(c)\right)\int\frac{\cosh(dx)}{x}dx}{18ab^2}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.54

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx =$$

$$-\frac{d\text{RootSum}\left[a+b\#1^3\&,\frac{2\cosh(c+d\#1)\text{Chi}(d(x-\#1))-2\text{Chi}(d(x-\#1))\sinh(c+d\#1)-2\cosh(c+d\#1)\text{Shi}(d(x-\#1))+2\sin}{}\right.}{}$$

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(d*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
 - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegra
l[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#
1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(
x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*
(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1
+ d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*Cosh[d*x]*(
-3*a*Cosh[c] + d*x*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 - (6*b*(d*x*(a + b*x
^3)*Cosh[c] - 3*a*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(a*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 2230, normalized size of antiderivative = 2.86

method	result	size
risch	Expression too large to display	2230

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*(sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*E
i(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c
^2*x^6+sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*E
i(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c^
2*x^6-2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+
4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*c*x^6-2*sum((_R1^2*b*c-2*_R1*b*
```

c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))*b^2*c*x^6+sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_
R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2
)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3))*b^2*x^6+2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*e
xp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))*a*b^2*c^2*x^3+3*exp(-d*x-c)*a*b^2*d*x^4+sum((_R1^2*b*c^2-_R1*a*d^3-2*_R
1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*
b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*x^6+2*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+
10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^2*x^3-3*exp(d*x+c)*a*b^2*d*x^4-4*sum((
_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c
)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*
c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*c*x^3-4*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c
^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)
*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*
c*x^3+2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10
*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*
Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*
x^3+sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,
-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c^2
+3*exp(-d*x-c)*a^2*b*d*x+2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b
*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))*a*b*x^3+sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+
c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))*a^2*b*c^2-3*exp(d*x+c)*a^2*b*d*x-2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d
^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp
(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)*a^2*c+9*exp(-d*x-c)*a^2*b-2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^
2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x
-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*c+9*exp(d
*x+c)*a^2*b+sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*
c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_
R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*
a^2+sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1
*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(
1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2)/a^2
/b^2/(b^2*x^6+2*a*b*x^3+a^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(559) = 1118$.

Time = 0.31 (sec) , antiderivative size = 2972, normalized size of antiderivative = 3.81

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(36*a^2*cosh(d*x + c) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*
d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*
a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 +
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d
^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) +
 ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x
^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6
 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x
^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^
2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-
3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^
2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*co
sh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6
 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x
+ c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 -
 (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*
cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(a*d^3
/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(
1/3)) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(
```

```
b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6
+ 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
 (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*
x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
 sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d
^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a
^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2
*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c
)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*si
nh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((
-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^
6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2
))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)
*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cos
h(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^
(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
 a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/
3)) + 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^
2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2
*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)
^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(a*b*d*x^4 +
 a^2*d*x)*sinh(d*x + c))/((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*cosh(d*x +
c)^2 - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 7*x*e^(2*c))*e^(d*x) - (d*x^2 - 7*x)*e^(-d*x))/(b^3*d
^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*e^c) + 1/2
*integrate((56*b*x^3*e^c - 9*a*d*x*e^c - 7*a*e^c)*e^(d*x)/(b^4*d^2*x^12 + 4
*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 1/2*i
ntegrate((56*b*x^3 + 9*a*d*x - 7*a)*e^(-d*x)/(b^4*d^2*x^12*e^c + 4*a*b^3*d^
2*x^9*e^c + 6*a^2*b^2*d^2*x^6*e^c + 4*a^3*b*d^2*x^3*e^c + a^4*d^2*e^c), x)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.111 $\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1147

$$
\begin{aligned}
\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = &-\frac{\cosh(c+dx)}{18ab^2x^4}+\frac{2\cosh(c+dx)}{9a^2bx}-\frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2}+\frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} \\
&-\frac{2(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{7/3}b^{2/3}} \\
&+\frac{\sqrt[3]{-1}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{5/3}b^{4/3}} \\
&+\frac{2\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{7/3}b^{2/3}} \\
&-\frac{(-1)^{2/3}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{5/3}b^{4/3}} \\
&-\frac{2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{7/3}b^{2/3}} \\
&-\frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{5/3}b^{4/3}} \\
&-\frac{2d\,\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^2b} \\
&-\frac{2d\,\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^2b} \\
&-\frac{2d\,\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^2b} \\
&+\frac{d\sinh(c+dx)}{18ab^2x^3}-\frac{d\sinh(c+dx)}{18b^2x^3\,(a+bx^3)} \\
&+\frac{2d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^2b} \\
&+\frac{2(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{7/3}b^{2/3}} \\
&-\frac{\sqrt[3]{-1}d^2\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{5/3}b^{4/3}} \\
&-\frac{2d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^2b} \\
&\quad 2\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)
\end{aligned}
$$

```
[Out] -2/27*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-
1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/
3)-2/27*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*
a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/54*(-1)^(1/3)*d^2*Chi((-1)^(1/3)*a^(1/
3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27
*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3
)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*(-1)^(2/3)*d^2*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/18*cosh
(d*x+c)/a/b^2/x^4+2/9*cosh(d*x+c)/a^2/b/x-1/6*cosh(d*x+c)/b/x/(b*x^3+a)^2+1
/18*cosh(d*x+c)/b^2/x^4/(b*x^3+a)-2/27*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*cosh(c-a^(1/3)*d/b^(
1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*Chi(a^(1/3)*d/b^
(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*Shi(a^(1/3)*d/b^(1/3)+d*x)*
sinh(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-2/27*d*Chi((-1)^(1/3)*a^(1/3)*
d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b-2/27*(-1)^(2/3)*S
hi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/
a^(7/3)/b^(2/3)+1/54*(-1)^(1/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*
sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-2/27*d*Chi(-(-1)^(2/3)
*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b+2/27*(-1
)^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))/a^(7/3)/b^(2/3)-1/54*(-1)^(2/3)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3
)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/18*d*sinh(d*x
+c)/a/b^2/x^3-1/18*d*sinh(d*x+c)/b^2/x^3/(b*x^3+a)
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.00, number of steps used = 89, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used

$= \{5399, 5401, 3378, 3384, 3379, 3382, 5400, 5398, 5389\}$

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^{5/3}b^{4/3}}$$
$$-\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}}$$
$$-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}}$$
$$-\frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d^2}{54a^{5/3}b^{4/3}}$$
$$-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}}$$
$$-\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d^2}{54a^{5/3}b^{4/3}}$$
$$-\frac{2\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b}$$
$$-\frac{2\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b}$$
$$-\frac{2\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b}$$
$$-\frac{\sinh(c+dx)d}{18b^2x^3\left(bx^3+a\right)}+\frac{\sinh(c+dx)d}{18ab^2x^3}$$
$$+\frac{2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)d}{27a^2b}$$
$$-\frac{2\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b}$$
$$-\frac{2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)d}{27a^2b}$$
$$+\frac{2\cosh(c+dx)}{9a^2bx}+\frac{\cosh(c+dx)}{18b^2x^4\left(bx^3+a\right)}-\frac{\cosh(c+dx)}{6bx\left(bx^3+a\right)^2}-\frac{\cosh(c+dx)}{18ab^2x^4}$$
$$-\frac{2(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{7/3}b^{2/3}}$$
$$+\frac{2\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}}$$

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/18*Cosh[c + d*x]/(a*b^2*x^4) + (2*Cosh[c + d*x])/(9*a^2*b*x) - Cosh[c +
d*x]/(6*b*x*(a + b*x^3)^2) + Cosh[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-
1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(1/3)*d^2*Cosh[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(54*a^(5/3)*b^(4/3)) + (2*(-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(27*a^(7
/3)*b^(2/3)) - ((-1)^(2/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Cos
hIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(5/3)*b^(4/3)) -
(2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
27*a^(7/3)*b^(2/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^2*b) - (2*d*CoshIntegr
al[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^
(1/3)])/(27*a^2*b) - (2*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^2*b) + (d*Sinh[c + d*x
])/(18*a*b^2*x^3) - (d*Sinh[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*Cosh[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3) - d*x])/(27*a^2*b) + (2*(-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/
3)) - ((-1)^(1/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c
- (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b)
- (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])
/(27*a^(7/3)*b^(2/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^
(1/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^2*b) + (2*(-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) - ((-1)^(2/3)
*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\text{integral} = -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} - \frac{\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx}{6b} + \frac{d\int \frac{\sinh(c+dx)}{x\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2\int \frac{\cosh(c+dx)}{x^5(a+bx^3)}\,dx}{9b^2} - \frac{d\int \frac{\sinh(c+dx)}{x^4(a+bx^3)}\,dx}{18b^2} - \frac{d\int \frac{\sinh(c+dx)}{x^4(a+bx^3)}\,dx}{6b^2} + \frac{d^2\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2\int \left(\frac{\cosh(c+dx)}{ax^5} - \frac{b\cosh(c+dx)}{a^2x^2} + \frac{b^2x\cosh(c+dx)}{a^2(a+bx^3)}\right)dx}{9b^2} - \frac{d\int \left(\frac{\sinh(c+dx)}{ax^4} - \frac{b\sinh(c+dx)}{a^2x} + \frac{b^2x^2\sinh(c+dx)}{a^2(a+bx^3)}\right)dx}{18b^2} - \frac{d\int \left(\frac{\sinh(c+dx)}{ax^4} - \frac{b\sinh(c+dx)}{a^2x} + \frac{b^2x^2\sinh(c+dx)}{a^2(a+bx^3)}\right)dx}{6b^2} + \frac{d^2\int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a(a+bx^3)}\right)dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{2\int \frac{\cosh(c+dx)}{x^5}\,dx}{9ab^2} - \frac{2\int \frac{\cosh(c+dx)}{x^2}\,dx}{9a^2b} - \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^3}\,dx}{18a^2} - \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^3}\,dx}{6a^2} - \frac{d\int \frac{\sinh(c+dx)}{x^4}\,dx}{18ab^2} - \frac{d\int \frac{\sinh(c+dx)}{x^4}\,dx}{6ab^2} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{18a^2b} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{6a^2b} + \frac{d^2\int \frac{\cosh(c+dx)}{x^3}\,dx}{18ab^2} - \frac{d^2\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{18ab}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} - \frac{d^2\cosh(c+dx)}{36ab^2x^2} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2}$$
$$+ \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2d\sinh(c+dx)}{27ab^2x^3} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)}$$
$$+ \frac{2\int\left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{9a^2}$$
$$- \frac{d\int\left(\frac{\sinh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{18a^2}$$
$$- \frac{d\int\left(\frac{\sinh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sinh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{6a^2}$$
$$+ \frac{d\int\frac{\sinh(c+dx)}{x^4}dx}{18ab^2} - \frac{(2d)\int\frac{\sinh(c+dx)}{x}dx}{9a^2b} - \frac{d^2\int\frac{\cosh(c+dx)}{x^3}dx}{54ab^2} - \frac{d^2\int\frac{\cosh(c+dx)}{x^3}dx}{18ab^2}$$
$$- \frac{d^2\int\left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{18ab}$$
$$+ \frac{d^3\int\frac{\sinh(c+dx)}{x^2}dx}{36ab^2} + \frac{(d\cosh(c))\int\frac{\sinh(dx)}{x}dx}{18a^2b} + \frac{(d\cosh(c))\int\frac{\sinh(dx)}{x}dx}{6a^2b}$$
$$+ \frac{(d\sinh(c))\int\frac{\cosh(dx)}{x}dx}{18a^2b} + \frac{(d\sinh(c))\int\frac{\cosh(dx)}{x}dx}{6a^2b}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{d^2\cosh(c+dx)}{108ab^2x^2} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2}$$
$$+ \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2d\text{Chi}(dx)\sinh(c)}{9a^2b} + \frac{d\sinh(c+dx)}{18ab^2x^3} - \frac{d^3\sinh(c+dx)}{36ab^2x}$$
$$- \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2d\cosh(c)\text{Shi}(dx)}{9a^2b} - \frac{2\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}}$$
$$+ \frac{\left(2\sqrt[3]{-1}\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}} - \frac{\left(2(-1)^{2/3}\right)\int\frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{7/3}\sqrt[3]{b}}$$
$$- \frac{d\int\frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54a^2b^{2/3}} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54a^2b^{2/3}} - \frac{d\int\frac{\sinh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{54a^2b^{2/3}}$$
$$- \frac{d\int\frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{18a^2b^{2/3}} - \frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{18a^2b^{2/3}} - \frac{d\int\frac{\sinh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{18a^2b^{2/3}}$$
$$+ \frac{d^2\int\frac{\cosh(c+dx)}{x^3}\,dx}{54ab^2} + \frac{d^2\int\frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{54a^{5/3}b} + \frac{d^2\int\frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{54a^{5/3}b}$$
$$+ \frac{d^2\int\frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{54a^{5/3}b} - \frac{d^3\int\frac{\sinh(c+dx)}{x^2}\,dx}{108ab^2} - \frac{d^3\int\frac{\sinh(c+dx)}{x^2}\,dx}{36ab^2}$$
$$+ \frac{d^4\int\frac{\cosh(c+dx)}{x}\,dx}{36ab^2} - \frac{(2d\cosh(c))\int\frac{\sinh(dx)}{x}\,dx}{9a^2b} - \frac{(2d\sinh(c))\int\frac{\cosh(dx)}{x}\,dx}{9a^2b}$$

= Too large to display

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.58

$$\int\frac{x\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx$$
$$= \frac{\text{RootSum}\left[a+b\#1^3\&, \frac{-ad^2\cosh(c+d\#1)\text{Chi}(d(x-\#1))+ad^2\text{Chi}(d(x-\#1))\sinh(c+d\#1)+ad^2\cosh(c+d\#1)\text{Shi}(d(x-\#1))}{}\right.}{}$$

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) +
 a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 4*b*C
osh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)]*Si
nh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshIntegral[
```

```
d*(x - #1)]*#1^2 - 4*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 4*b
*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (a*d^2*Cosh[c
+ d*#1]*CoshIntegral[d*(x - #1)] + a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c +
d*#1] + a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + a*d^2*Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)] - 4*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1
 - 4*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1 - 4*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
4*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 4*b*d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1^2 + 4*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
]*#1^2 + 4*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*
b*Cosh[d*x]*(b*x^2*(7*a + 4*b*x^3)*Cosh[c] + a*d*(a + b*x^3)*Sinh[c]))/(a +
 b*x^3)^2 + (6*b*(a*d*(a + b*x^3)*Cosh[c] + b*x^2*(7*a + 4*b*x^3)*Sinh[c])*
Sinh[d*x])/(a + b*x^3)^2)/(108*a^2*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 1416, normalized size of antiderivative = 1.23

method	result	size
risch	Expression too large to display	1416

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*(-sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*
Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*
c*d*x^6-sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*
Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3*c
*d*x^6+sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4
*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*d*x^6+sum((_R1^2*b*c-2*_R1*b*c^2
-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))*b^2*d*x^6-2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*
exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c
^3))*a*b^2*c*d*x^3-2*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^
2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))*a*b^2*c*d*x^3-12*exp(-d*x-c)*b^3*x^5-12*exp(d*x+c)*b^3*x^5+3*exp(-d
*x-c)*a*b^2*d*x^3-3*exp(d*x+c)*a*b^2*d*x^3+2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d
^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp
(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)*a*b*d*x^3+2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*
b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*d*x^3-sum((_R1^2-2*_R1*c+
```

```
c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c*d-sum((_R1^2-2*_R1*c+c^2
+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*b*c*d-21*exp(-d*x-c)*a*b^2*x^2-
21*exp(d*x+c)*a*b^2*x^2+3*exp(-d*x-c)*a^2*b*d-3*exp(d*x+c)*a^2*b*d+sum((_R1
^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(
_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3
*_Z*b*c^2+a*d^3-b*c^3))*a^2*d+sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^
2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x
-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*d)/a^2/b^
2/(b^2*x^6+2*a*b*x^3+a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4691 vs. $2(843) = 1686$.

Time = 0.33 (sec) , antiderivative size = 4691, normalized size of antiderivative = 4.09

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 8*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*(a*d^3/b)^(
2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^
2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6
+ 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqr
t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*Ei(d*x
 - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 4*(-a*d^3/
b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((a*b^2*d^3
*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3
+ sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*E
i(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sq
rt(-3) + 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x
+ c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*
(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b
^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3
```

)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*
d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^
3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)
^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d
x + c)^2 - 8(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 +
 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2
*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x
 + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) - 1) + c) - 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*(
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^
6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3)
)*cosh(c + (-a*d^3/b)^(1/3)) + 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d
x + c)^2 + 4(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((
a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*c
osh(-c + (a*d^3/b)^(1/3)) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x +
c)^2 - 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^
6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
- sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1
/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 +
 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh
(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x
 + c)^2 + 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^
3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/
b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*
x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*
b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))
*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3

```
+ a^3*d^3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
sinh(d*x + c)^2 - 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (
a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^
2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 +
2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + s
qrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b
^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2)
 - (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x
^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(4*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3)*sinh(d*x + c)^2 - 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
+ (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x +
c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*
x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(4*(a*b^2*d^3*x^6 + 2*
a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*sinh(d*x + c)^2 + 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
+ (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c
)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x
+ (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(4*a*b^2*d^2*x^5 + 7*a^2
*b*d^2*x^2)*cosh(d*x + c) - 12*(a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c))/((a
^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*cosh(d*x + c)^2 - (a^3*b^3*
d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx^3+a\right)^3}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a
^2*b*d*x^3*e^c + a^3*d*e^c) + 1/2*integrate((8*b*x^3*e^c - a*e^c)*e^(d*x)/(
b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x) -
 1/2*integrate((8*b*x^3 - a)*e^(-d*x)/(b^4*d*x^12*e^c + 4*a*b^3*d*x^9*e^c +
 6*a^2*b^2*d*x^6*e^c + 4*a^3*b*d*x^3*e^c + a^4*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx^3+a\right)^3}\,dx$$

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\cosh(c+d\,x)}{\left(b\,x^3+a\right)^3}\,dx$$

```
[In] int((x*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x^3)^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```